University of Stavanger

Faculty of Science and Technology

Date: 31st August 2017.

Exam in: PET 540, Natural Gas Reservoir and Production Engineering.
Duration: 4 hours.
Supporting materials: Use of simple calculator is permitted.
Content: 4 exercises on 4 pages.
Annotation: Answer all questions.
Course Responsible: Yen Adams Sokama-Neuyam.

Exercise 1

1. On a P-T diagram, show that the state of equilibrium of a two-phase system will fall on a line.
2. Use a diagram to explain the effect of retrograde condensation on well productivity.
3. Use a P-T diagram to explain the difference between a wet gas and a gas condensate.

Isothermal gas compressibility, c is defined as:

$$c = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

Where V is the volume, p is pressure and T is constant absolute temperature.

4. Show that the compressibility of a real gas, c_{real} and an ideal gas, c_{ideal} can be derived respectively as:

$$c_{real} = \frac{1}{p} - \frac{1}{Z}\left(\frac{\partial Z}{\partial p}\right)_T$$

$$c_{ideal} = \frac{1}{p}$$

5. By approximating the derivative $\left(\frac{\partial Z}{\partial p}\right)$ with a central difference, calculate the gas compressibility at 400 bars given the following data.

P(bar)	Z
425	1.101
400	1.062
295	0.95

6. Calculate the ideal gas compressibility at 400 bars and evaluate the relative deviation from the real gas compressibility.

Exercise 2

The Laplace equation is given by:

$$\nabla . \nabla m = 0$$

Where m is the pseudo-pressure.

1. State the significance of the Laplace equation and outline its underlying assumptions.

Given the Kirchhoff's transformation:

$$m(p) = \frac{1}{(\rho/\mu)_r} \int_{p_r}^{p} \frac{\rho}{\mu} dp$$

Where ρ is the gas density, μ is the viscosity and p_r is a reference pressure.

2. Show that the pseudo-pressure for an ideal gas, $m(p)$ is given by:

$$m(p) = \frac{1}{2}\left(\frac{p^2}{p_r} - p_r\right)$$

The wet gas formation volume factor, B_g is given by:

$$B_g = \frac{\rho_{sc}}{M_G}\frac{M}{\rho}(1 + R_{MLG})$$

Where ρ_{sc} and ρ are the gas density at standard conditions and reservoir conditions, M and M_G are the average molecular mass of the gas at reservoir and standard conditions and R_{MLG} is the molar condensate-gas-ratio.

3. If $R_{MLG} = R_{MLGi}$, show that the Kirchhoff's transformation can also be written as:

$$m(p) = \left(\mu\beta_g\right)_r \int_{p_r}^{p} \frac{1}{\left(\mu\beta_g\right)} dp$$

Where R_{MLGi} is the initial molar condensate-gas-ratio.

For radial flow, it can be shown that:

$$\frac{dm}{dr} = \frac{1}{r}\frac{m_e - m_w}{\ln\left(\frac{r_e}{r_w}\right)}$$

$$\frac{dp}{dr} = \frac{(\mu\beta)}{(\mu\beta)_r}\frac{dm}{dr}$$

Where m_e and m_w are the pseudo pressures evaluated at the reservoir boundary, r_e and the well, r_w respectively.

4. Use Darcy's law to show that the steady-state radial flow equation can be derived as:

$$m_e - m_w = \frac{q_{sc}(\mu\beta)_r}{2\pi kh}\ln\left(\frac{r_e}{r_w}\right)$$

Where kh is the permeability-thickness ratio.

Exercise 3

For constant diffusivity coefficient, D_Q, the radial heat flow in a well can be expressed in cylindrical coordinates as:

$$\frac{1}{D_Q}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Where $\frac{\partial T}{\partial t}$ is the rate of change of temperature and $\frac{\partial T}{\partial r}$ is the temperature gradient. The equation is solved by splitting the solution into the uploading period, the transient period and the steady-state period.

1. Explain the behaviour of the equation during the uploading and the steady-state periods.
2. Show that the general solution of the steady-state radial heat flow can be derived as:

$$T = a\ln(r) + b$$

Where a and b are integration constants.

Given Fourier's law:

$$J_Q = -\kappa_T\frac{\partial T}{\partial r}$$

Where J_Q is the specific heat flow and κ_T is the thermal conductivity of the material.

3. Show that, the integration constant, a, can be expressed as:

$$a = -\frac{r}{\kappa_T}U_Q(T - T_s)$$

Where $J_Q = U_Q(T - T_s)$, $(T - T_s)$ is the difference between the well temperature and the reservoir temperature and U_Q is the heat transfer coefficient.

4. With appropriate well and reservoir boundary conditions, show that the temperature difference between the well and the formation is given by:

$$T - T_s = a\ln\left(\frac{r}{r_s}\right)$$

Exercise 4

1. Explain mist flow.

The density of a water-gas mixture can be expressed as:

$$\rho_m = \rho_g\left(1 - \frac{q_w}{q_w + q_g} + \frac{\rho_w}{\rho_g}\frac{q_w}{q_w + q_g}\right)$$

Where q_w and q_g are the volumetric flow rate of the aqueous phase and the gas mixture respectively.

2. By making justifiable assumptions, show that ρ_m can also be expressed as

$$\rho_m = \rho_g F_w$$

Where $F_w = 1 + \frac{w_w}{w_g}$ is the water correction factor and w_w and w_g are the water and gas phase mass flow rate respectively.

3. Explain the difference between water influx and water channelling.
4. Construct a uniform inflow performance curve for varying reservoir pressure.

Well deliverability can be expressed as:

$$\bar{p} - p_{wh} = \Delta p_{inflow} + \Delta p_{tubing}$$

Where Δp_{inflow} and Δp_{tubing} are the inflow and tubing performance respectively and $\bar{p}$ and p_{wh} are the average reservoir pressure and the wellhead pressure respectively.

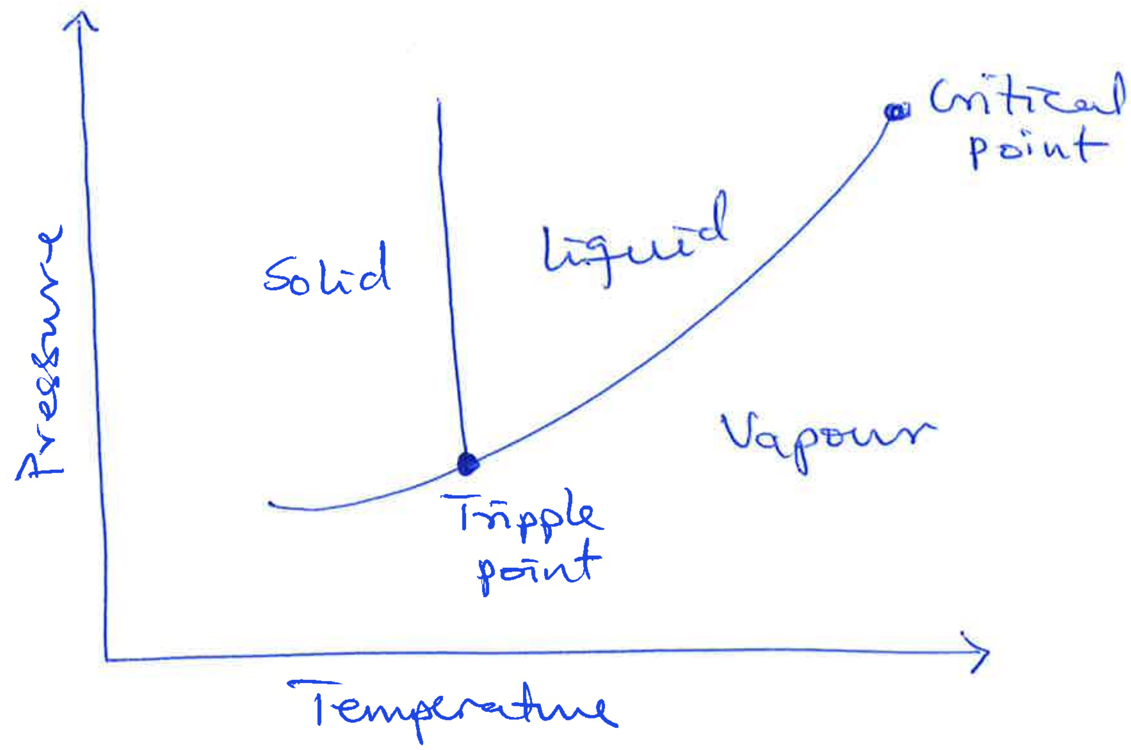
5. Given that $p_{bh} \sim p_{wh} e^{N_{gp}}$, construct a typical well deliverability curve and show the maximum well deliverability.
6. With the aid of a deliverability curve, explain the effect of compression on gas delivery.

Best of Luck – Yen.

Proposed Solutions to Exam Exercises PET 540, August 2017.

Exercise 1

1.



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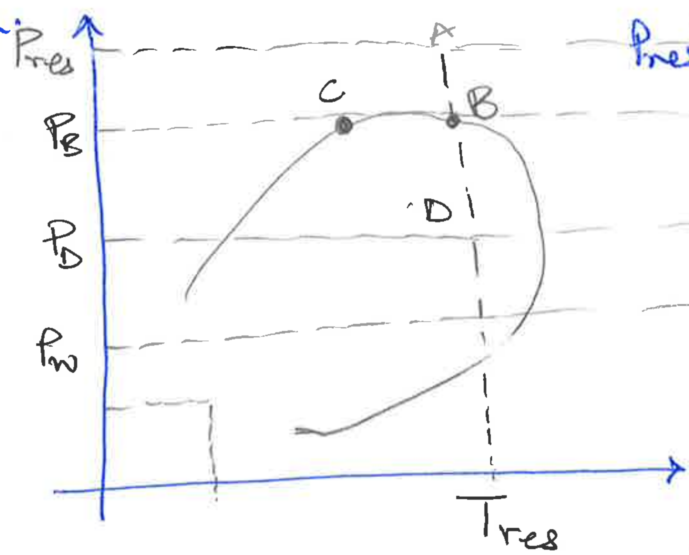
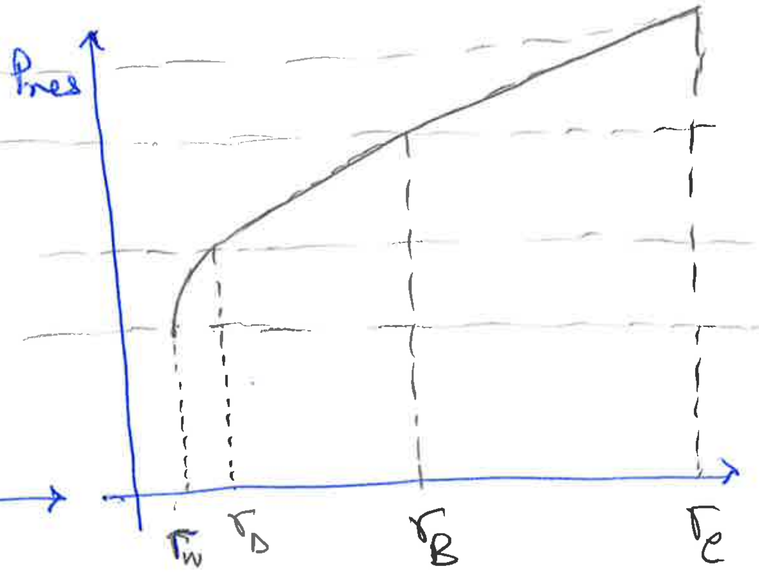
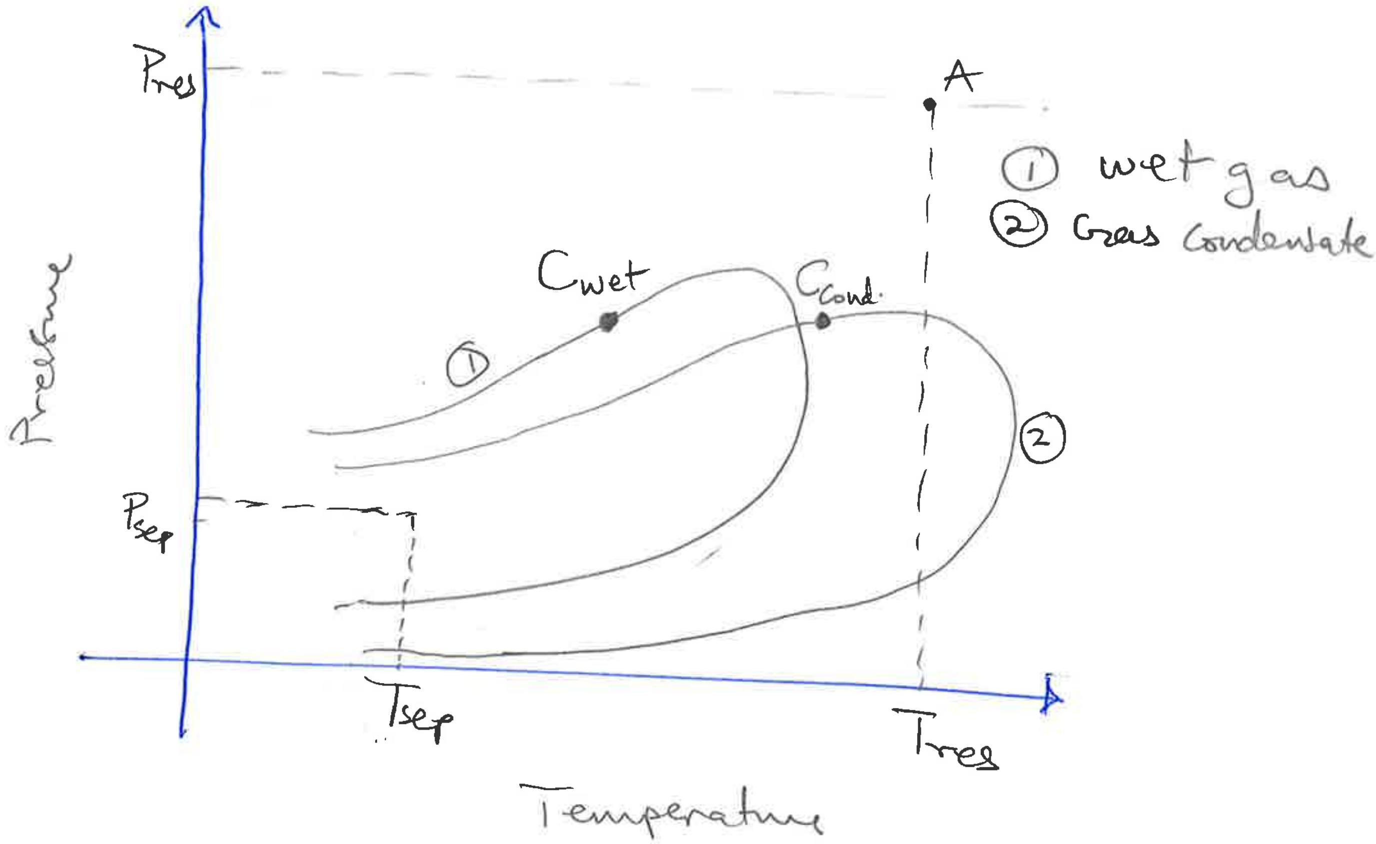




- As the reservoir pressure drops from A to D, liquid condensate drops into the pores in the wellbore vicinity.
- Pressure drop in the wellbore increases significantly which impairs permeability to gas and reduces well productivity.

3.



4. Given that:

$$c = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

The real gas equation:

$$PV = ZnRT$$

Taking differentials:

$$p\,\partial V + V\,\partial p = nRT\,\partial z$$

rearranging and dividing by $\partial p \cdot p$

$$\frac{\partial V}{\partial p} + \frac{V}{p} = \frac{\partial z}{\partial p} \cdot \frac{nRT}{p}$$

But $\frac{nRT}{p} = \frac{V}{z}$

$$\Rightarrow \frac{\partial V}{\partial p} = -\frac{V}{p} + \frac{V}{z}\frac{\partial z}{\partial p}$$

$$C_{real} = -\frac{1}{V} \cdot \left(-\frac{V}{p} + \frac{V}{z}\frac{\partial z}{\partial p}\right)$$

$$\underline{C_{real} = \frac{1}{p} - \frac{1}{z}\left(\frac{\partial z}{\partial p}\right)}$$

For ideal gas: $z = 1$

thus $C_{ideal} = \frac{1}{p} - \frac{1}{(1)}\left(\frac{\partial (1)}{\partial p}\right)$

$$\underline{C_{ideal} = \frac{1}{p}}$$

5. For central difference approximation

$$\left(\frac{\partial z}{\partial p}\right)_{p=p_i} = \frac{Z(P_{i+1}) - Z(P_{i-1})}{P_{i+1} - P_{i-1}}$$

$$C_{400} = \frac{1}{400} - \frac{1}{1.062}\left(\frac{1.101 - 0.95}{425 - 295}\right)$$

$$= \underline{0.001406}$$

6. For ideal gas:

$$C_{400} = \frac{1}{400} = 0.0025$$

Deviation: $\frac{\Delta C}{C} = \frac{0.0025 - 0.001406}{0.001406}$

$$= \underline{\underline{77.81\%}}$$

Exercise 2

1. The Laplace Equation describes incompressible steady-state flow of liquids in porous media.

other assumptions

- negligible gravitational effect
- constant rock permeability.
- Darcy flow conditions.

2. For ideal gas equation, gas density can be derived as

$$\rho = \frac{m_g}{nRT} p$$

The pseudo-pressure given by

$$m(p) = \frac{1}{(\rho/\mu)_r} \int_{p_r}^{p} \frac{\rho}{\mu} dp$$

$$m(p) = \frac{nRT}{m_g} \left(\frac{\mu}{p}\right)_r \int_{p_r}^{p} \frac{m_g}{nRT} \frac{p}{\mu} dp$$

$$m(p) = \left(\frac{\mu}{p}\right)_r \int_{p_r}^{p} \frac{p}{\mu} dp$$

If μ is assumed constant for $p \in [p_r, p]$

⑥

$$m(p) = \mu \cdot \frac{1}{p_r} \cdot \frac{1}{\mu} \int_{p_r}^{p} p \, dp$$

$$= \frac{1}{p_r} \cdot \left. \frac{p^2}{2} \right|_{p_r}^{p} = \frac{1}{p_r} \left(\frac{p^2}{2} - \frac{p_r^2}{2} \right)$$

$$m(p) = \frac{1}{2} \left(\frac{p^2}{p_r} - p_r \right)$$

③ Given that

$$B_g = \frac{\rho_{sc}}{M_G} \cdot \frac{M}{\rho} (1 + R_{MLG})$$

for $R_{MLG} = R_{MLG\,i}$

$$M_G = M (1 + R_{MLG})$$

$$\Rightarrow B_g = \frac{\rho_{sc}}{\rho} \Rightarrow \frac{\rho}{\mu} = \left(\frac{\rho_{sc}}{\mu B_g} \right)$$

Substituting into the Kircchoff's transformation

$$m(p) = \frac{1}{\left(\rho_{sc} / \mu B_g \right)_r} \int_{p_r}^{p} \frac{\rho_{sc}}{\mu B_g} \, dp$$

$$m(p) = \left(\mu B_g \right)_r \int_{p_r}^{p} \frac{1}{(\mu B_g)} \, dp$$

ρ_{sc} is constant with pressure

4 Given

$$\frac{dm}{dr} = \frac{1}{r} \frac{m_e - m_w}{\ln(r_e/r_w)}$$

$$\frac{dp}{dr} = \frac{(\mu B)}{(\mu B)_r} \frac{dm}{dr}$$

Darcy law for steady-state radial flow

$$q_{sc} = \frac{q}{B} = \frac{2\pi r h K}{\mu B} \frac{dp}{dr}$$

substituting for $\frac{dp}{dr}$!

$$q_{sc} = \frac{2\pi r h K}{\mu B} \cdot \frac{\mu B}{(\mu B)_r} \frac{dm}{dr}$$

substituting for dm/dr

$$q_{sc} = \frac{2\pi r h K}{\mu B} \cdot \frac{\mu B}{(\mu B)_r} \cdot \frac{1}{r} \frac{m_e - m_w}{\ln(r_e/r_w)}$$

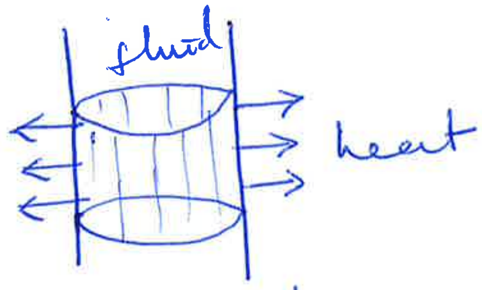
$$\Rightarrow \quad q_{sc} = \frac{2\pi h K}{(\mu B)_r} \cdot \frac{m_e - m_w}{\ln(r_e/r_w)}$$

Rearranging gives

$$m_e - m_w = \frac{q_{sc}(\mu B)_r}{2\pi K h} \ln\left(\frac{r_e}{r_w}\right)$$

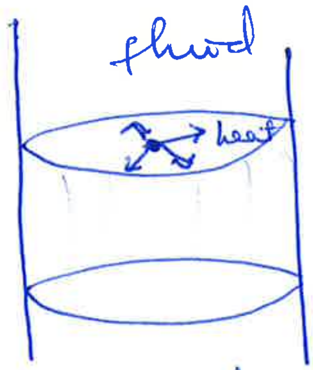
Exercise 3

1) Uploading period



- No temperature gradient yet.
- uploading of heat to the surroundings.

Steady-state period



- temperature gradient is established.
- heat begins to flow within the fluid from the centre of the well.

2) Given the general equation:

$$\frac{1}{D_\phi}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

under steady-state conditions

$$\frac{\partial T}{\partial t} = 0$$

$$\Rightarrow \quad \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

$$r\frac{\partial T}{\partial r} = a \qquad a = \text{constant.}$$

$$\partial T = a\,\frac{\partial r}{r}$$

integrating the equation

$$\int \partial T = a \int \frac{\partial r}{r}$$

$$\underline{T = a \ln(r) + b}$$ b = integration constant.

3 Given

$$J_\varphi = -K_T \frac{\partial T}{\partial r}$$

Differentiating the general equation

$$\frac{\partial T}{\partial r} = \frac{a}{r}$$

Combining the two equations ~~yields~~

$$\frac{\partial T}{\partial r} = -\frac{J_\varphi}{K_T}$$

and $\frac{\partial T}{\partial r} = \frac{a}{r}$ yields

$$a = -r \frac{J_\varphi}{K_T}$$

But $J_\varphi = U_\varphi (T - T_s)$ which gives

$$\underline{a = -\frac{r}{K_T} U_\varphi (T - T_s)}$$

4 Given that

$$T = a \ln(r) + b$$

At reservoir:

$$r = r_s \quad T = T_s$$

$$\Rightarrow \quad T_s = a \ln(r_s) + b$$

At the well:

$$r = r \quad T = T$$

$$T = a \ln(r) + b$$

Substracting the two equations

$$T - T_s = a \ln(r) - a \ln(r_s)$$

$$T - T_s = a \ln\left(\frac{r}{r_s}\right)$$

Exercise 4

(1) <u>Mist flow:</u> small liquids, homogeneously dispersed in the gas which is flowing at a very high rate.

(2) Given that

$$\rho_m = \rho_g \left(1 - \frac{q_w}{q_w + q_g} + \frac{\rho_w}{\rho_g} \frac{q_w}{q_w + q_g} \right)$$

For $q_w << q_g$

$$\left(1 - \frac{q_w}{q_w + q_g} \right) \simeq 1 \quad , \quad \frac{q_w}{q_w + q_g} \simeq \frac{q_w}{q_g}$$

$$\Rightarrow \quad \rho_m = \rho_g \left(1 + \frac{\rho_w q_w}{\rho_g q_g} \right)$$

mass flow rate $w = \rho q$

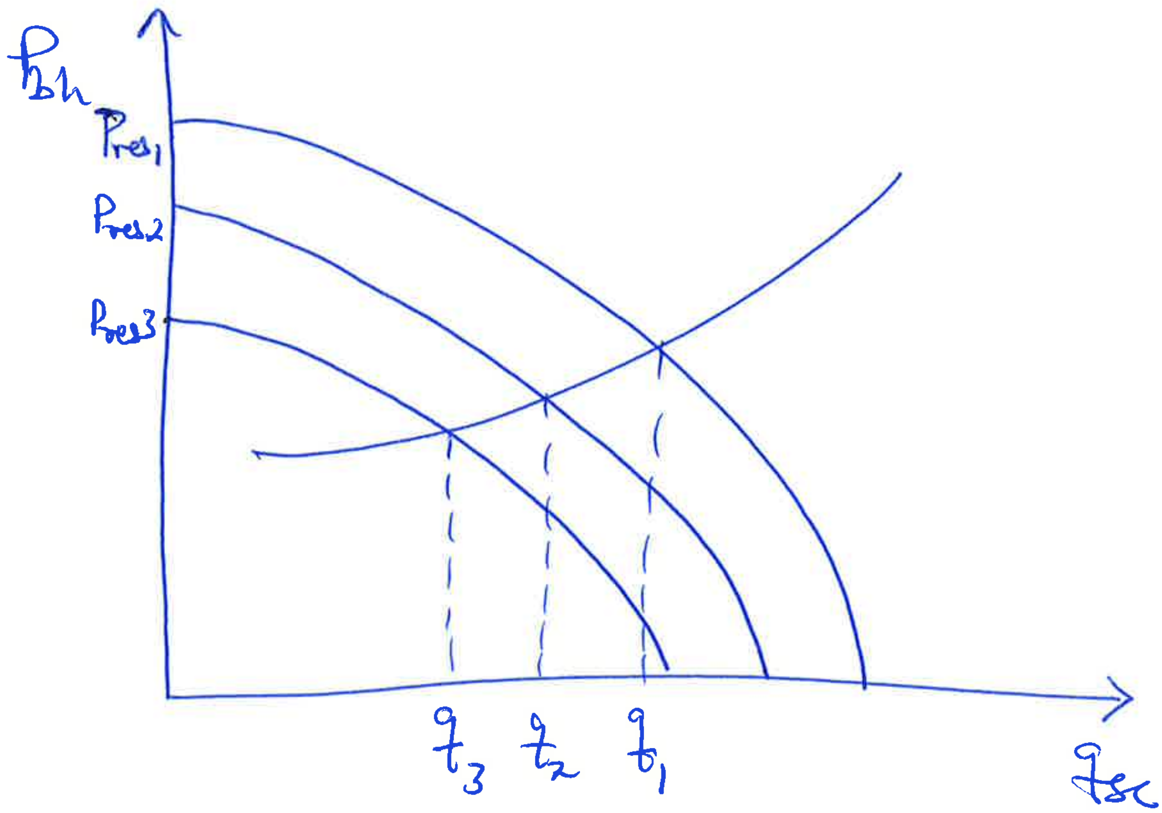
$$\rho_m = \rho_g \left(1 + \frac{w_w}{w_g} \right)$$

$$\Rightarrow \quad \rho_m = \rho_g F_w \quad \text{for} \quad F_w = 1 + \frac{w_w}{w_g}$$

3 **Water influx:** when aquifer water invade the reservoir and pockets of gas are isolated by the surpassing water. Areas containing large volumes of gas will not be produced.

Water Channelling: When water by following reservoir heterogeneities, flows directly towards the well and cause drastic increase in the water cut.

4

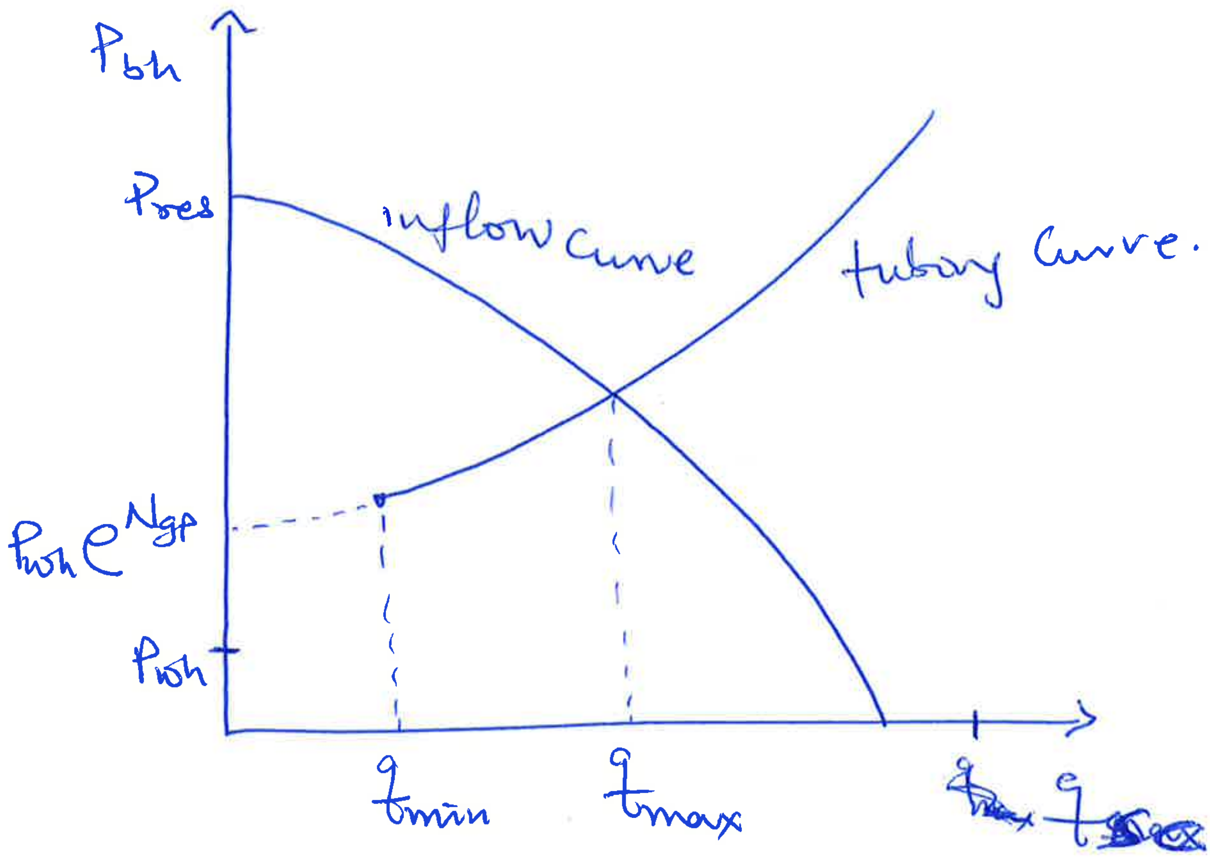


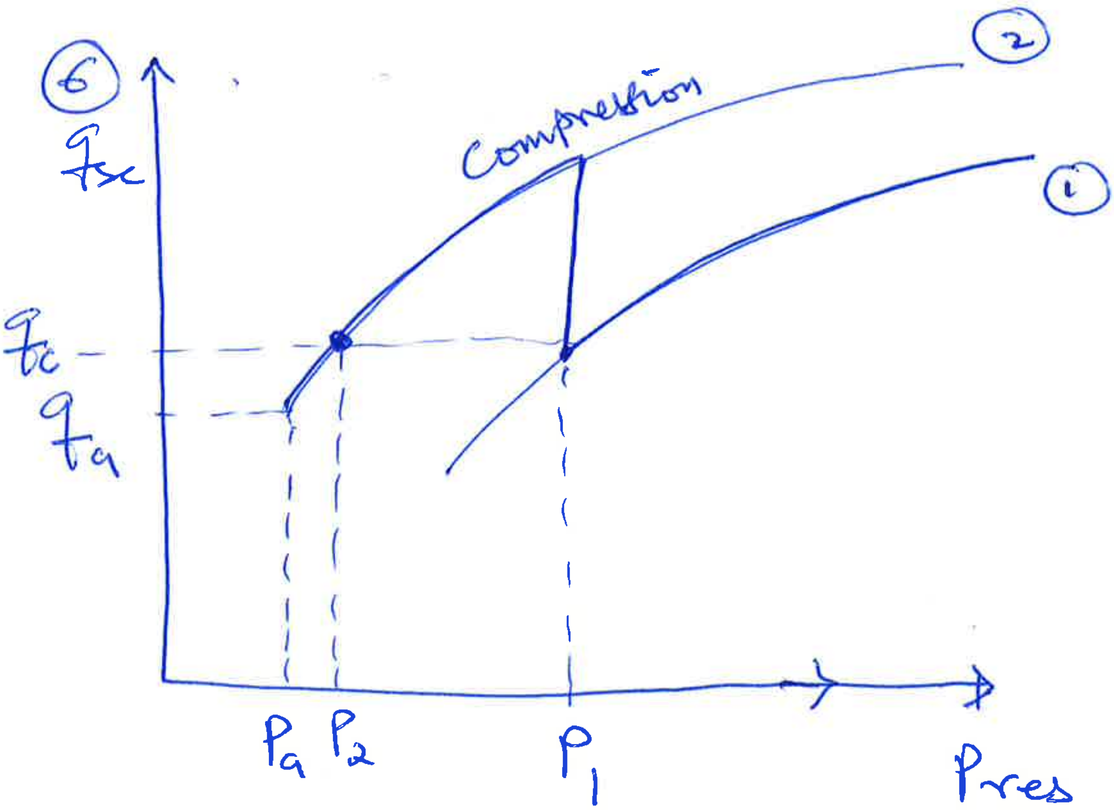
5 Given that

$$\bar{P} - P_{wh} = \Delta P_{inflow} + \Delta P_{tubing}$$

and

$$P_{bh} \approx P_{wh}\, e^{N_{gp}}$$





- Reservoir pressure decreases to P_1 along curve 1, meeting the minimum acceptable flow rate.
- Compression takes production along curve 2, allowing further production down to P_2.