Solution: MPE 340 Reservoir simulation, introduction
DATE: June. 9, 2006

Problem 1

a) See course material.

b) See course material.

c) Mass balance equations

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o}\rho_o x_1 \frac{\partial p_o}{\partial x}\right)+\frac{\partial}{\partial x}\left(\frac{kk_{rg}}{\mu_g}\rho_g y_1 \frac{\partial p_g}{\partial x}\right)+q_1=\frac{\partial}{\partial x}\left[\varphi\left(S_o\rho_o x_1+S_g\rho_g y_1\right)\right]$$

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o}\rho_o x_2 \frac{\partial p_o}{\partial x}\right)+\frac{\partial}{\partial x}\left(\frac{kk_{rg}}{\mu_g}\rho_g y_2 \frac{\partial p_g}{\partial x}\right)+q_2=\frac{\partial}{\partial x}\left[\varphi\left(S_o\rho_o x_2+S_g\rho_g y_2\right)\right]$$

Unknowns: p_o, p_g, S_o, S_g, x_1, x_2, y_1, y_2

Equations
- 2 mass balance equations
- $S_o + S_g = 1$
- $P_{cgo}(S_g) = p_g - p_o$
- $x_1 + x_2 = 1, y_1 + y_2 = 1$
- Equilibrium conditions: $K_1 = y_1 /x_1$, $K_2 = y_2 /x_2$

Solution procedure
- use the last 6 constraint equations to eliminate 6 unknowns
- solve for the remaining 2 unknowns using mass balance equations
- use the constraint equations to update the 6 eliminated unknowns

Problem 2

a) Absolute permeability
- heterogeneous
- anisotropic
- independent of time

Porosity
- heterogeneous
- isotropic
- pressure dependent, $\phi = \phi_0[1+c(p-p_0)]$

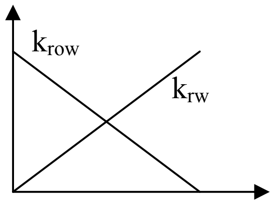
b) Fluid input parameters
- reference densities for gas, oil, water, ρ_{gs}, ρ_{os}, ρ_{ws}
- volume factors for gas, oil, water, B_g, B_o, B_w
- gas in oil solution ratio R_s

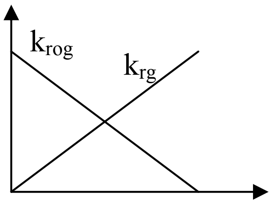
Oil component density: $\rho_{oc} = \rho_{os} V_{o,STC}/ V_{o,RC} = \rho_{os}/ B_o$
Oil phase density: $\rho_{op} = (\rho_{os} V_{o,STC} + \rho_{gs} V_{dg})/ V_{o,RC} = \rho_{oc}+ \rho_{gs} R_s/ B_o$

c) See course material.

Problem 3

a) Input normalized functions:





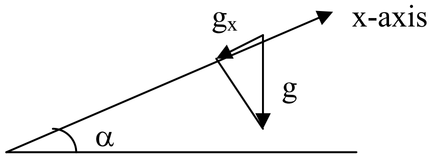
Given saturation end points S_{wr}, S_{gr}, S_{ogr} and *KRG* (max value of k_{rg}), *KRO* (max value of k_{rog})
Given gas saturation S. $k_{rg}(S) = KRG\,(S- S_{gr})/(1- S_{wr} - S_{gr})$, $S\geq S_{gr,}$ and $k_{rg}(S) =0$, $S< S_{gr}$
Given gas saturation S. $k_{rog}(S) = KRO\,(1-[S/(1- S_{wr} – S_{ogr})])$, $S < 1- S_{wr} – S_{ogr}$ and $k_{rg}(S) =0$, $S \geq 1- S_{wr} – S_{ogr}$

b) Given gas saturation S. $k_{rg}(S) = KRG\,(S- S_{gr})/(1- S_{wr} - S_{gr})$, $S\geq S_{gr,}$ and $k_{rg}(S) =0$, $S< S_{gr}$
Given gas saturation S. $k_{rog}(S) = KRO\,(1-[S/(1- S_{wr} – S_{ogr})])$, $S < 1- S_{wr} – S_{ogr}$ and $k_{rog}(S) =0$, $S \geq 1- S_{wr} – S_{ogr}$

Problem 4

a) Assumptions:
- incompressible fluids
- incompressible rock
- zero capillary pressure
- homogeneous reservoir

b)



$$\gamma_{l,x} = \rho_l g_x = \rho_l g \sin\alpha$$

$$u_l = -\frac{kk_{rl}}{\mu_l}\left(\frac{\partial p_l}{\partial x} \pm \rho_l g \sin\alpha\right)$$

The part of the fluid velocity caused by gravity is given by $\frac{kk_{rl}}{\mu_l}\rho_l g \sin\alpha$

If α is positive the gravity g_x acts in the negative x-direction as depicted in the figure, Hence, the + sign must be selected

$$u = -\frac{kk_{rl}}{\mu_l}\left(\frac{\partial p_l}{\partial x} + \rho_l g \sin\alpha\right)$$

c) See course material.

d) Parameters effecting F but not f are

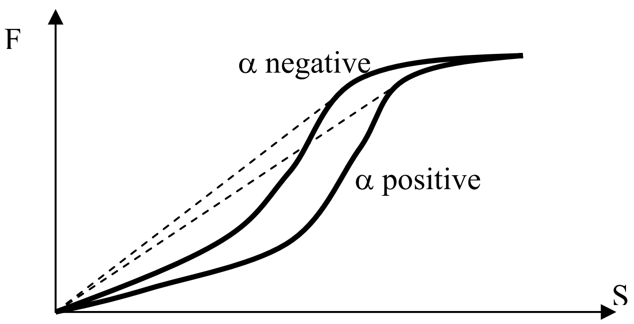
- flow rate
- absolute permeability
- density difference
- dip angle

F is negative if

$Gk_{ro} > 1$.

If max k_{ro} is 1, then F will take negative values if G > 1.

e) Water injection at the lowest part will be most efficient. In this case α will be positive and G will be positive. For the opposite case, G will be negative.



The fractional flow curve for positive G will always be less than the curve for negative G. Hence, the font saturation for G positive is larger than the front saturation for G negative as depicted in the figure. Larger front saturation will result in higher recovery.

Problem 5

See Exercises with solutions on It's learning.