From Gas to Electricity

Course material for PET640

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List of content

1. Introduction

From gas to electricity is dealing with various energy conversion technologies, using gaseous fuels, primarily natural gas (NG), to produce electricity. Heat is a byproduct of this process that can be utilized in different ways to enhance the so called “fuel utilization factor”, which defines the amount of useful energy versus the total heat released during combustion of the fuel.

Combustion can be explained as conversion of the chemically bound energy of the fuel to heat, which also results in release of emissions of different kinds to the atmosphere. Improved efficiency of the energy conversion technologies involved, results in reduced specific emission, i.e. emission per produced unit of electricity and useful heat. Therefore, there will be focus on methods and measures, which results in efficiency improvements for various types of energy conversion technologies presented in this booklet.

Widening the course title “from gas to electricity”, to become more inclusive one can refer to “thermal power engineering”, meaning conversion of heat from any source of fuel to electricity. The generalized setup could be described by the figure below:

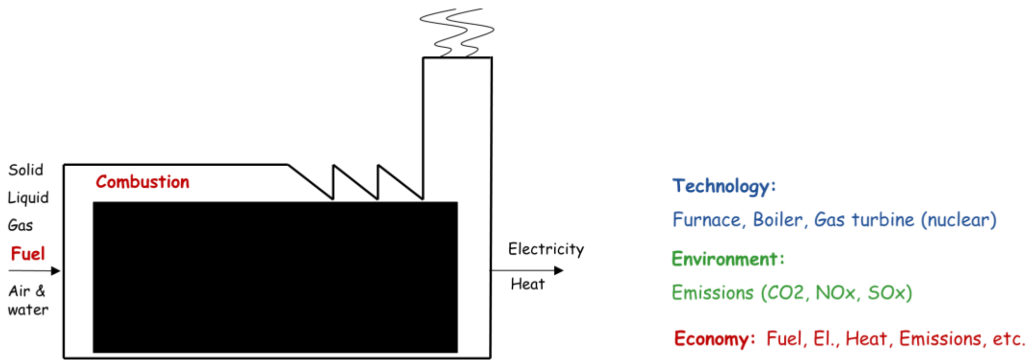


Figure 1: Thermal power plant as general black-box setup

As shown by the figure, any kind of energy conversion technology that converts released heat from combustion to mechanical work, in terms of a rotating shaft, can produce electricity via a generator. Therefore, it is important to take advantage of this opportunity as an engineer, to widen the general understanding to cover thermal power engineering.

The “black box” approach presented here, emphasizes the conversion of heat to electricity independent of the energy conversion technology employed. A major consequence of heat generation via combustion is the release of the unwanted emissions (NOx, Sox and CO2) to the atmosphere. Detailed schemes for combustion of various fuels will be discussed in coming chapters. It can also be mentioned here that nuclear power plants are also thermal power plants, but without combustion. The heat release in nuclear power plants is due to nuclear reactions, which will not be considered in this booklet.

The course material is presented in three chapters, which follow this introduction, starting with steam cycle, followed by gas turbine and finally the combined cycle. The reason for selection of this structure is that in chapter 2, steam cycle, the well-defined working fluid steam/water is used to introduce the basic thermodynamic rules as well as the function of various components of the system. Thereby a base will be established for the coming captures, despite the fact that natural gas is not the most common fuel in steam cycles. Given the fact that exhaust gas from combustion of fuel is not directly used as working medium in the power cycle (steam is used as working medium), one can allow use of more difficult and not so clean fuels in steam cycles. Solid, liquid and gas fuels all can be combusted in a boiler, due to the large volume and ease of material cooling. In the gas turbine, where the exhaust gas is the working medium of the cycle, purity of the fuel is very important to allow use of advanced cooling methods, which results in improved efficiency.

In summary:

- Energy conversion technologies are needed for electricity (and heat) generation from different fuels
- Combustion is the process needed to convert the chemical energy in the fuel to heat, where the emissions are formed (CO_2, NOx, SOx, etc.)
- Electrical efficiency depends on technology level and power range (plant cost)
- Combined heat and power, when possible, results in much higher fuel utilization factor
- Detail plant and component analysis, function, design, mathematical modeling and calculations are the focus of this course.
- You need to keep the big picture in mind for orientation when digging into the details of different components!

2. The steam cycle

Steam cycle can be defined as a closed cycle where water at high pressure is converted to high-pressure steam in a boiler, expanded in a turbine (which drives a generator to produce electricity), condensed to water at low pressure in a condenser and pressurized by a pump to be delivered back to the boiler. The figure below shows the schematic of the simple steam cycle.

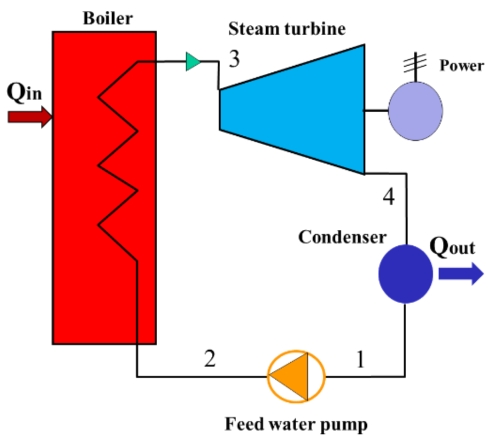


Figure 2: Schematic of a simple steam cycle

To be able to analyze the steam cycle further, one need to learn more about the properties of steam as working fluid. The subchapters below introduce temperature-entropy, enthalpy-entropy diagrams as analysis tool for understanding of the processes that take place in different components of the steam cycle. For definitions and detailed explanation of enthalpy and entropy the readers are referred to standard thermodynamic books.

Heat and mass balance can be applied to the parts of the plant or to the whole plant using the principle of control volume, where the properties of the flows crossing the borders of the control volume need to be in balance according to the following equations:

The mass balance $\sum_i \dot{m}_{in_i} = \sum_i \dot{m}_{out_i}$

The energy balance $\sum_i (\dot{m} \cdot h)_{in_i} = \sum_i (\dot{m} \cdot h)_{out_i}$

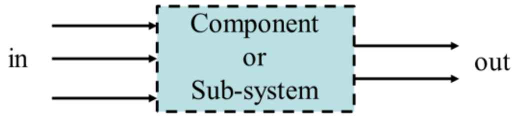


Figure 3: The concept of control volume

This principle can be applied to any thermodynamic system. It is clear from figure 3 that information about mass flows and enthalpies at inlet and outlet of the control volume are needed for calculation of mass and energy balances.

2.1. Steam properties

Steam is a well-defined working medium and the energy related properties of steam are available in standard steam tables, and also presented graphically in temperature vs. entropy diagrams (TS-diagram) and enthalpy vs. entropy diagrams (hS-diagram). The main element of orientation in both tables and diagrams for steam is the saturation line. The state of saturation is defined by:

- saturated liquid, where addition of heat at constant pressure would result in evaporation and steam generation
- saturated steam, where removal of heat at constant pressure would result in condensation and water formation.

In general, TS-diagram is a useful tool for system description, meaning that operation of various components of the steam cycle can be visualized in a TS-diagram. The hS-diagram is especially useful for visualization and analysis of the expansion in the steam turbine and compression in the pump, as well as definition of the component isentropic efficiency.

The figure below shows the schematic TS- diagram for water, with saturation line (the dashed black line), as well as the constant pressure lines, i.e. the isobars (P_A & P_B). Definition of the critical point, defined by the critical pressure and temperature for water, i.e. $P_{critical}$ = 221.2 bar & $T_{critical}$ = 374.15 °C [properties of water and steam 3rd printing], establishes the criteria for sub- and super critical steam cycles. The critical point is marked with a blue circle in the figure, on top of the dashed bell of the saturation line. The characteristic of the subcritical steam cycles (which constitute a much larger share of the existing steam cycles) is that conversion of water to steam is marked by a distinct boiling process (shown by the horizontal line between saturated water and saturated steam side in the figure). The specific characteristic of boiling process is that additional heat input results in steam formation at constant pressure and temperature, so that the point indicating the state of the working fluid is moving from left to right and the amount of steam and water is calculated by the rule of leverage. Point 4 in the figure, which indicates turbine exit is placed within the so called wet region (steam-water mixture). The state of the working fluid before reaching saturated water is called sub-cooled water and the state of steam after the saturation line is called super-heated steam (indicated by point 3, which is the inlet of the turbine). T_B in the figure below is the mean value of the temperatures T_2 and T_3.

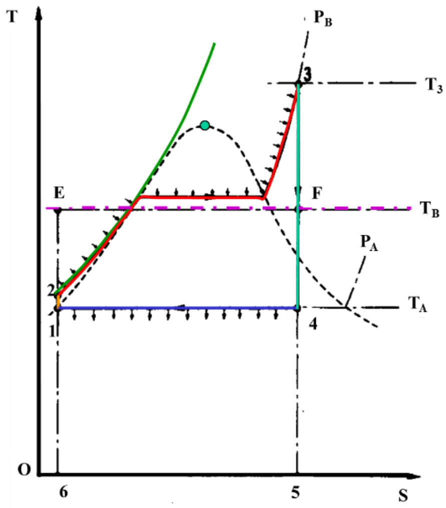


Figure 4: Steam cycle components in a TS-diagram

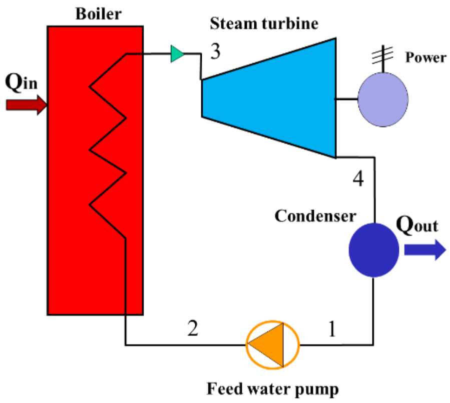


Figure 5: Simple steam cycle

Figure 5 illustrates the components of the simple steam cycle, which are shown in the TS-diagram following same numbering as in figure 4. P_A & P_B indicate the highest and lowest pressure level of the cycle.

2.2. Simple steam cycle components and their function

The figure 4 above illustrates the position of the components of the ideal simple steam cycle in a TS-diagram. It is assumed that the highest and lowest pressures and temperatures of the cycle are known. The assumption made here is that the system is free from losses (ideal), i.e. no pressure or heat losses in the components of the system and that the isentropic efficiency of the pump and the turbine is 100%, indicated by vertical lines between points 1 and 2, and between points 3 and 4 respectively in figure 4. The boiler's position is indicated in TS-diagram between the points 2 and 3. As one can notice the pressurized water after the pump is heated up in the boiler to reach the saturation temperature for the given pressure (sub-cooled region). All the heat input after saturation point is consumed for evaporation of the liquid at constant temperature (and pressure) until the flow reaches the state of saturated steam. Further heat input in the boiler results in increased steam temperature (super-heated steam) prior to boiler exit which is the same as the turbine inlet.

In the following sections axial turbine is assumed as the base for all analysis (if nothing else is indicated), since it is the dominating type of turbines in the modern power plants. In an axial turbine the working fluid (steam) moves parallel to the turbine shaft throughout the machine.

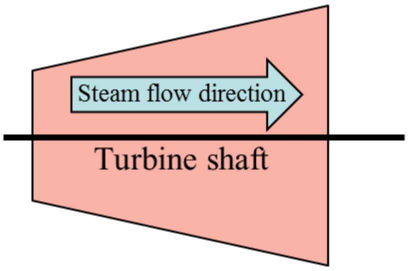


Figure 6: Axial steam turbine with steam flow direction

The figure below shows the impact of deviation of the components' isentropic efficiencies from 100% (i.e. $\eta_s < 100\%$), which always results in increased entropy between inlet and outlet of the component.

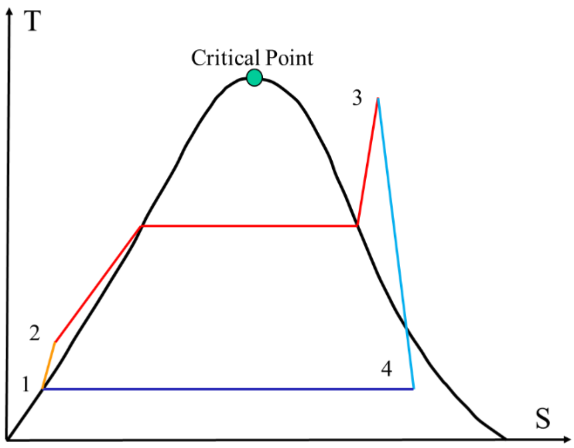


Figure 7: Impact of turbine and pump efficiencies on expansion and compression lines (increasing entropy between inlet and outlet of the component)

The real TS and hS diagram for water-steam with colored lines indicating constant pressure, enthalpy, specific volume and steam quality are shown below:

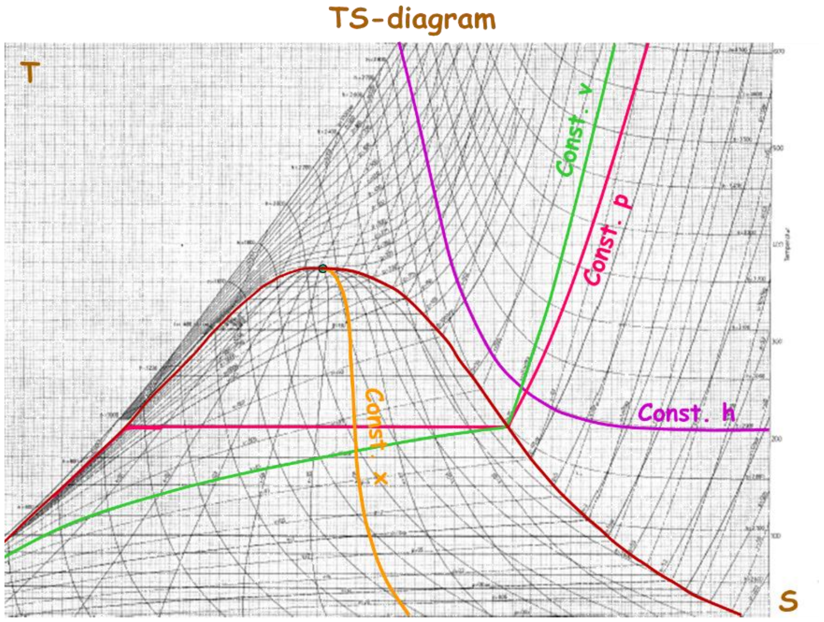


Figure 8: TS-diagram for water-steam, with highlighted constant pressure, enthalpy, specific volume and steam quality or dryness factor

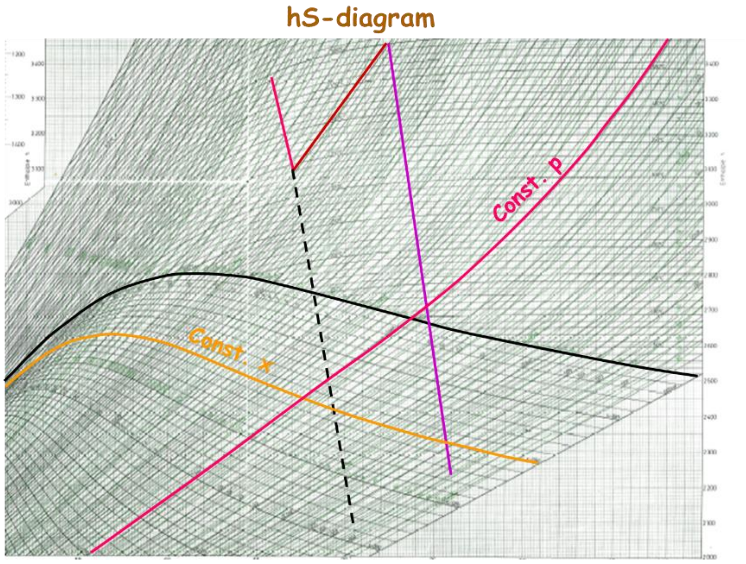


Figure 9: hS-diagram for water-steam, with highlighted constant pressure, and steam quality or dryness factor as well as expansion lines for single (dotted) and reheat turbine

2.3. Efficiency and efficiency improvement

2.3.1. Component efficiencies

Component efficiencies are generally defined as the ratio between the real net output and the ideal maximum possible output (always less than or equal to 1!). However, various settings fits various components and therefore the differences in the equations' outlook. Component efficiencies are the indicators of the technology level of the component, i.e. high efficiency means high technology level.

2.3.1.1. Boiler

Boiler efficiency is defined as the total temperature difference between the maximum combustion temperature (T_{max}) and the exhaust gas temperature at the chimney ($T_{chimney}$), divided by the temperature difference between the maximum temperature (T_{max}) and the atmospheric temperature (T_{atm}).

$$\eta = \frac{T_{max} - T_{chimney}}{T_{max} - T_{atm}}$$

Generally speaking, reduction of the exhaust gas temperature released to the atmosphere increases the boiler's efficiency. However, one should also take into account the acid dew point, e.g. in case of sulfur containing fuels the exhaust temperature should not be lower than 130 °C to avoid acid corrosion attack on the chimney.

2.3.1.2. Steam turbine and pump

As shown in previous section, efficiency of the turbomachinery components (i.e. turbine and pump), which depends on their detailed design, has certain impact on the cycle efficiency. The best way to define and visualize the expansion and compression efficiencies of these components is using the enthalpy-entropy diagram (hS-diagram).

For the turbine, the real expansion line is shown in figure 10 by the solid red line connecting the P_{in} and P_{out} isobars, and the ideal expansion line (η_s = 100%) is shown by the vertical red dotted line indicating constant entropy between inlet and outlet of the turbine. Deviation from 100% efficiency results in increased entropy at the outlet of the turbine as it is indicated in the hS-diagrams below ($S_{out} > S_{in}$):

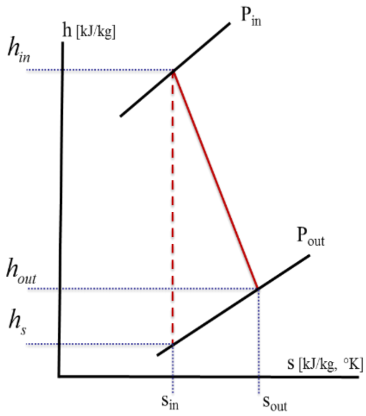


Figure 10: Expansion in the turbine, between inlet and outlet pressure

Using the notifications given in the figure above, the isentropic expansion efficiency can be expressed as:

$$\eta_s = \frac{h_{in} - h_{out}}{h_{in} - h_s}$$

which is always less than or equal to 1. It expresses that the real enthalpy change between inlet and outlet of the turbine (useful work indicator, considering constant mass flow rate through the turbine) divided by the total available enthalpy difference between inlet and outlet at 100% efficiency, which is a measure of effectiveness for the turbine, also called the isentropic expansion efficiency η_s.

In the same way, one can express the isentropic efficiency of a pump. It should be observed that certain portion of the input power is converted to pressure rise in the working fluid, and the outlet pressure will therefore be higher than the inlet pressure:

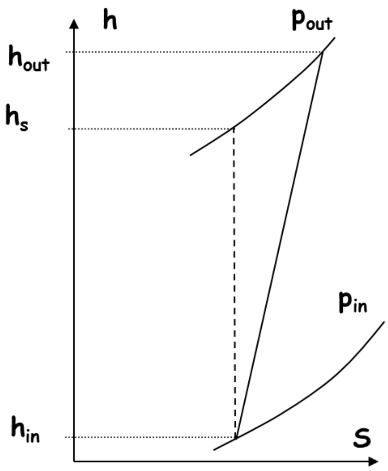


Figure 11: compression in the pump, between inlet and outlet pressure

$$\eta_s = \frac{h_s - h_{in}}{h_{out} - h_{in}}$$

Beside improved component efficiencies, which can be realized by improved design of the component, one can also improve the system design via alternative modifications to the system layout. Here we will focus on two methods applied to the cycle, namely regenerative cycle and reheat cycle.

Before analysis of these two methods, one need to present an expression for the system efficiency so that we can identify the source of the improvement. System efficiency or cycle efficiency is defined as the net power generated, divided by the total energy input to the cycle. Usually the control volume method is used to illustrate the system boundaries and thereby the amount of energy input and power output that crosses the system boundary (see the figure below).

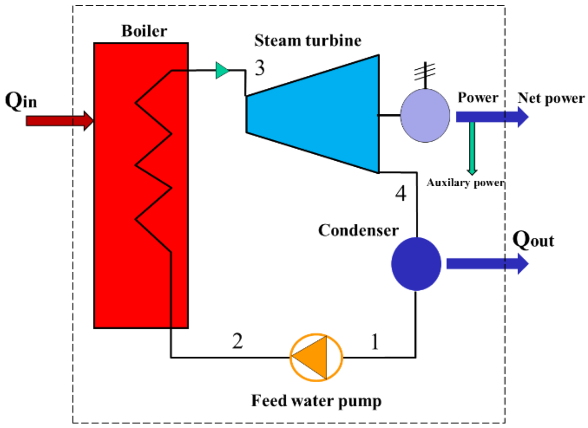


Figure 12: Energy exchange over the system boundary, covering only the steam side

$$\eta_{steam\ cycle} = \frac{Net\ power\ out}{Q_{in}}$$

It means that the electricity generated minus the auxiliary power, e.g. used for running the feed water pump, etc., divided by heat input by the fuel (expressed as fuel mass flow rate multiplied with the fuels lower heating value) minus the heat rejected to the surrounding in the condenser expresses the cycle efficiency.

2.3.2. Cycle efficiency improvement

2.3.2.1. Pre-heating (Regenerative cycle)

Regeneration, means using the internal energy of the system to boost the system's efficiency. Feed water pre-heating is a regenerative method used in steam cycles. Steam is bled from the turbine at various pressure levels and used to heat the feed water before entering the boiler. Hereby the water temperature is increased at entrance to the boiler so that less heat (i.e. fuel input) would be required to reach the final steam temperature at the boiler's outlet. However, steam bleed-off results in less power output, but the total impact leads to efficiency improvement.

The figure below illustrates the regenerative steam cycle with reheat:

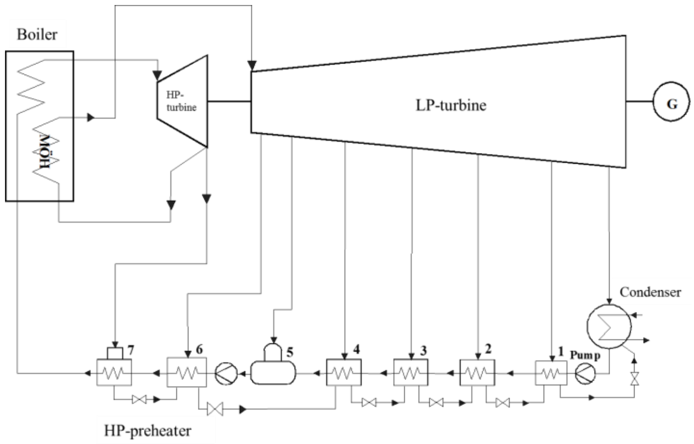


Figure 13: Regenerative cycle with reheat

One could also illustrate the impact on the efficiency using the TQ-diagram, where the y-axis shows the temperature and the x-axis shows the amount of heat transferred between the exhaust gas from the combustion and the water-steam side:

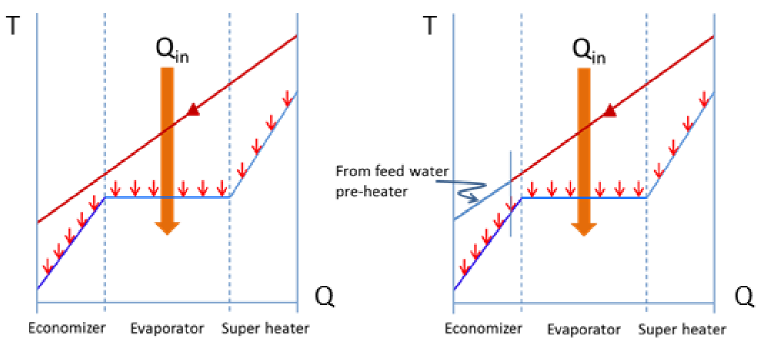


Figure 14: Heat transfer from exhaust gas to water side in the boiler

Left: all heat is provided by combustion Right: part of the heat is provided by pre-heater

As can be seen in figure 14, the heat input, i.e. the amount of fuel needed to reach the final steam temperature is reduced due to feed water preheating. Obviously the amount of the steam that is expanding in the turbine is also reduced, reducing the net power output, but the impact of pre-heating on system efficiency is higher, resulting in a net improvement of the cycle efficiency.

2.3.2.2. Reheat cycle

Higher steam data, i.e. pressure and temperature, results also in improved efficiency. However, the maximum allowable temperature is limited by the material, but pressure can generally be increased, resulting in size reduction and performance improvement. The consequence of increased steam pressure is that the steam quality (dryness) at the turbine exit becomes low, with risk for erosion damage to the turbine blades. To avoid this risk, one needs to reheat the steam in the boiler, after partial expansion in the turbine. The hS-diagram below illustrates the reheat process, showing that if the expansion in the HP-turbine had proceed to P_{Cond}, it should result in excessive liquid content at the turbine outlet (the dotted line in the figure below).

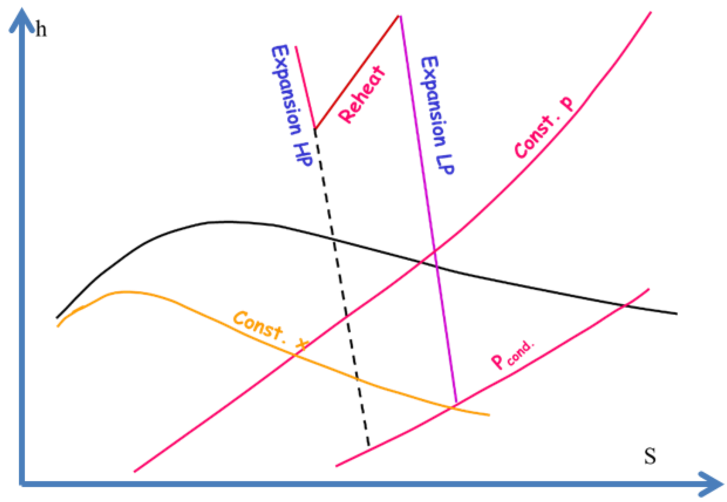


Figure 15: Expansion in the turbine and steam quality at the outlet

Interrupting the expansion after the HP-turbine and heating the steam in the boiler to higher temperature, i.e. reheating the steam, reduces the liquid content of the steam at the turbine outlet, avoiding erosion problems on the turbine blades. Figure 16 shows the reheat process in a real hS-diagram.

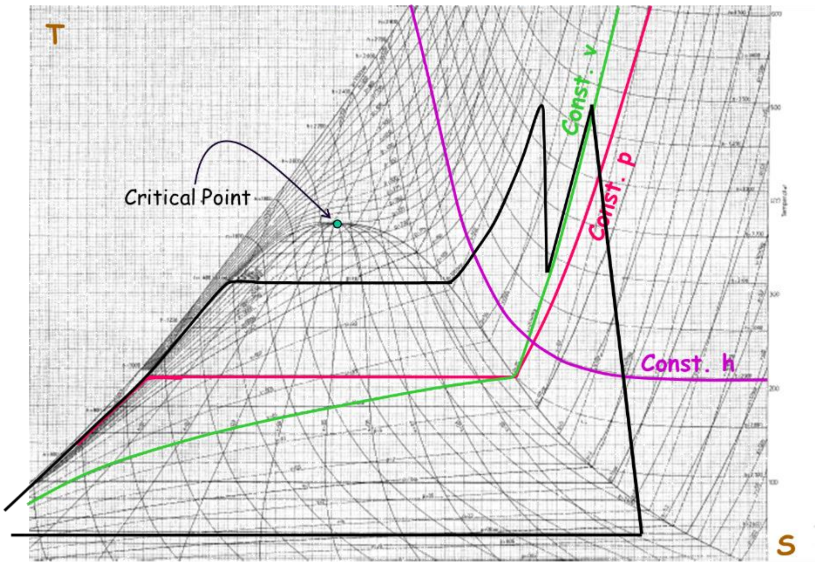


Figure 16: Reheat cycle in TS-diagram

2.4. Condensing and backpressure turbine

Steam cycles can be designed for electricity generation only, with so called condensing turbines, or for combined electricity and heat production, with backpressure turbine. The difference between these two setups is that condensing turbines carry out the expansion to the condenser pressure level, which is dictated by the cooling water temperature, but in backpressure turbines, expansion is interrupted at an elevated pressure that provides steam at a given temperature at the turbine outlet. This steam can be used for either district heating or as heat source for other industries, such as pulp and paper or chemical industry.

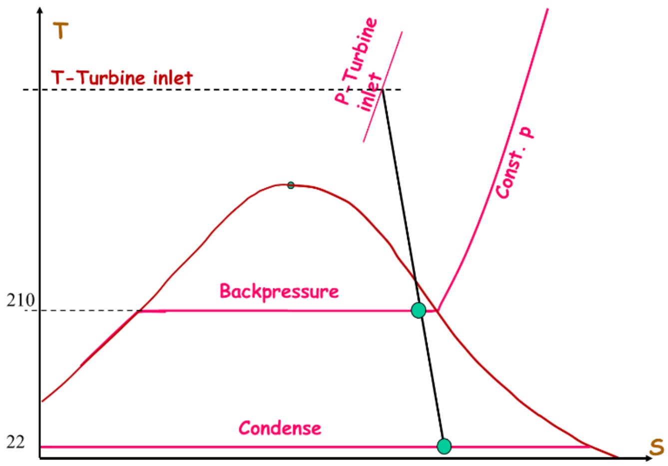


Figure 16: The backpressure turbine deliverables steam at 210 C, and the condensing turbine has temperature of 22 C at turbine outlet

2.4.1. Stage theory

The axial steam turbine consists of nozzle or stator rows (nonmoving blades) followed by a rotating row of blades (rotor blades), which together constitutes a turbine stage. A steam turbine might have many stages, depending on the pressure difference between inlet and outlet of the turbine, and the turbine efficiency. Having few turbine stages, results in higher stage load and consequently lower efficiency due to higher flow velocities and thereby higher losses. There is a gap between the rotating blades tip and the housing, which is called tip clearance, defining the flow leakage over the blades. Turbine stage can be designed as action or reaction stage. The degree of reaction, defining the share of expansion over the stator and the rotor blades in a turbine stage, indicates the pressure change over the rotor blades to the pressure change over the whole stage. The figure below shows a turbine stage.

Since change of flow velocities over the stator and rotor blades is an indicator of the energy exchange between the working medium (steam) and the structure (blades) one can analyze the conversion of energy content in the working fluid to mechanical work by using the velocity triangles. Since the change of radius between inlet and outlet of an axial stage is negligible, the setup below for driving the equations uses a centrifugal pump as base for the analysis, showing the generality of the approach for all kind of turbo machineries.

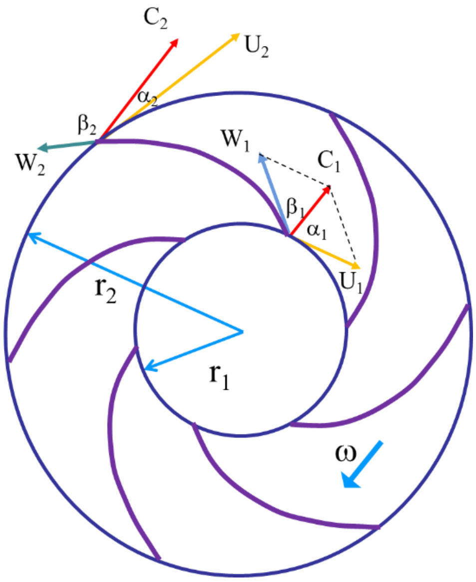
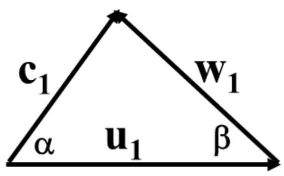


Figure 17: Velocity vectors at inlet and outlet of a centrifugal pump

The figure above shows the velocity vectors, C: absolute velocity, U: tangential or blade velocity and W: relative velocity, r_1 and r_2 are the radius at inlet and outlet respectively. Index 1 indicates the inlet and index 2 the outlet of the blade.

With the notifications from the figure above, one can isolate the velocity triangle at the inlet to express the geometrical relationships (see the figure below).Using C_{1r} and C_{1u}, that are the radial and tangential component of the absolute velocity vector, one can setup the following equations:



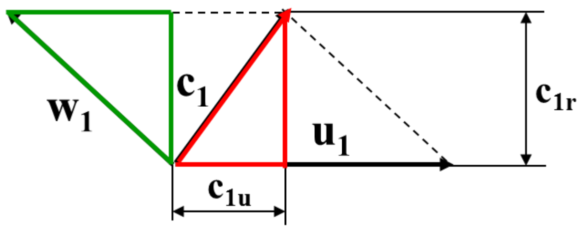


Figure 18: Velocity triangles

Using the red triangle one ca write: $c_{1r}^2 = c_1^2 - c_{1u}^2$

Using the green triangle (with the sides W1, C1r and U1-c1u) we can write:

$$c_{1r}^2 = w_1^2 - \left(u_1 - c_{1u}\right)^2$$

Combining the two equations, putting write hand sides equal to each other one can drive the following relation:

$$c_1^2 - c_{1u}^2 = w_1^2 - u_1^2 - c_{1u}^2 + 2 \cdot u_1 \cdot c_{1u}$$

That can finally be expressed as:

$$\Rightarrow \quad u_1 \cdot c_{1u} = \frac{1}{2}\left(c_1^2 + u_1^2 - w_1^2\right)$$

Using the same analogy for the outlet velocity triangle:

$$u_2 \cdot c_{2u} = \frac{1}{2}\left(c_2^2 + u_2^2 - w_2^2\right)$$

Written in compact form between inlet and outlet, we can see that the total enthalpy change (or work per mass flow rate) of Euler's turbomachinery equation can be expressed using the velocity parameters:

$$\Rightarrow \quad \Delta h_0 = u_1 \cdot c_{1u} - u_2 \cdot c_{2u} = \frac{1}{2}\left[c^2 + u^2 - w^2\right]_1^2$$

Expressed as specific effect (effect per mass flow rate) we get the following equation:

$$\frac{P}{\dot{m}} = \frac{1}{2}[(c_1^2 - c_2^2) + (u_1^2 - u_2^2) + (w_2^2 - w_1^2)$$

The term with absolute velocity C, expresses change in the fluid's kinetic energy, i.e. dynamic pressure. The term with tangential velocity U, expresses energy exchange between fluid and the rotor, caused by changes in centrifugal forces between inlet and outlet, i.e. static pressure energy, and the term with relative velocity W, expresses the change in fluid's energy, equal to change of its static pressure energy. Therefore, the degree of reaction can be expressed using this equation as:

$$R = \frac{\left(u_1^2 - u_2^2\right) + \left(w_2^2 - w_1^2\right)}{\left(c_1^2 - c_2^2\right) + \left(u_1^2 - u_2^2\right) + \left(w_2^2 - w_1^2\right)}$$

2.4.1.1. Action stage

An action stage is defined by degree of reaction being equal to zero. This means that the expansion of the steam (i.e. the pressure change between the inlet and outlet of the turbine stage) occurs only over the stator row (nozzle blades). Therefore, there is no pressure change over the rotor blades, which means equal pressure between inlet and outlet of the rotor. This makes the design of the stage easier from a production point of view, since the sealing and the tip clearance are not an issue any more. However, large pressure change over the stator results in higher flow velocities and thereby higher losses. Therefor the stage efficiency for an action stage is lower than stage efficiency for reaction stage with similar data. The figure below shows the difference between expansion in an action stage and the reaction stage in a hS-diagram.

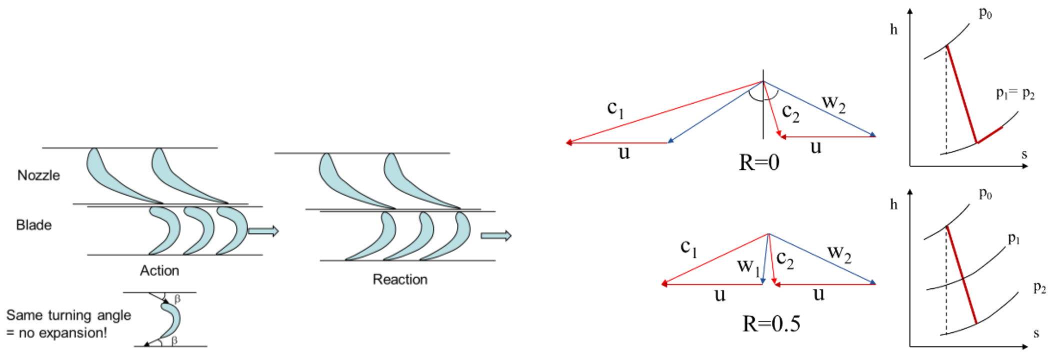


Figure 19A: Action and reaction blading. No expansion (pressure change) over blades of action stage only turning the flow

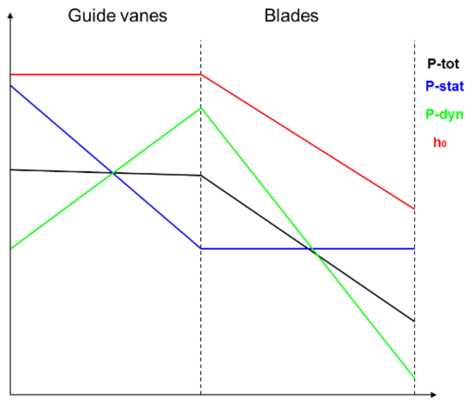


Figure 19B: Property change over an action stage

2.4.1.2. Reaction stage

In a reaction stage the working fluid expands both over the stator and rotor blades, resulting in a pressure difference over the rotating blades. Therefore it is important to minimize the gap between the blade tip and the housing, i.e. the tip clearance, to reduce the leakage of steam over the blade tip, to maintain high stage efficiency. The degree of reaction might be anything between zero and one, but it is usually about 0.5, so that the load is shared evenly over the stator and the rotor blades. This will result in lower flow velocities over both stator and rotor, and thereby reduced flow losses.

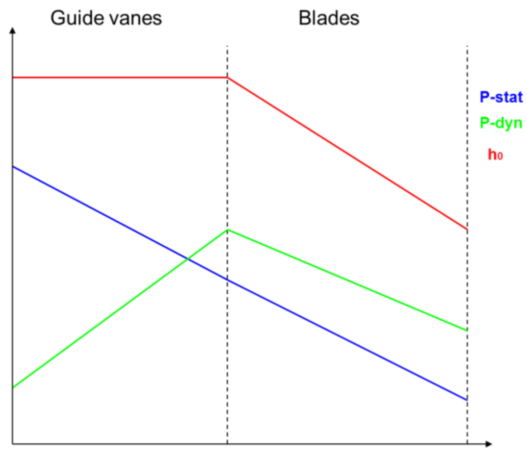


Figure 19C: Property change over an action stage

Figure 20: A typical steam turbine rotor

2.5. Problems: Vapour power cycles

Learning objectives:

- Analysing vapour power cycles in which the working fluid is alternately vaporized and condensed.
- Analysing power generation coupled with process heating called co-generation.
- Investigating ways to modify the basic Rankine vapour power cycle to increase the cycle thermal efficiency.
- Analysing the reheat and regenerative vapour power cycles.
- Analysing power cycles that consist of two separate cycles known as combined cycles.

I. Examples 1- 7 are mostly connected to each other and review the “vapour and combined power cycles” by the following purposes:

1. The Simple Ideal Rankine Cycle.
2. An Actual Steam Power Cycle.
3. Effect of Boiler Pressure and Temperature on Efficiency
4. The Ideal Reheat Rankine Cycle.
5. The Ideal Regenerative Rankine Cycle.
6. The Ideal Reheat–Regenerative Rankine Cycle.
7. An Ideal Cogeneration Plant**.**

II. Examples 8- 14 are classified and dispersed examples from the above-mentioned subjects.

III. Example 15 is “steam regenerative Rankine cycle with two feed- water heaters”, besides its EES code for thermodynamic simulation.

IV. Examples 16-18 are geothermal power plants.

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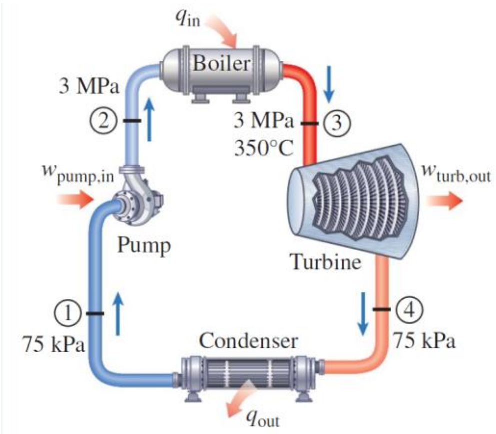
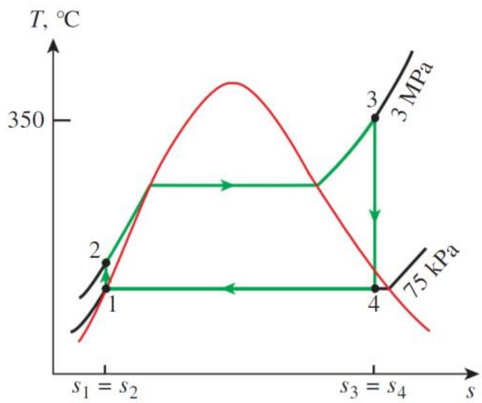
The Simple Ideal Rankine Cycle

1. Consider a steam power plant operating on the simple ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350 °C and is condensed in the condenser at a pressure of 75 kPa. Determine the thermal efficiency of this cycle.

[η_{th} = 26%]





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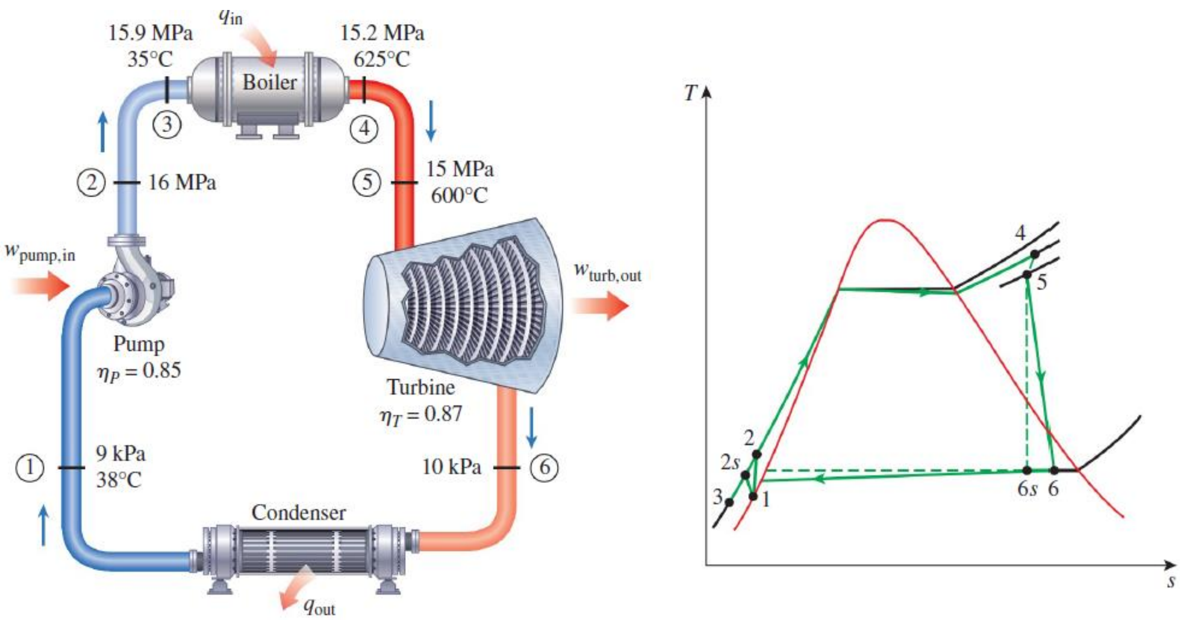
An Actual Steam Power Cycle

2. A steam power plant operates on the cycle shown in the figure. If the isentropic efficiency of the turbine is 87 percent and the isentropic efficiency of the pump is 85 percent, determine (a) the thermal efficiency of the cycle and (b) the net power output of the plant for a mass flow rate of 15 kg/s.

[η_{th} = 36.1%, W_{out} =18.9 MW]



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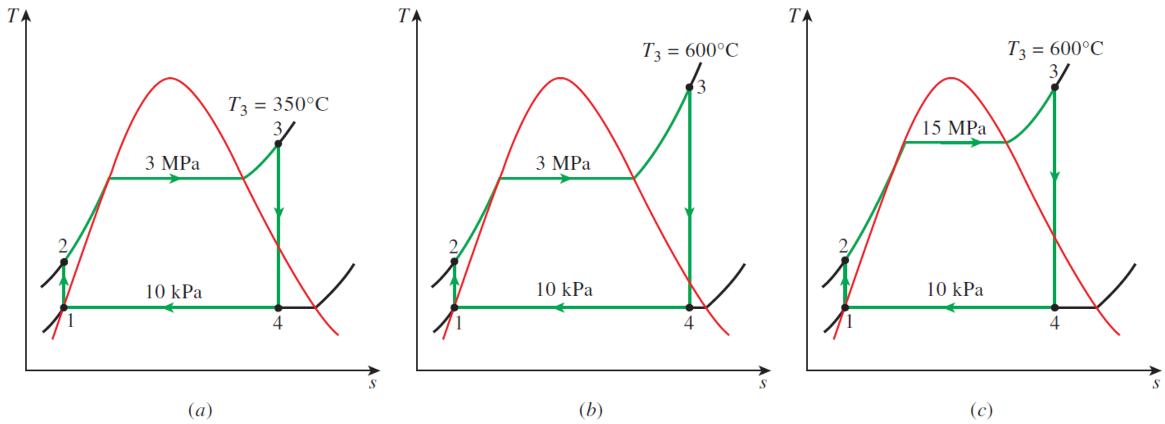
Effect of Boiler Pressure and Temperature on Efficiency

3. Consider a steam power plant operating on the ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350 °C and is condensed in the condenser at a pressure of 10 kPa. Determine (a) the thermal efficiency of this power plant, (b) the thermal efficiency if steam is superheated to 600 °C instead of 350 °C, and (c) the thermal efficiency if the boiler pressure is raised to 15 MPa while the turbine inlet temperature is maintained at 600 °C.

[$\eta_{th,1}$ = 33.4%, $\eta_{th,2}$ = 37.3%, $\eta_{th,3}$ = 43%]



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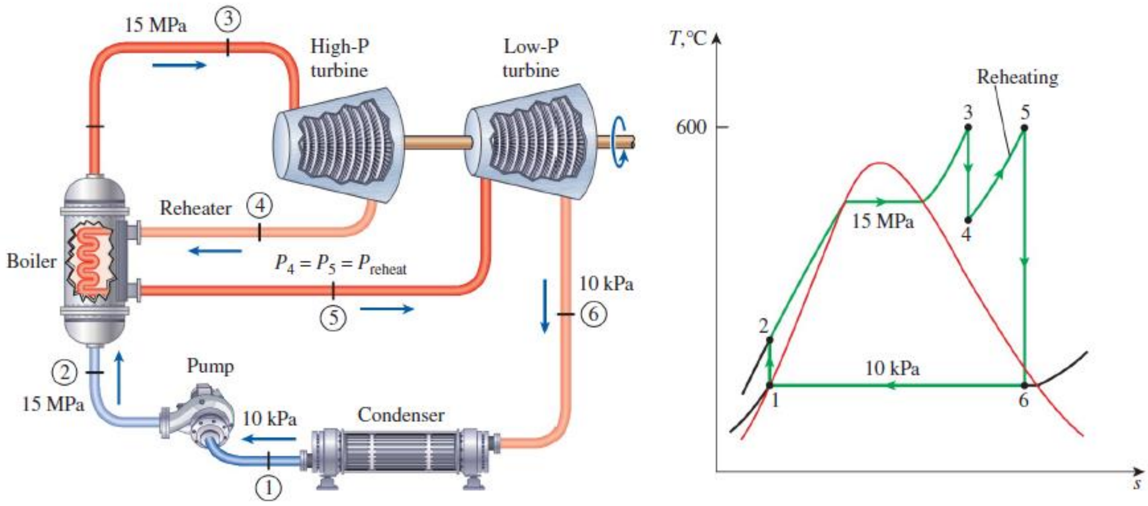
The Ideal Reheat Rankine Cycle

4. Consider a steam power plant operating on the ideal reheat Rankine cycle. Steam enters the high-pressure turbine at 15 MPa and 600 °C and is condensed in the condenser at a pressure of 10 kPa. If the moisture content of the steam at the exit of the low-pressure turbine is not to exceed 10.4 percent, determine (a) the pressure at which the steam should be reheated and (b) the thermal efficiency of the cycle. Assume the steam is reheated to the inlet temperature of the high-pressure turbine.

[P_{reheat}= 4 MPa, η_{th} = 45%]



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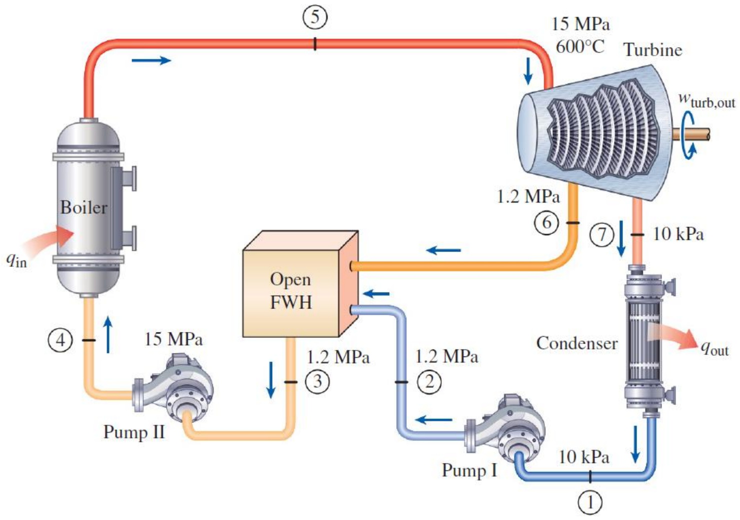
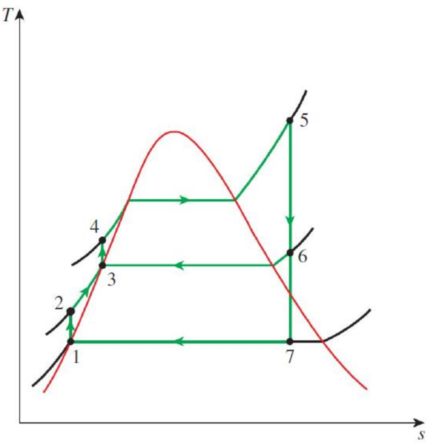
The Ideal Regenerative Rankine Cycle

5. Consider a steam power plant operating on the ideal regenerative Rankine cycle with one open feed-water heater. Steam enters the turbine at 15 MPa and 600 °C and is condensed in the condenser at a pressure of 10 kPa. Some steam leaves the turbine at a pressure of 1.2 MPa and enters the open Feed-water heater. Determine the fraction of steam extracted from the turbine and the thermal efficiency of the cycle.

[y=0.2270, η_{th} = 46.3%]





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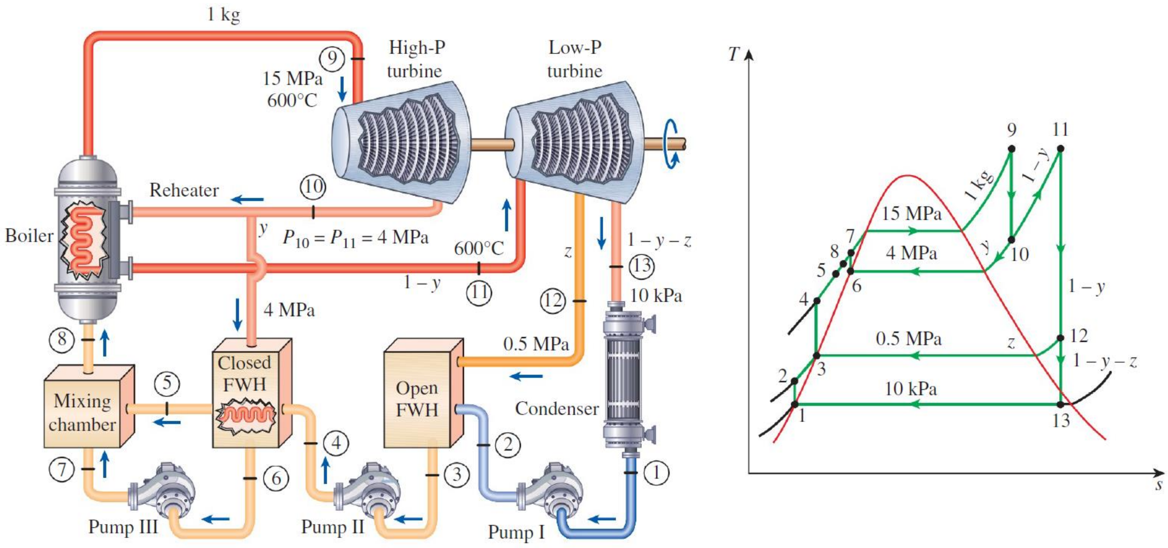
The Ideal Reheat–Regenerative Rankine Cycle

6. Consider a steam power plant that operates on an ideal reheat-regenerative Rankine cycle with one open feed-water heater, one closed feed-water heater, and one re-heater. Steam enters the turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. Some steam is extracted from the turbine at 4 MPa for the closed feed-water heater, and the remaining steam is reheated at the same pressure to 600°C. The extracted steam is completely condensed in the heater and is pumped to 15 MPa before it mixes with the feed-water at the same pressure. Steam for the open feed-water heater is extracted from the low-pressure turbine at a pressure of 0.5 MPa. Determine the fractions of steam extracted from the turbine as well as the thermal efficiency of the cycle.

   [y=0.1766, z= 0.1306, η_{th} = 49.2%,]



The enthalpies at the various states and the pump work per unit mass of fluid flowing through them are:

h_1= 191.81 kJ/kg	h_9 = 3155.0 kJ/kg
h_2 = 192.30 kJ/kg	h_{10} = 3155.0 kJ/kg
h_3 = 640.09 kJ/kg	h_{11} = 3674.9 kJ/kg
h_4 = 643.92 kJ/kg	h_{12} = 3014.8 kJ/kg
h_5 = 1087.4 kJ/kg	h_{13} = 2335.7 kJ/kg
h_6 = 1087.4 kJ/kg	$W_{pump\ I,in}$ = 0.49 kJ/kg
h_7 = 1101.2 kJ/kg	$W_{pump\ II.in}$ = 3.83 kJ/kg
h_8 = 1089.8 kJ/kg	$W_{pump\ III,in}$ = 13.77 kJ/kg

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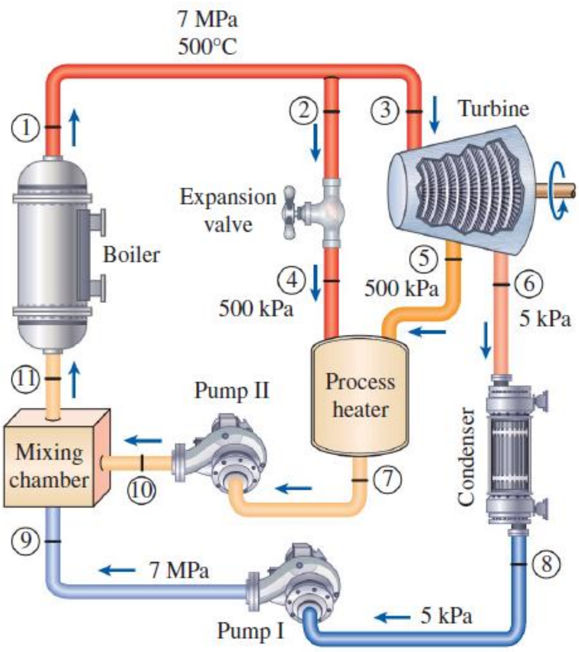
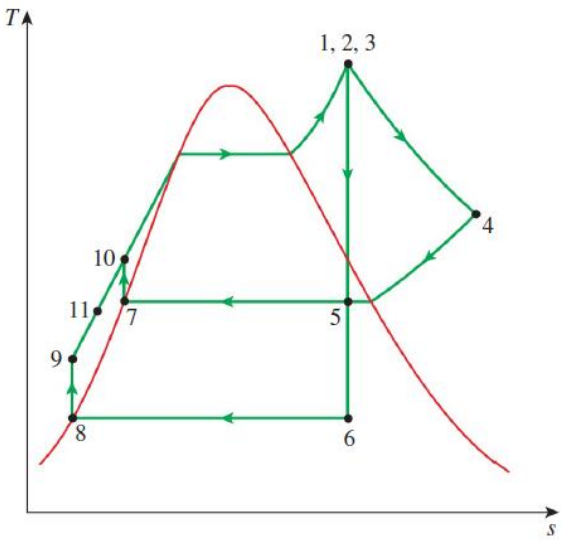
An Ideal Cogeneration Plant

7. Consider the cogeneration plant shown in the following figure. Steam enters the turbine at 7 MPa and 500 °C. Some steam is extracted from the turbine at 500 kPa for process heating. The remaining steam continues to expand to 5 kPa. Steam is then condensed at constant pressure and pumped to the boiler pressure of 7 MPa. At times of high demand for process heat, some steam leaving the boiler is throttled to 500 kPa and is routed to the process heater. The extraction fractions are adjusted so that steam leaves the process heater as a saturated liquid at 500 kPa. It is subsequently pumped to 7 MPa. The mass flow rate of steam through the boiler is 15 kg/s. Disregarding any pressure drops and heat losses in the piping and assuming the turbine and the pump to be isentropic, determine (a) the maximum rate at which process heat can be supplied, (b) the power produced and the utilization factor when no process heat is supplied, and (c) the rate of process heat supply when 10 percent of the steam is extracted before it enters the turbine and 70 percent of the steam is extracted from the turbine at 500 kPa for process heating.

[Q_{max} = 41570 kW, W_{out} = 20.0 MW, ε=40.8%, Q = 26.2 MW]





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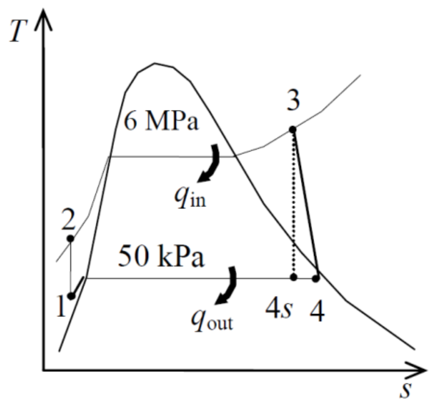
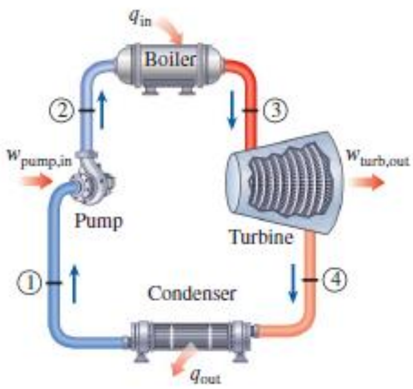
Simple Steam Cycle:

8. A simple Rankine cycle uses water as the working fluid. The boiler operates at 6000 kPa and the condenser at 50 kPa. At the entrance to the turbine, the temperature is 450°C. The isentropic efficiency of the turbine is 94 percent, pressure and pump losses are negligible, and the water leaving the condenser is subcooled to 6.3°C less than saturated temperature at 50 kPa. The boiler is sized for a mass flow rate of 20 kg/s. Determine the rate at which heat is added in the boiler, the power required to operate the pumps, the net power produced by the cycle, and the thermal efficiency.

[Q= 59660 kW, W_{Pump}= 122 kW, $W_{net, out}$ = 18050 kW, η_{th} = 30.25%]





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9. Consider a steam power plant that operates on a simple ideal Rankine cycle and has a net power output of 45 MW. Steam enters the turbine at 7 MPa and 500°C and is cooled in the condenser at a pressure of 10 kPa by running cooling water from a lake through the tubes of the condenser at a rate of 2000 kg/s. Show the cycle on a T-s diagram with respect to saturation lines, and determine:

a) The thermal efficiency of the cycle, the mass flow rate of the steam, and the temperature rise of the cooling water when the isentropic efficiency of the components of the cycle assumed to be 100%.

b) Assuming an isentropic efficiency of 87 percent for both the turbine and the pump.

[$\eta_{th,a}$ = 38.9 %, $m_{steam,a}$ = 36 kg/s, $\Delta T_{cooling,a}$ =8.4 °C]

[$\eta_{th,b}$ = 33.8%, $m_{steam,b}$ = 41.43 kg/s, $\Delta T_{cooling,b}$ = 10.5 °C]

a)

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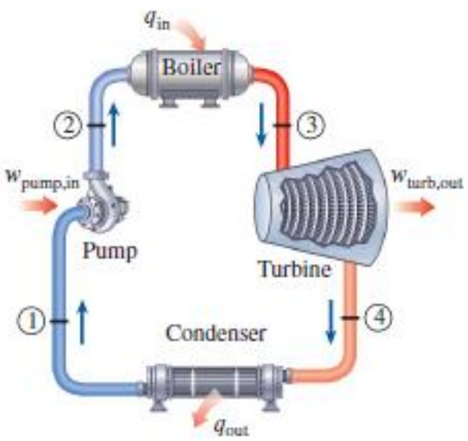
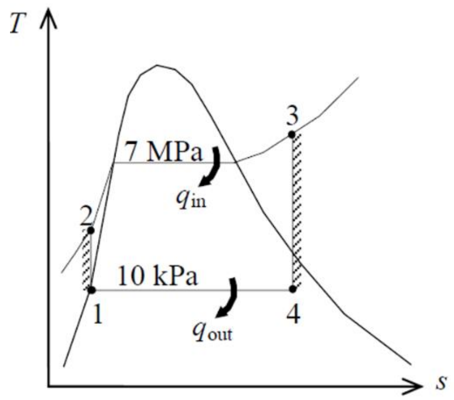
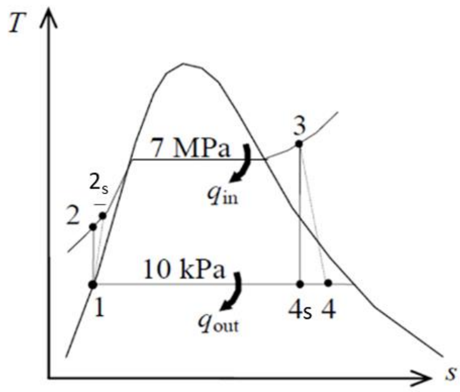




b)



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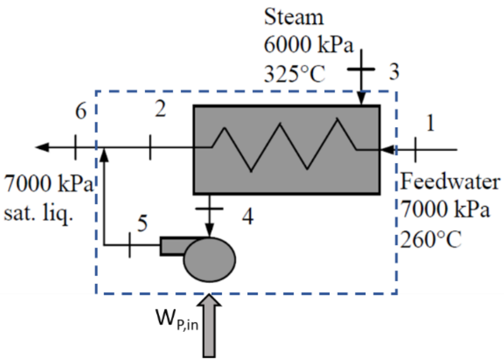
Regenerative Steam Cycle:

10. The closed feed-water heater of a regenerative Rankine cycle is designed to heat feed-water at 7 MPa from 260°C to a saturated liquid. The turbine supplies bleed steam at 6000 kPa and 325°C to this unit. The steam is condensed to saturated liquid before entering the pump. Calculate the amount of bleed steam required to heat 1 kg of feed-water in this unit to the desired condition.

[m_3 =0.0779 kg/s]



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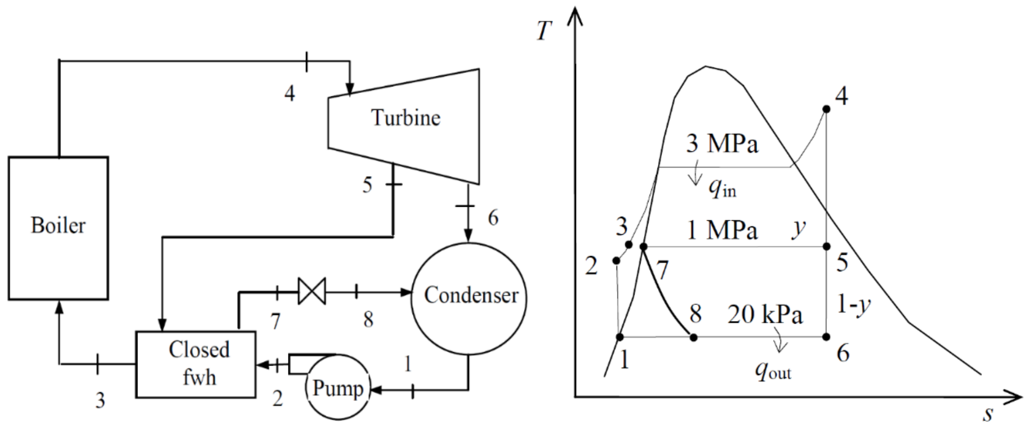
11. Consider a steam power plant that operates on the ideal regenerative Rankine cycle with a closed feedwater heater as shown in the figure. The plant maintains the turbine inlet at 3000 kPa and 350°C; and operates the condenser at 20 kPa. Steam is extracted at 1000 kPa to serve the closed feedwater heater, which discharges into the condenser after being throttled to condenser pressure. Calculate the work produced by the turbine, the work consumed by the pump, and the heat supply in the boiler for this cycle per unit of boiler flow rate.

[$W_{out,\ turbine}$= 740.9 kJ/kg, W_{pump} = 3.03 kJ/kg, Q_{in} =2353 kJ/kg]



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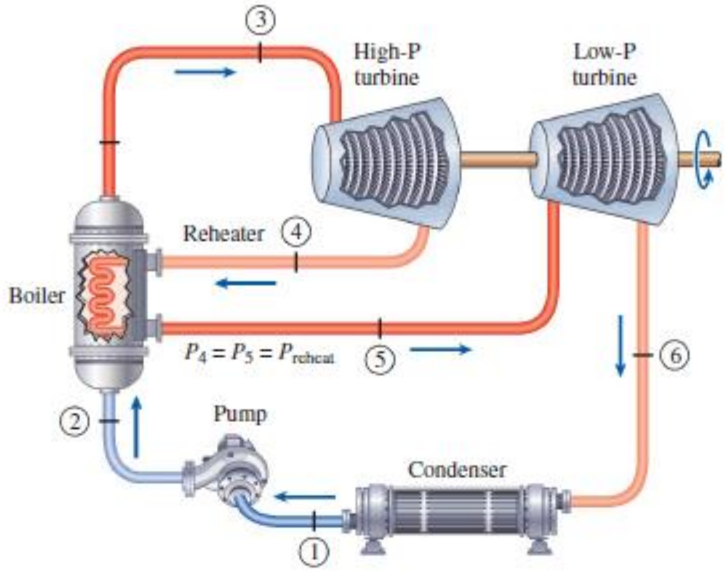
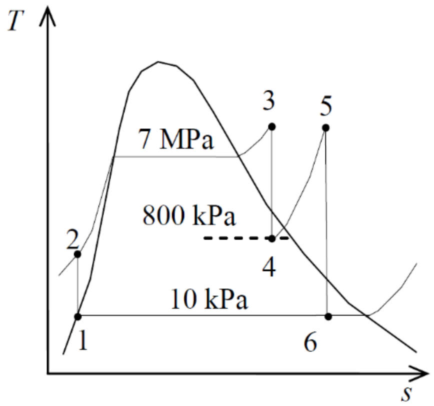
Reheat Steam Cycle

12. Consider a steam power plant that operates on the ideal reheat Rankine cycle. The plant maintains the boiler at 7000 kPa, the reheat section at 800 kPa, and the condenser at 10 kPa. The mixture quality at the exit of both turbines is 93 percent. Determine the temperature at the inlet of each turbine and the cycle's thermal efficiency.

[T_3 = 373.3 °C, T_5 = 416.2 °C, η_{th} =37.6 %]





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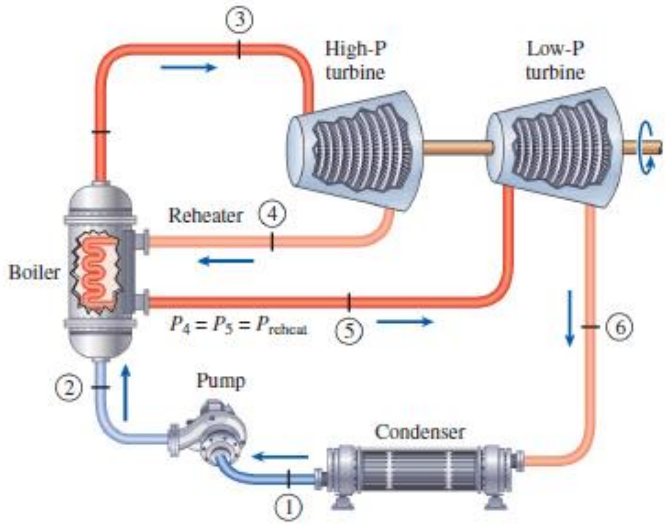
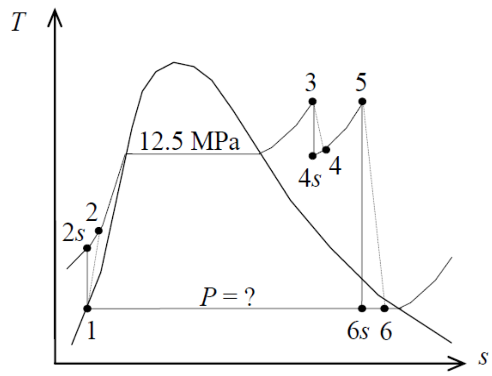
13. A steam power plant operates on the reheat Rankine cycle. Steam enters the high-pressure turbine at 12.5 MPa and 550°C at a rate of 7.7 kg/s and leaves at 2 MPa. Steam is then reheated at constant pressure to 450°C before it expands in the low-pressure turbine. The isentropic efficiencies of the turbine and the pump are 85 percent and 90 percent, respectively. Steam leaves the condenser as a saturated liquid. If the moisture content of the steam at the exit of the turbine is not to exceed 5 percent, determine (a) the condenser pressure, (b) the net power output, and (c) the thermal efficiency.

[P_{con} = 9.73 kPa, $W_{net,out}$ = 10242 kW, η_{th} = 36.9 %]





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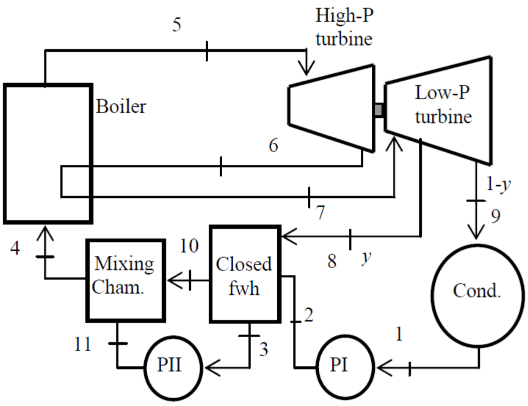
14. A steam power plant operates on the reheat-regenerative Rankine cycle with a closed feed-water heater. Steam enters the turbine at 8 MPa and 500°C at a rate of 15 kg/s and is condensed in the condenser at a pressure of 20 kPa. Steam is reheated at 3 MPa to 500°C. Some steam is extracted from the low-pressure turbine at 1.0 MPa, is completely condensed in the closed feedwater heater, and pumped to 8 MPa before it mixes with the feedwater at the same pressure. Assuming an isentropic efficiency of 88 percent for both the turbine and the pump, determine (a) the temperature of the steam at the inlet of the closed feedwater heater, (b) the mass flow rate of the steam extracted from the turbine for the closed feedwater heater, (c) the net power output, and (d) the thermal efficiency.

[T_8 = 349.9 °C, m_8 = 2.637 kg/s, $W_{net, out}$ = 16206 kW, η_{th} =36.7%]



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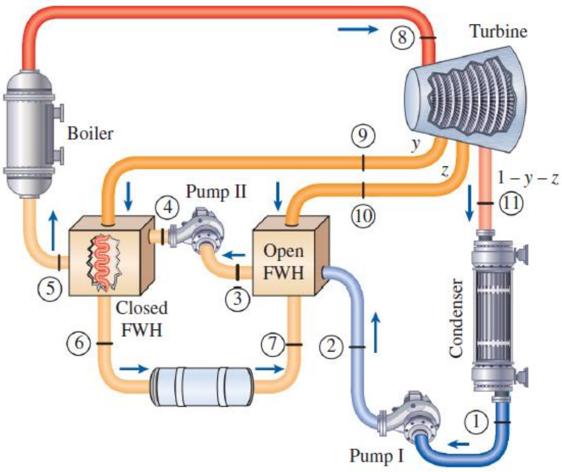
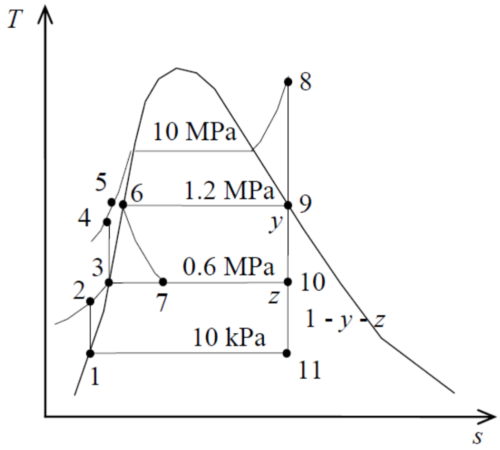
15. Consider an ideal steam regenerative Rankine cycle with two feed-water heaters, one closed and one open. Steam enters the turbine at 10 MPa and 600 °C and exhausts to the condenser at 10 kPa. Steam is extracted from the turbine at 1.2 MPa for the closed feed-water heater and at 0.6 MPa for the open one. The feed-water is heated to the condensation temperature of the extracted steam in the closed feed-water heater. The extracted steam leaves the closed feed-water heater as a saturated liquid, which is subsequently throttled to the open feed-water heater. Show the cycle on a T-s diagram with respect to saturation lines, and determine (a) the mass flow rate of steam through the boiler for a net power output of 400 MW and (b) the thermal efficiency of the cycle.

[m= 313 kg/s, η_{th} = 45.2 %]





EES Code:

```
"Input Data"
P[8] = 10000 [kPa]
T[8] = 600 [C]
P[9] = 1200 [kPa]
P_cfwh=600 [kPa]
P[10] = P_cfwh
P_cond=10 [kPa]
P[11] = P_cond
W_dot_net=400 [MW]*Convert(MW, kW)
Eta_turb= 100/100 "Turbine isentropic efficiency"
Eta_turb_hp = Eta_turb "Turbine isentropic efficiency for high pressure stages"
Eta_turb_ip = Eta_turb "Turbine isentropic efficiency for intermediate pressure stages"
Eta_turb_lp = Eta_turb "Turbine isentropic efficiency for low pressure stages"
Eta_pump = 100/100 "Pump isentropic efficiency"

"Condenser exit pump  or Pump 1 analysis"
Fluid$='Steam_IAPWS'
P[1] = P[11]
P[2]=P[10]
h[1]=enthalpy(Fluid$,P=P[1],x=0) {Sat'd liquid}
v1=volume(Fluid$,P=P[1],x=0)
```

```
s[1]=entropy(Fluid$,P=P[1],x=0)
T[1]=temperature(Fluid$,P=P[1],x=0)
w_pump1_s=v1*(P[2]-P[1])"SSSF isentropic pump work assuming constant specific volume"
w_pump1=w_pump1_s/Eta_pump "Definition of pump efficiency"
h[1]+w_pump1= h[2]  "Steady-flow conservation of energy"
s[2]=entropy(Fluid$,P=P[2],h=h[2])
T[2]=temperature(Fluid$,P=P[2],h=h[2])

"Open Feedwater Heater analysis"
z*h[10] + y*h[7] + (1-y-z)*h[2] = 1*h[3] "Steady-flow conservation of energy"
h[3]=enthalpy(Fluid$,P=P[3],x=0)
T[3]=temperature(Fluid$,P=P[3],x=0) "Condensate leaves heater as sat. liquid at P[3]"
s[3]=entropy(Fluid$,P=P[3],x=0)

"Boiler condensate pump  or Pump 2 analysis"
P[5]=P[8]
P[4] = P[5]
P[3]=P[10]
v3=volume(Fluid$,P=P[3],x=0)
w_pump2_s=v3*(P[4]-P[3])"SSSF isentropic pump work assuming constant specific volume"
w_pump2=w_pump2_s/Eta_pump "Definition of pump efficiency"
h[3]+w_pump2= h[4]  "Steady-flow conservation of energy"
s[4]=entropy(Fluid$,P=P[4],h=h[4])
T[4]=temperature(Fluid$,P=P[4],h=h[4])

"Closed Feedwater Heater analysis"
P[6]=P[9]
 y*h[9] + 1*h[4] = 1*h[5] + y*h[6] "Steady-flow conservation of energy"
h[5]=enthalpy(Fluid$,P=P[6],x=0) "h[5] = h(T[5], P[5]) where T[5]=Tsat at P[9]"
T[5]=temperature(Fluid$,P=P[5],h=h[5]) "Condensate leaves heater as sat. liquid at P[6]"
s[5]=entropy(Fluid$,P=P[6],h=h[5])
h[6]=enthalpy(Fluid$,P=P[6],x=0)
T[6]=temperature(Fluid$,P=P[6],x=0) "Condensate leaves heater as sat. liquid at P[6]"
s[6]=entropy(Fluid$,P=P[6],x=0)

"Trap analysis"
P[7] = P[10]
y*h[6] = y*h[7]  "Steady-flow conservation of energy for the trap operating as a throttle"
T[7]=temperature(Fluid$,P=P[7],h=h[7])
s[7]=entropy(Fluid$,P=P[7],h=h[7])

"Boiler analysis"
q_in + h[5]=h[8]"SSSF conservation of energy for the Boiler"
h[8]=enthalpy(Fluid$, T=T[8], P=P[8])
s[8]=entropy(Fluid$, T=T[8], P=P[8])

"Turbine analysis"
ss[9]=s[8]
hs[9]=enthalpy(Fluid$,s=ss[9],P=P[9])
Ts[9]=temperature(Fluid$,s=ss[9],P=P[9])
h[9]=h[8]-Eta_turb_hp*(h[8]-hs[9])"Definition of turbine efficiency for high pressure stages"
```

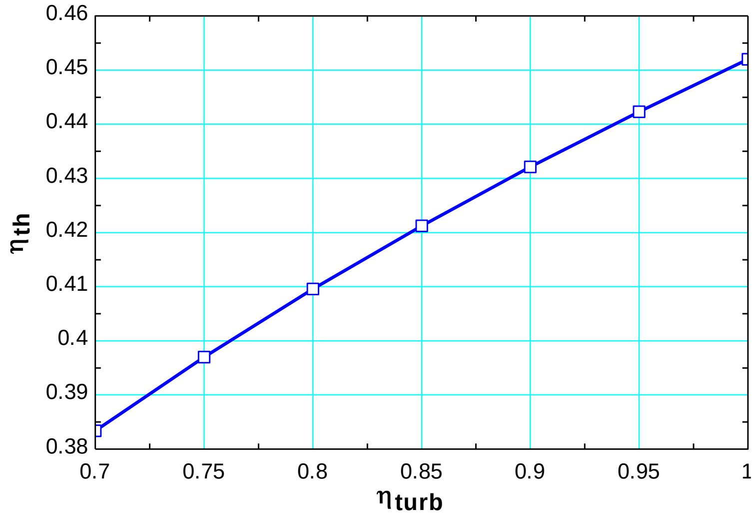
```
T[9]=temperature(Fluid$,P=P[9],h=h[9])
s[9]=entropy(Fluid$,P=P[9],h=h[9])
ss[10]=s[8]
hs[10]=enthalpy(Fluid$,s=ss[10],P=P[10])
Ts[10]=temperature(Fluid$,s=ss[10],P=P[10])
h[10]=h[9]-Eta_turb_ip*(h[9]-hs[10])"Definition of turbine efficiency for Intermediate pressure stages"
T[10]=temperature(Fluid$,P=P[10],h=h[10])
s[10]=entropy(Fluid$,P=P[10],h=h[10])
ss[11]=s[8]
hs[11]=enthalpy(Fluid$,s=ss[11],P=P[11])
Ts[11]=temperature(Fluid$,s=ss[11],P=P[11])
h[11]=h[10]-Eta_turb_lp*(h[10]-hs[11])"Definition of turbine efficiency for low pressure stages"
T[11]=temperature(Fluid$,P=P[11],h=h[11])
s[11]=entropy(Fluid$,P=P[11],h=h[11])
h[8] =y*h[9] + z*h[10] + (1-y-z)*h[11] + w_turb "SSSF conservation of energy for turbine"

"Condenser analysis"
(1-y-z)*h[11]=q_out+(1-y-z)*h[1]"SSSF First Law for the Condenser"

"Cycle Statistics"
w_net=w_turb - ((1-y-z)*w_pump1+ w_pump2)
Eta_th=w_net/q_in
W_dot_net = m_dot * w_net
```

η_{turb}	η_{th}	m [kg/s]
0.7	0.3834	369
0.75	0.397	356.3
0.8	0.4096	345.4
0.85	0.4212	335.8
0.9	0.4321	327.4
0.95	0.4423	319.8
1	0.452	313

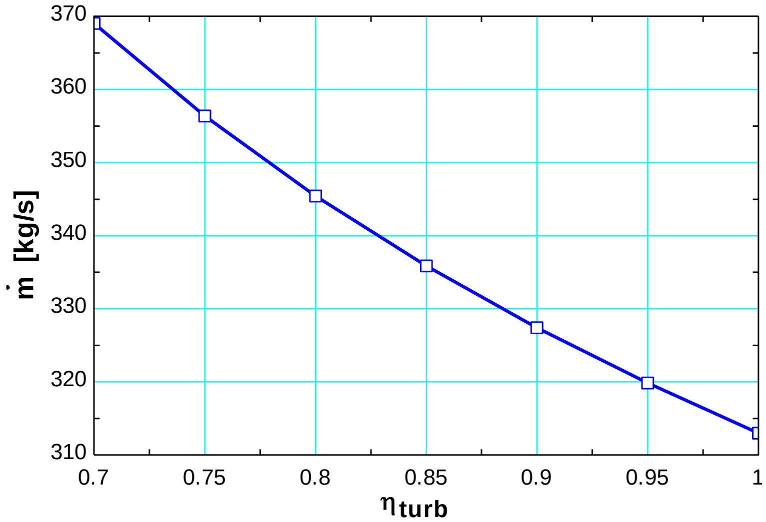
η_{pump}	η_{th}	m [kg/s]
0.7	0.4509	313.8
0.75	0.4511	313.6
0.8	0.4513	313.4
0.85	0.4515	313.3
0.9	0.4517	313.2
0.95	0.4519	313.1
1	0.452	313

P_{cfwh} [kPa]	y	z
100	0.1702	0.05289
200	0.1301	0.09634
300	0.1041	0.1226
400	0.08421	0.1418
500	0.06794	0.1569
600	0.05404	0.1694
700	0.04182	0.1801
800	0.03088	0.1895

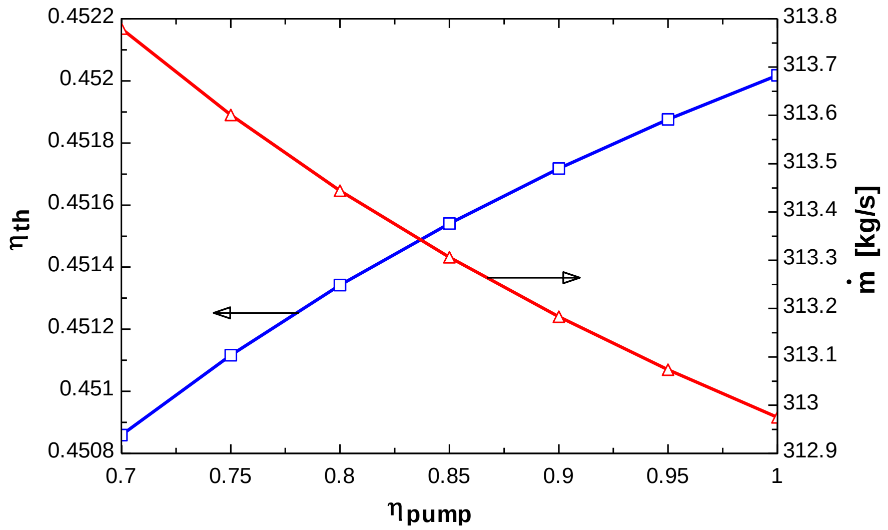
900	0.02094	0.1979
1000	0.01179	0.2054



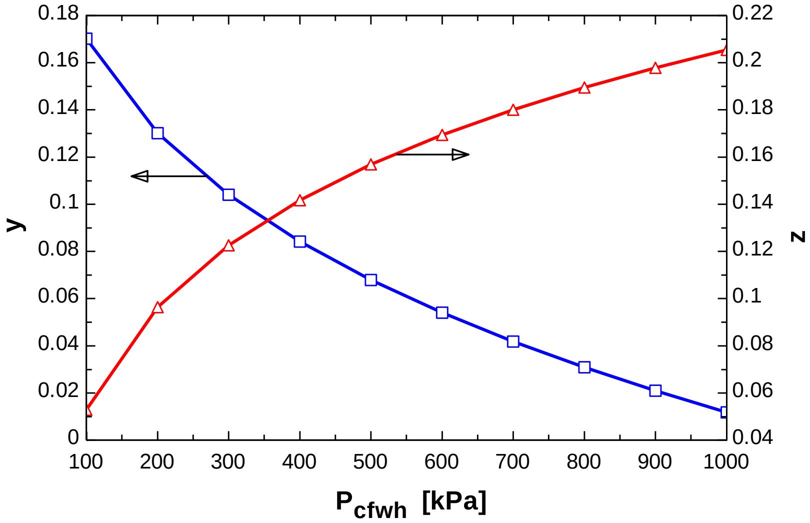
The effect of turbine efficiency on the cycle thermal efficiency.



The effect of turbine efficiency on the mass flow rate.



The effect of pump efficiency on the cycle thermal efficiency.



The effect of closed feed-water heater on y and z.

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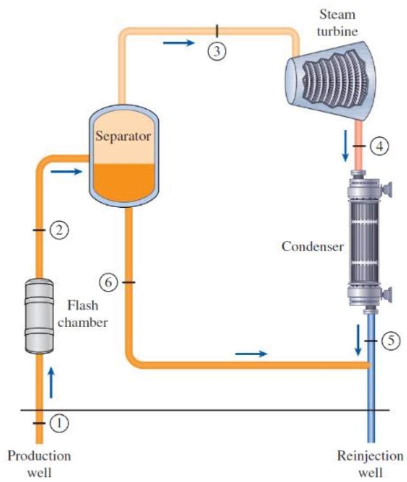
16. The schematic of a single-flash geothermal power plant with state numbers is given in the figure. Geothermal resource exists as saturated liquid at 230 ^{o}C. The geothermal liquid is withdrawn from the production well at a rate of 230 kg/s and is flashed to a pressure of 500 kPa by an essentially isenthalpic flashing process where the resulting vapor is separated from the liquid in a separator and is directed to the turbine. The steam leaves the turbine at 10 kPa with a moisture content of 10 percent and enters the condenser where it is condensed; it is routed to a reinjection well along with the liquid coming off the separator. Determine

(a) Mass flow rate of steam through the turbine,
(b) The isentropic efficiency of the turbine,
(c) The power output of the turbine,
(d) The thermal efficiency of the plant (the ratio of the turbine work output to the energy of the geothermal fluid relative to standard ambient conditions).

[$m_{turbine}$ = 38.2 kg/s, $\eta_{turbine}$ = 68.6%, $W_{turbine}$ = 15410 kW, η_{th} = 7.6%]



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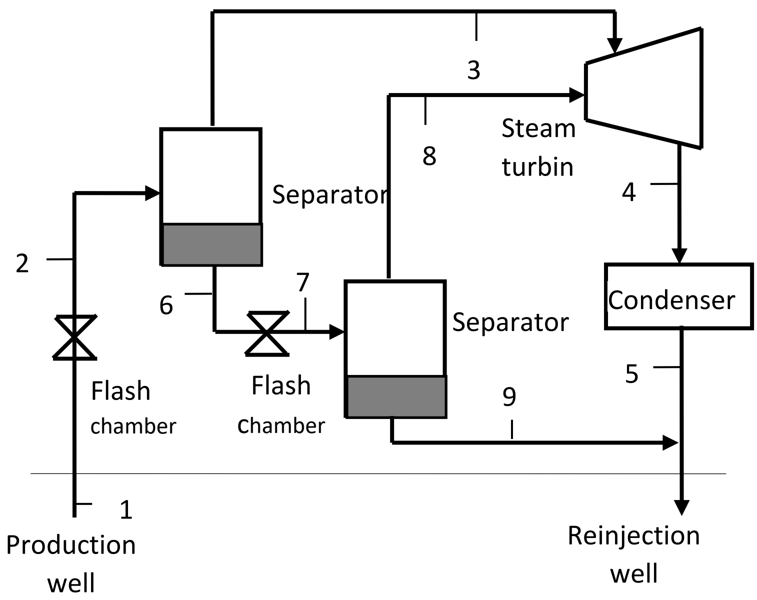
17. Reconsider Prob. 16. Now, it is proposed that the liquid water coming out of the separator be routed through another flash chamber maintained at 150 kPa, and the steam produced be directed to a lower stage of the same turbine. Both streams of steam leave the turbine at the same state of 10 kPa and 90 percent quality. Determine (a) the temperature of steam at the outlet of the second flash chamber, (b) the power produced by the lower stage of the turbine, and (c) the thermal efficiency of the plant.

[T_7 = 11.35 °C, $W_{T2,\,out}$ = 5191 kW, η_{th} = 10.1 %]



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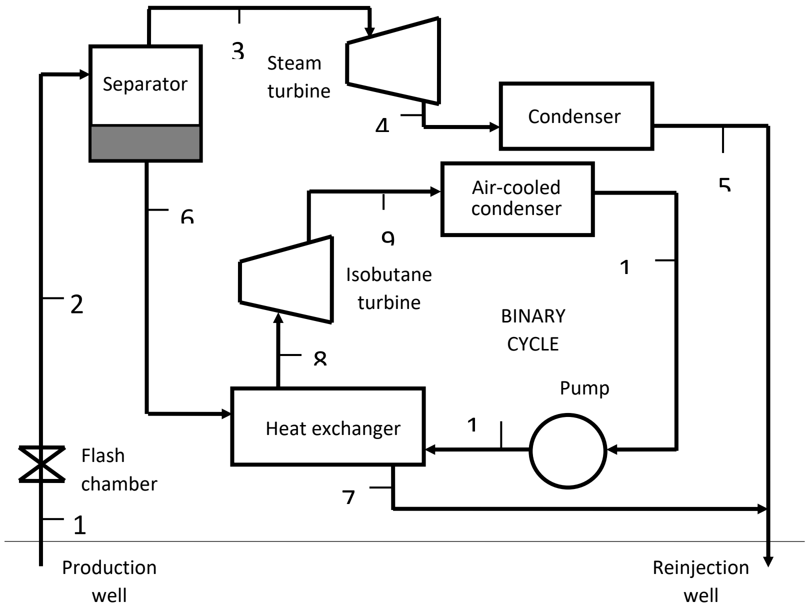
18. Reconsider Prob. 16. Now, it is proposed that the liquid water coming out of the separator be used as the heat source in a binary cycle with isobutane as the working fluid. Geothermal liquid water leaves the heat exchanger at 90°C while isobutane enters the turbine at 3.25 MPa and 145°C and leaves at 80°C and 400 kPa. Isobutane is condensed in an air-cooled condenser and then pumped to the heat exchanger pressure. Assuming an isentropic efficiency of 90 percent for the pump, determine (a) the mass flow rate of isobutane in the binary cycle, (b) the net power outputs of both the flashing and the binary sections of the plant, and (c) the thermal efficiencies of the binary cycle and the combined plant. The properties of isobutane may be obtained from EES.

[$m_{isobutane}$ = 105.46 kg/s, $W_{net, steam}$ = 15410 kW, $W_{net, binary}$ = 6139 kW, η_{th} = 10.6 %]



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3. Boiler and heat transfer

There are many technical solutions for boiler design, depending on fuel type and the circulation principle. For solid fuel based boilers, fuel feeding system is usually the challenging part from a practical point of view. However, for gas fired boilers the fuel handling part is not an issue. Combustion of natural gas is presented in the next chapter, gas turbine. It is more important to design the heat transfer surfaces correctly, since the radiation from gas fuels is much lower than the liquid and solid fuels.

3.1. Boiler

Generally a boiler can be characterized by the hot gas passage and placing of the heat transfer surfaces (i.e. evaporator, super heaters, economizer and the air preheater) in the hot gas passage. What is important here is that there is sufficient material cooling for heat transfer surfaces, which depends on the heat transfer between the exhaust gas and various phases of water-steam. Since highest energy absorption by water-steam takes place during the boiling, the evaporator surfaces are placed at the hottest part of the boiler, i.e. the combustion zone. The super heater surfaces and the economizer are placed in sequence after the evaporator. The schematic figure below shows the boiler with various heat transfer surfaces in a drum based boiler.

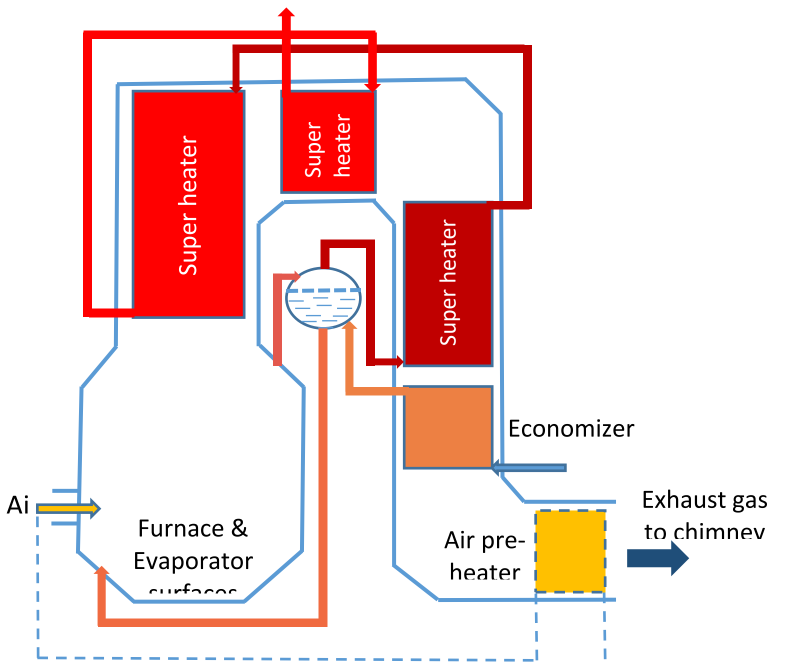


Figure 21: Schematic drawing of a boiler with heat transfer surfaces

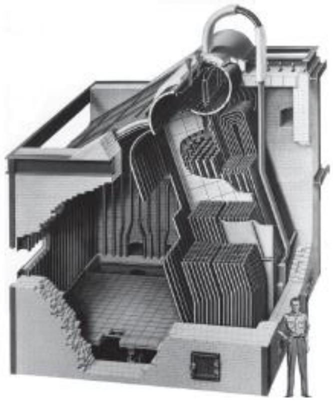


Figure 22: Internally fired boiler, 1933

The drum is a vessel where water and steam are separated after the evaporator, so that dry steam is delivered to the super heaters and the water is recirculated to the evaporator. The drum based boiler enables natural circulation, which is based on the principle of density difference as function of temperature. Since cold water has higher density, the hot water-steam is rising in the tubes causing circulation, as shown in the schematic figure below. The circle indicates cut through of the drum, which is a cylinder shaped pressure vessel.

Figure 23: Schematic figure of natural circulation, and picture of a drum

3.1.1. Heat transfer

In this section a brief presentation of the heat transfer calculations is given for orientation purposes. For detail explanation of the impact of various factors, such as shape and geometry of the heat transfer surface the readers are referred to text books on heat transfer.

Heat transfer through a flat plate can be expressed by following equations and is illustrated by the figure below.

$$\dot{Q} = \dot{m} \cdot \Delta h = \dot{m} \cdot c_p \cdot \Delta t$$

$$\dot{Q} = K \cdot A \cdot \theta_m$$

$$\frac{1}{K} = \frac{1}{\alpha_1} + \frac{1}{\lambda} + \frac{1}{\alpha_2}$$

$$\mathrm{Re} = w \cdot d_i \cdot \rho / \eta$$

$$\mathrm{Pr} = \eta \cdot C_p / \lambda$$

$$Nu = \alpha \cdot d_i / \lambda$$

$$Nu = A \cdot \mathrm{Re}^B \cdot \mathrm{Pr}^C \cdot D$$

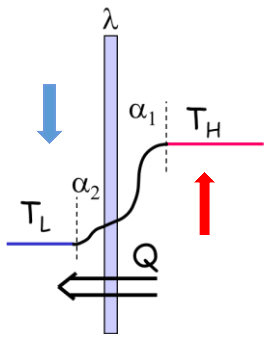


Figure 24: Schematic figure for heat transfer through a flat plate

For turbulent flow in a pipe with warming up water: $Nu = 0.032 \cdot \mathrm{Re}^{0.8} \cdot \mathrm{Pr}^{0.37} \left(\frac{d_i}{L} \right)^{0.054}$

For flow over a tube bundle in Re= 103 to 2.105 the heat transfer coefficient due to convection can be calculated using: $Nu = 0.254 \cdot \mathrm{Re}^{0.632} \cdot \mathrm{Pr}^{0.33}$

$$\theta_m = \frac{(t_{h-in} - t_{c-out}) - (t_{h-out} - t_{c-in})}{ln \frac{(t_{h-in} - t_{c-out})}{(t_{h-out} - t_{c-in})}}$$

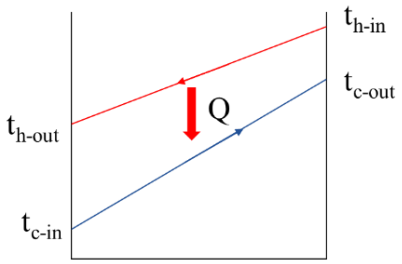


Figure 25: Schematic figure of natural circulation, and picture of a drum

Where α = Heat convection coefficient [W/m2.°C]
λ = Thermal conductivity [W/m .°C]
w = Flow velocity [m/s]
η = Dynamic viscosity [kg/m.s]
ρ = Density [kg/m^3], $\dot{Q}$=Heat flux [kW], $\dot{m}$=mass flow rate [kg/s],
di=inner dimeter [m], θ_m=logarithmic mean temperature
Re: Reynolds number, Pr: Prandtl number, Nu: Nusselt number

The thickness of the boundary layers on both side of flat wall, as well as the type of the fluid on each side plays important role for the effectiveness of the heat transferred through the surface. Generally speaking, boiling water has highest heat absorption capacity, but calculation of the heat transfer coefficients in boiling region is rather complicated. Liquid water is the second best heat absorber followed by steam as coolant. Therefore, the resistance for heat transfer is highest for gas to gas heat exchangers, resulting in big and bulky units. In the figure above it is assumed that hot exhaust gas and water are exchanging heat through a flat plate.

Using the parameters presented in the figure, one can setup the equations needed for heat transfer calculations.

3.1.2. Air pre-heater

As discussed in section 2.3.2.2, steam cycles are designed with a number of pre-heaters. Consequently, the incoming water to the boiler is arriving at higher temperature, which reduces the opportunity for cooling the exhaust gases efficiently. This could result in lower boiler efficiency, if the exhaust gas wouldn't be cooled by an air pre-heating. The combustion air is heated in an air pre-heater after the economizer surfaces (following the exhaust gas flow through the boiler towards the chimney). Higher air temperature means lower fuel consumption to reach the final steam temperature which contributes to improved efficiency. The figure below illustrates the impact of the air pre-heater, using a TS-diagram.

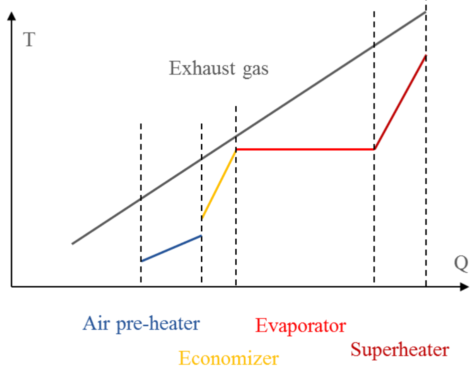


Figure 26: Heat transfer in various parts of the boiler, including air pre-heater

I should be observed that in heat recovery steam generators (HRSG), as will be discussed in combined cycle section, no combustion takes place in HRSG and thereby the number of feed-water pre-heaters are limited to one dearator, where non-condensable gases are separated from the steam-water in a direct contact heat exchanger. The incoming steam and water are at same pressure and leave the component at same temperature.

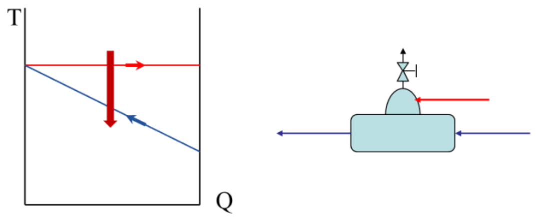


Figure 27: Schematic figure of dearator and the heat transfer process

3.1.3. Problem: Heat Transfer

Learning Objectives:

- Recognize the role of heat exchanger.
- Heat transfer coefficient.
- Obtain a relation for the logarithmic mean temperature difference for use in the LMTD method.
- Effect of heat transfer area.
- Efficiency of the heat exchanger.
- Perform a general energy analysis on heat exchangers.

I. Example 1 & 2 are provided to introduce the concept of one dimensional "heat transfer coefficient" and "heat resistance".

II. Example 3-9 are mostly connected in calculation of heat transfer coefficient and required area for heat transfer as well as required amount of utility to achieve desired condition in heat exchangers.

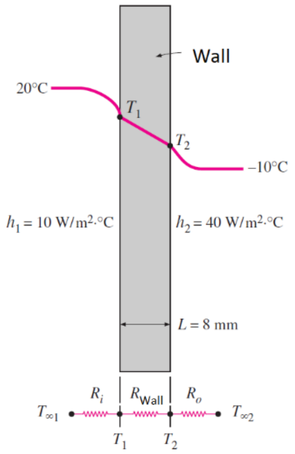
III. Example 10 is "Heat Exchanger fed by geothermal source", besides its EES code for thermodynamic simulation.

****

The concept of heat transfer and heat resistance

1. Consider a 0.8-m-high and 1.5-m-wide wall with a thickness of 8 mm and a thermal conductivity of k = 0.78 W/m · °C. Determine the steady rate of heat transfer through this wall and the temperature of its inner surface fluid is maintained at 20°C while the temperature of the outer surface fluid is -10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be h_1 = 10 W/m2 · °C and h_2 = 40 W/m2 · °C, which includes the effects of radiation.

[Q= 226 W, $T_{inner\ surface}$ = -2.2 °C]



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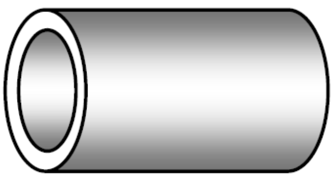
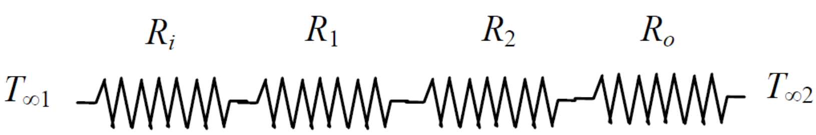
2. Steam at 320°C flows in a stainless steel pipe (k _15 W/m · °C) whose inner and outer diameters are 5 cm and 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation (k _ 0.038 W/m · °C). Heat is lost to the surroundings at 5°C by natural convection and radiation, with a combined natural convection and radiation heat transfer coefficient of 15 W/m2 · °C. Taking the heat transfer coefficient inside the pipe to be 80 W/m2 · °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

[Q=93.9 W, ΔT_{pipe}= 0.095°C, $\Delta T_{insulation}$ = 290°C]





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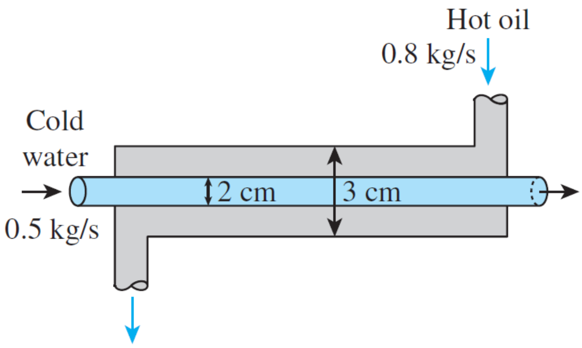
Overall Heat Transfer Coefficient of a Heat Exchanger

3. Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the

shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.
[U= 74.5 W/m^2. K]



Nusselt number for fully developed laminar flow in a circular annulus with one surface insulated and the other isothermal (Kays and Perkins, 1972)

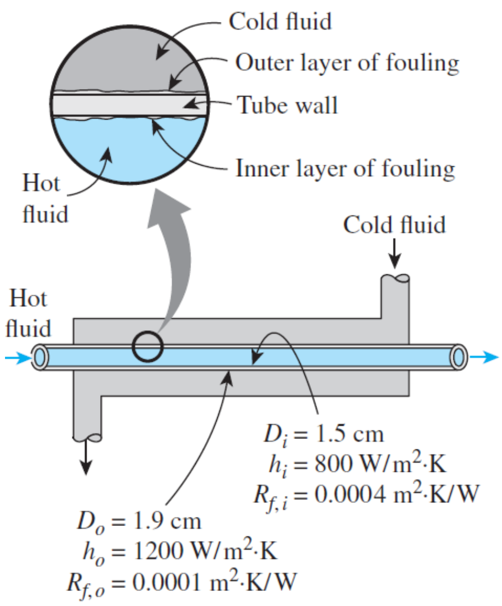
D_i/D_o	Nu_i	Nu_o
0.00	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

****

Effect of Fouling on the Overall Heat Transfer Coefficient

4. A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel (k = 15.1 W/m.K) inner tube of inner diameter D_i = 1.5 cm and outer diameter D_o = 1.9 cm and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be h_i = 800 W/m^2.K on the inner surface of the tube and h_o = 1200 W/m^2.K on the outer surface. For a fouling factor of $R_{f,\,i}$ = 0.0004 m^2.K/W on the tube side and $R_{f,\,o}$ = 0.0001 m^2.K/W on the shell side, determine (a) the thermal resistance of the heat exchanger per unit length and (b) the overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.

[R=0.0532 °C/W, U_i = 399 W/m^2 .K, U_o = 315 W/m^2 .K]



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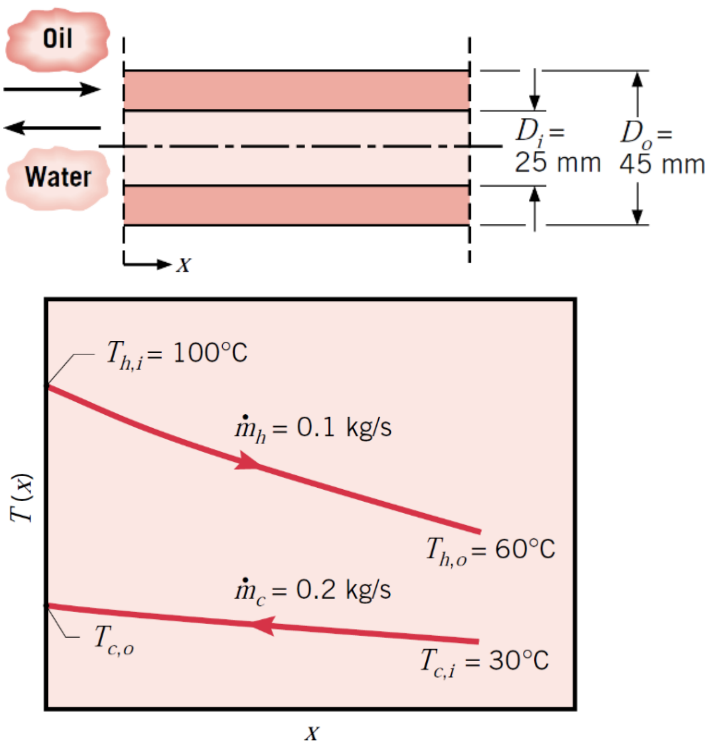
Logarithmic Mean Temperature Difference

5. A counter current concentric tube heat exchanger is used to cool the lubricating oil for a large industrial gas turbine engine. The flow rate of the cooling water through the inner tube (D_i = 25 mm) is 0.2 kg/s, while the flowrate of the oil through the outer annulus (D_o =45 mm) is 0.1 kg/s. the oil and the water are enter at temperatures of 100 °C and 30 °C respectively. How long must the tube be to made if the outlet temperature of the oi; is to be 60 °C?

[L=65.9 m]



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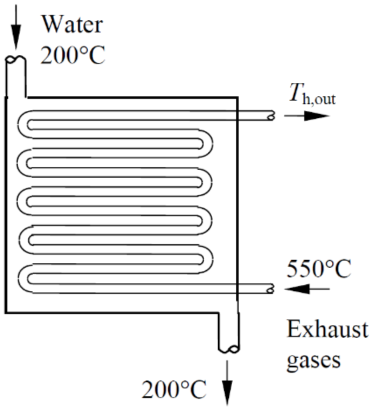
6. Hot exhaust gases of a stationary diesel engine are to be used to generate steam in an evaporator. Exhaust gases (C_p = 1051 J/kg · °C) enter the heat exchanger at 550ºC at a rate of 0.25 kg/s while water enters as saturated liquid and evaporates at 200ºC (h_{fg} = 1941 kJ/kg). The heat transfer surface area of the heat exchanger based on water side is 0.5 m^2 and overall heat transfer coefficient is 1780 W/m^2 · ºC. Determine the rate of heat transfer, the exit temperature of exhaust gases, and the rate of evaporation of water.

[Q= 88.85 kW, $T_{ehaust,out}$ = 211.8°C, m_{eva} = 0.0485 kg/s]



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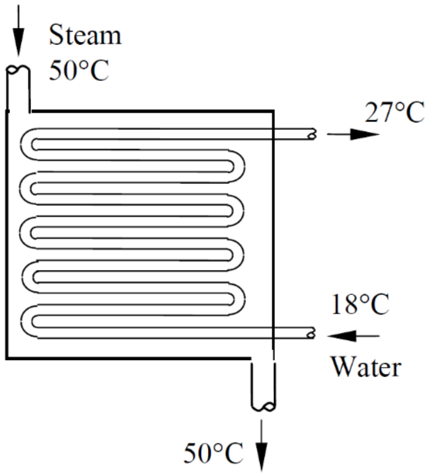
7. Steam in the condenser of a steam power plant is to be condensed at a temperature of 50°C (h_{fg} = 2305 kJ/kg) with cooling water (C_p =4180 J/kg·°C) from a nearby lake, which enters the tubes of the condenser at 18°C and leaves at 27°C. The surface area of the tubes is 58 m^2, and the overall heat transfer coefficient is 2400 W/m^2 · °C. Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.

[$m_{cooling\ water}$ = 73.1 kg/s, $m_{condensation}$ =1.15 kg/s]



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8. Water at the rate of 3.783 kg/s is heated the temperature of 55 °C in a shell-and-tube heat exchanger. On the shell side one pass is used with water as the heating fluid, 1.892 kg/s, entering the exchanger at 93.33 °C. The overall heat-transfer coefficient is 1419 W/m2 · °C, and the average water velocity in the 1.905-cm diameter tubes is 0.366 m/s. the efficiency of the heat exchanger is reduced to 60% due to fouling phenomenon. Because of space limitations, the tube length must not be longer than 8 ft [2.438 m]. Calculate the minimum allowable temperature of the cold stream entering to the heat exchanger, the number of passes, the number of tubes, and the length of the tubes, consistent with this restriction.

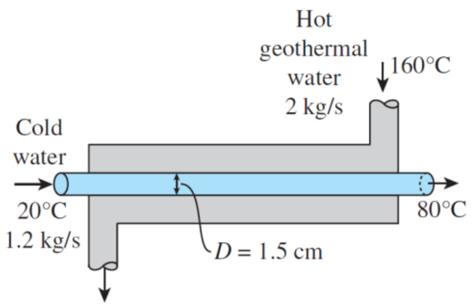
[T_{min} =37.78 °C, N_{pass}=2, N_{tubes} =36, L_{tubes} =1.646 m]

****

Heating Water in a Counter-Flow Heat Exchanger

9. A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m^2 .K, determine the length of the heat exchanger required to achieve the desired heating.

[L=109 m]



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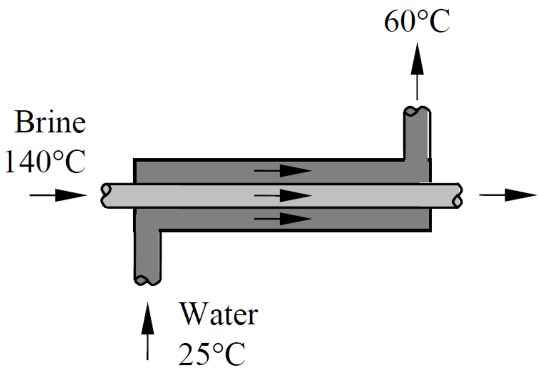
10. A double-pipe parallel-flow heat exchanger is to heat water (cp = 4180 J/kg.K) from 25°C to 60°C at a rate of 0.2 kg/s. The heating is to be accomplished by geothermal water (cp = 4310 J/kg.K) available at 140°C at a mass flow rate of 0.3 kg/s. The inner tube is thin-walled and has a diameter of 0.8 cm. If the overall heat transfer coefficient of the heat exchanger is 550 $W/m^2.K$, determine the length of the tube required to achieve the desired heating.

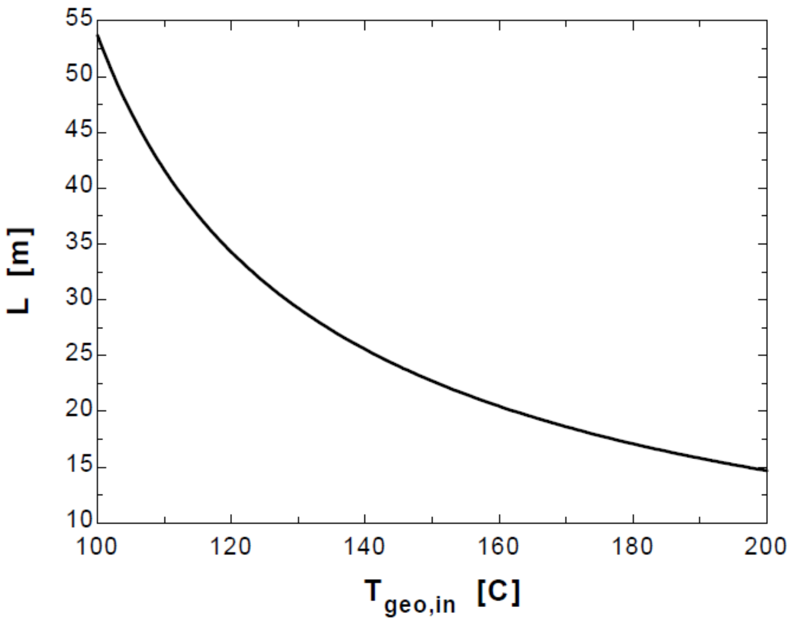
[L=25.5 m]



The problem is solved using EES, and the solution is given below.

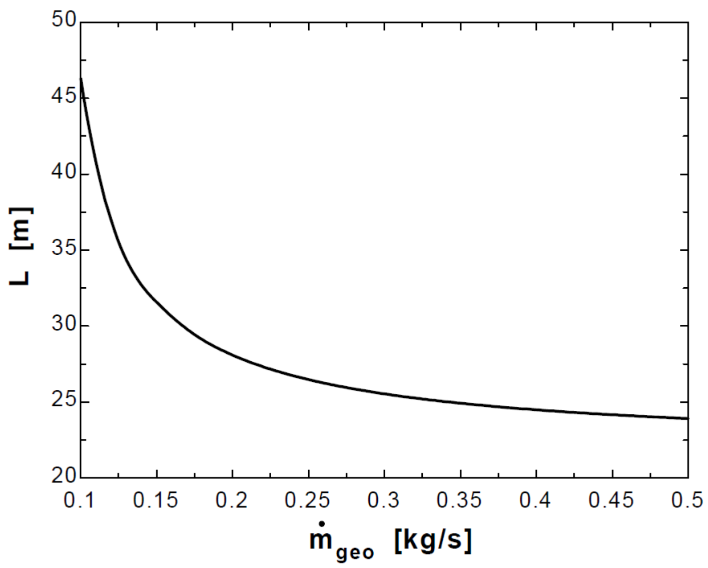
```
"GIVEN"
T_w_in=25 [C]
T_w_out=60 [C]
m_dot_w=0.2 [kg/s]
C_p_w=4.18 [kJ/kg-C]
T_geo_in=140 [C]
m_dot_geo=0.3 [kg/s]
C_p_geo=4.31 [kJ/kg-C]
D=0.008 [m]
U=0.55 [kW/m^2-C]

"ANALYSIS"
Q_dot=m_dot_w*C_p_w*(T_w_out-T_w_in)
Q_dot=m_dot_geo*C_p_geo*(T_geo_in-T_geo_out)
DELTAT_1=T_geo_in-T_w_in
DELTAT_2=T_geo_out-T_w_out
DELTAT_lm=(DELTAT_1-DELTAT_2)/ln(DELTAT_1/DELTAT_2)
Q_dot=U*A*DELTAT_lm
A=pi*D*L
```

$T_{geo,in}$ [C]	L [m]
100	53.73
105	46.81
110	41.62
115	37.56
120	34.27
125	31.54
130	29.24
135	27.26
140	25.54
145	24.04
150	22.7
155	21.51
160	20.45
165	19.48
170	18.61
175	17.81
180	17.08
185	16.4
190	15.78
195	15.21
200	14.67



The effect of geothermal temperature on the required length for heat transfer

m_{geo} [kg/s]	L [m]
0.1	46.31
0.125	35.52
0.15	31.57
0.175	29.44
0.2	28.1
0.225	27.16
0.25	26.48
0.275	25.96
0.3	25.54
0.325	25.21
0.35	24.93
0.375	24.69
0.4	24.49
0.425	24.32
0.45	24.17
0.475	24.04
0.5	23.92



The effect of geothermal mass flow rate on the required length for heat transfer

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4. The gas cycle

Gas turbines are fast starting, compact energy conversion technologies, utilizing many advanced technologies to maintain high performance and lifetime. Since the compressor takes in air from the atmosphere and the exhaust gas from the expander is rejected to the atmosphere, the gas turbine cycle is called open cycle or air breading machine.

The figure below shows schematic of a simple gas turbine, consisting of a compressor, combustion chamber and expander (also called turbine). Air at atmospheric pressure is compressed to high-pressure level in the compressor. Mixture of fuel and air is combusted in the combustion chamber generating hot exhaust gas at high pressure, which is expanded in the turbine, generating mechanical work (as rotating shaft) that drives the compressor and the surplus is converted to electricity via a generator.

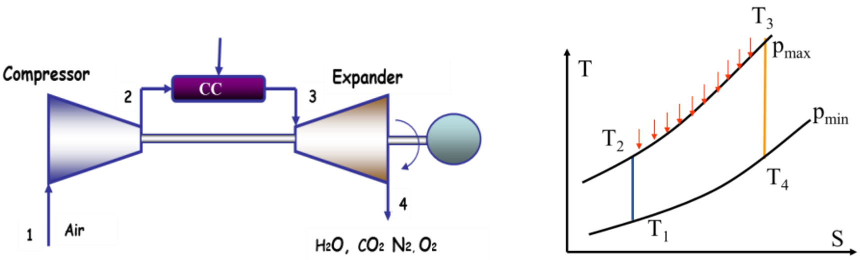


Figure 28: Ideal gas turbine cycle with 100% compressor and expander efficiency (ΔS=0)

4.1. Gas properties

Different from the steam cycle which has a well-defined working fluid, the working fluid in both compressor (air with changing properties such as humidity, pressure and temperature) and expander (exhaust gas composition is affected by amount and composition of fuel, as well as amount of cooling air) are changing by load and environmental conditions. Therefore, system analysis for gas cycles requires calculation of thermodynamic properties of the working fluid for each specific setting. This will be discussed more in detail in the chapter on combustion, where the calculation of thermodynamic properties of gas mixtures will be carried out in an example.

The basic thermodynamic relations needed for simple calculations are listed below:

Total enthalpy can be expressed as $h_0 = c_p \cdot T_0$

And also as sum of the static and dynamic part $h_0 = h + \frac{c^2}{2}$

For a perfect gas, the static enthalpy can be written as $h = c_p \cdot T$

Therefore the total temperature can be expressed as $T_0 = \frac{h_0}{c_p} = T + \frac{c^2}{2c_p}$

These equations are used to explain changes in fluid properties through the turbomachinery stage. As one can see the relation between temperature and enthalpy can be considered as a constant multiplier. Therefore, system analysis for gas turbine cycles are usually presented in TS-diagrams only.

4.2. Simple ideal gas turbine cycle

The simple open gas turbine cycle of figure X above is presented in the TS-diagram:

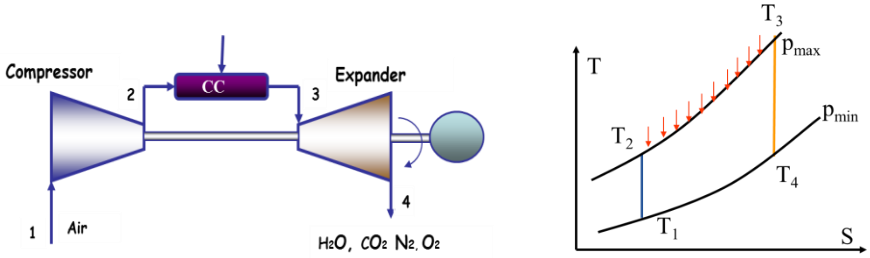


Figure 29: Ideal gas turbine cycle with 100% compressor and expander efficiency (ΔS=0)

The assumptions for the ideal gas turbine presented above are no pressure loss, no heat loss, 100% component's efficiency, constant mass flow (neglecting the fuel mass flow), negligible change in kinetic energy and constant c_p-value for the working fluids between inlet and outlet of the components.
Using the control volume principle and the basic equations presented above one can express the energy exchange per mass flow rate as change of temperature over the components. Using the numbers in the figure above as index, we can write:

Compressor: $W_c = T_2 - T_1$

Combustion chamber: $Q_{cc} = T_3 - T_2$

Turbine (Expander): $W_t = T_4 - T_3$

Using the definition of cycle efficiency : $\eta = \frac{Net\ work\ output}{Heat\ input} = \frac{(T_4 - T_3) - (T_2 - T_1)}{T_3 - T_2}$

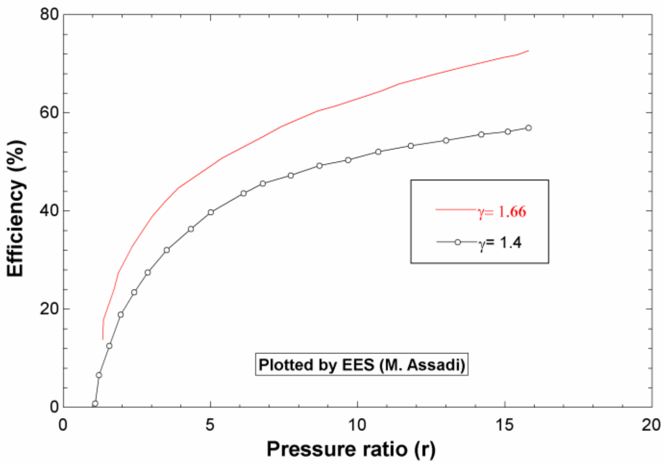
Introducing the pressure ratio as $\pi = p_{max}/p_{min}$ and the $\gamma = c_p/c_v$ we can express the cycle efficiency as (see appendix 1 for driving the final expression):

$$\eta = 1 - \frac{1}{\pi^{(\gamma-1)/\gamma}}$$

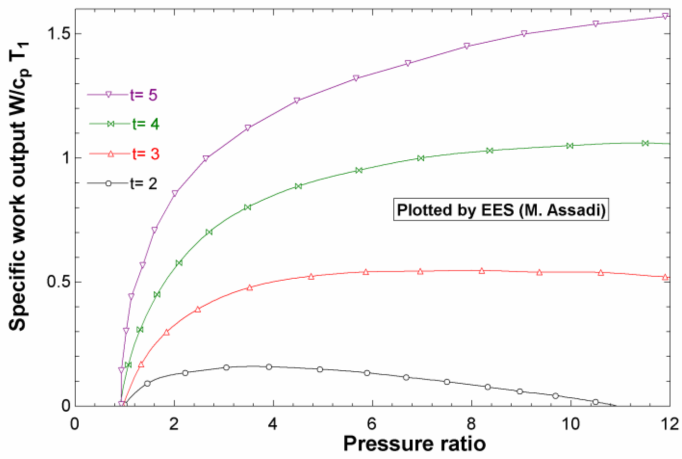
The specific work, using the temperature ration $T_3/T_1 = t$, can be expressed as:

$$\frac{W}{c_p \cdot T_1} = t\left(1 - \frac{1}{\pi^{(\gamma-1)/\gamma}}\right) - \left(\pi^{(\gamma-1)/\gamma} - 1\right)$$

Parameter variation for pressure ratio and the temperature ratio shows the performance variation as function of π with t as parameter (π=1-12 and t=1-5).



(a)



(b)

Figure 30: Efficiency and specific work output

The figure shows that for an ideal cycle, efficiency is increasing with increasing π. For specific power, which gives an indication about plant size versus power output, there is an optimum value for each t and certain pressure ratio.

4.3. Problems: Gas Power Cycle

Learning Objectives:

- Solve problems based on the Brayton cycle; the Brayton cycle with regeneration.
- Net work output and thermal efficiency of the cycle.
- Net Power Generation and cycle's Thermal Efficiency.

V. Examples 1- 4 are mostly connected to each other and review the "Brayton cycle" by the following purposes:

8. The Simple Ideal Brayton Cycle.
9. An Actual Gas Power Cycle.
10. An Actual Gas Turbine with Regeneration
11. A Gas Turbine with Reheating and Intercooling

VI. Examples 5 and 6 are classified and dispersed examples from the above-mentioned subjects.

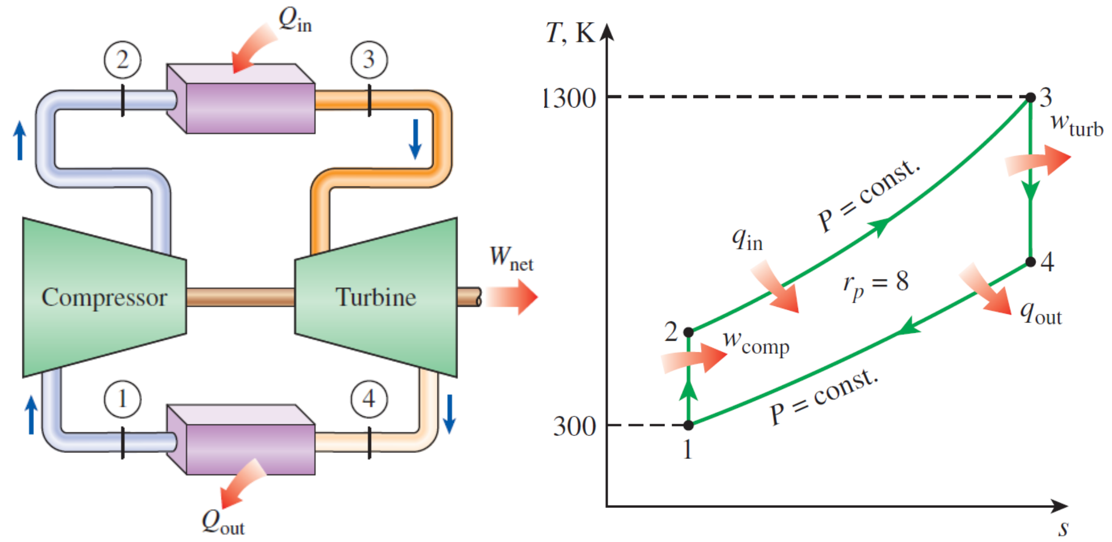
VII. Example 7 is "An Actual Gas Turbine with Regeneration", besides its EES code for thermodynamic simulation.

****

The Simple Ideal Brayton Cycle

1. A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet. Utilizing the air-standard assumptions, determine (a) the gas temperature at the exits of the compressor and the turbine, (b) the back work ratio, and (c) the thermal efficiency.

[T_2 = 540 K, T_4 = 770 K, r_{bw} = 0.403, η_{th} = 42.6%]



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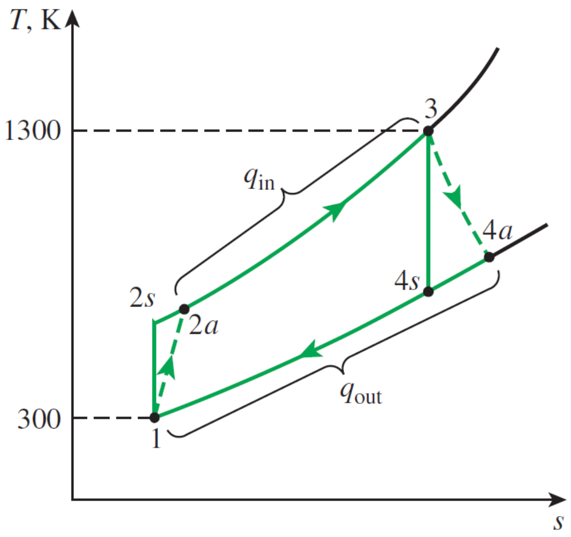
An Actual Gas-Turbine Cycle

2. Assuming a compressor efficiency of 80 percent and a turbine efficiency of 85 percent, determine (a) the back work ratio, (b) the thermal efficiency, and (c) the turbine exit temperature of the gas-turbine cycle discussed in previous example.

[r_{wb} = 0.592, η_{th} = 26.6 %, T_4 = 853 K]



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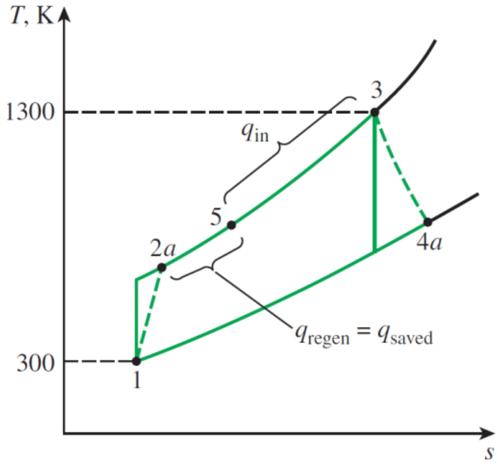
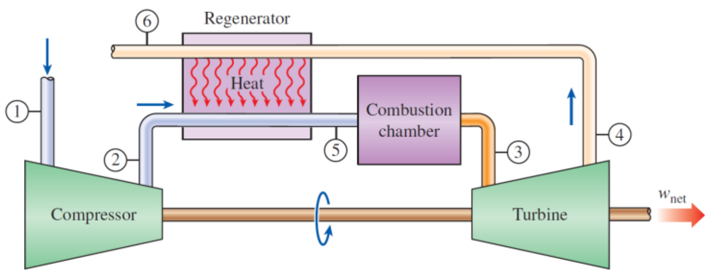
Actual Gas-Turbine Cycle with Regeneration

3. Determine the thermal efficiency of the gas-turbine described in previous if a regenerator having an effectiveness of 80 percent is installed.

   [η_{th} =36.9 %]





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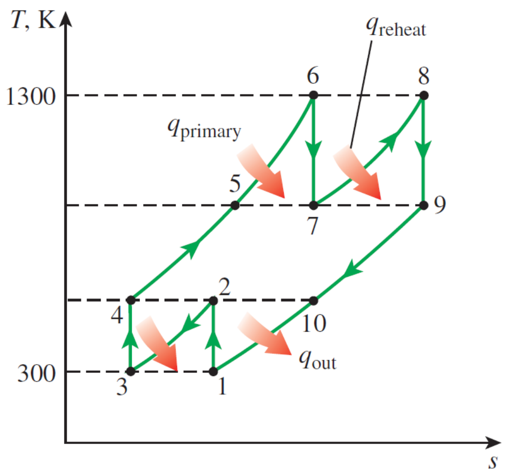
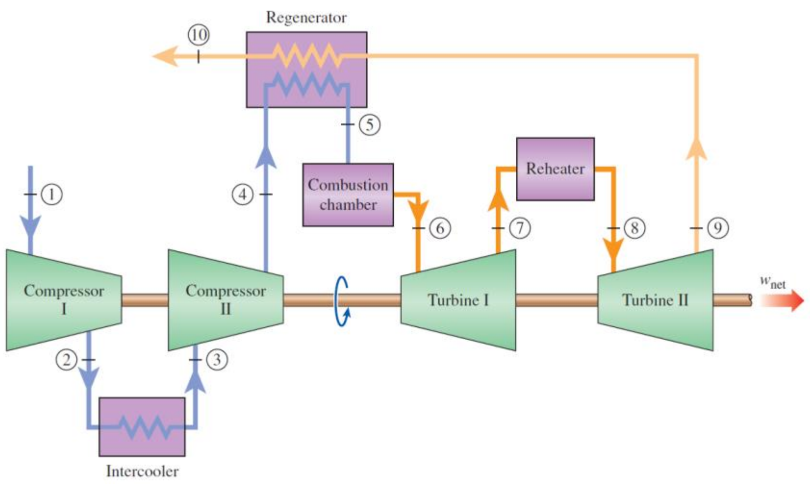
A Gas Turbine with Reheating and Intercooling

4. An ideal gas-turbine cycle with two stages of compression and two stages of expansion has an overall pressure ratio of 8. Air enters each stage of the compressor at 300 K and each stage of the turbine at 1300 K. Determine the back work ratio and the thermal efficiency of this gas-turbine cycle, assuming (a) no regenerators and (b) an ideal regenerator with 100 percent effectiveness. Compare the results with those obtained in previous example.

   [$r_{bw,1}$ = 0.304, $\eta_{th,1}$ = 35.8%, $r_{bw,2}$ = $r_{bw,1}$, $\eta_{th,2}$ = 69.6 %]





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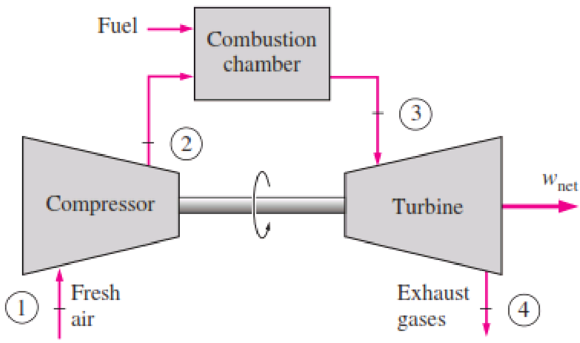
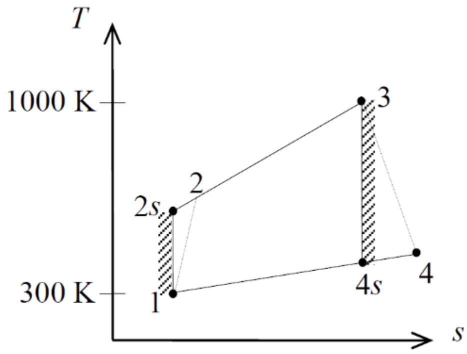
The Simple Ideal Brayton Cycle

5. Air is used as the working fluid in a simple ideal Brayton cycle that has a pressure ratio of 12, a compressor inlet temperature of 300 K, and for the specific fuel composition, the air flue ratio is 82 (the air fuel ratio vs. turbine inlet temperature is shown in the diagram below) (and a turbine inlet temperature of 1000 K). Determine the required mass flow rate of air for a net power output of 70 MW, assuming both the compressor and the turbine have an isentropic efficiency of (a) 100 percent and (b) 85 percent. Assume constant specific heats at room temperature.

[m_{ideal} = 352 kg/s, m_{actual} = 1037 kg/s]





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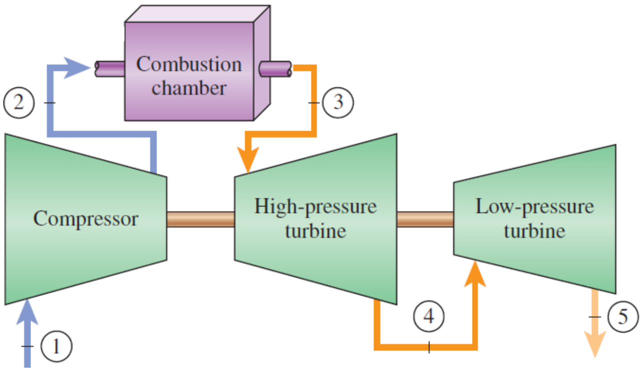
6. A gas-turbine power plant operates on a modified Brayton cycle shown in the figure with an overall pressure ratio of 8. Air enters the compressor at 0°C and 100 kPa. The air fuel ratio for that specific fuel adding into the combustion chamber is 47 (The maximum cycle temperature is 1500 K). The compressor and the turbines are isentropic. The high pressure turbine develops just enough power to run the compressor. Assume constant properties for air at 300 K with c_v = 0.718 kJ/kg. K, c_p = 1.005 kJ/kg. K, R = 0.287 kJ/kg. K, k = 1.4. (a) Sketch the T-s diagram for the cycle. Label the data states. (b) Determine the temperature and pressure at state 4, the exit of the high pressure turbine. (c) If the net power output is 200 MW, determine mass flow rate of the air into the compressor, in kg/s.

[T_4 = 1278.5 K, P_4 = 457.3 kPa, m_{air} = 441.8 kg/s]



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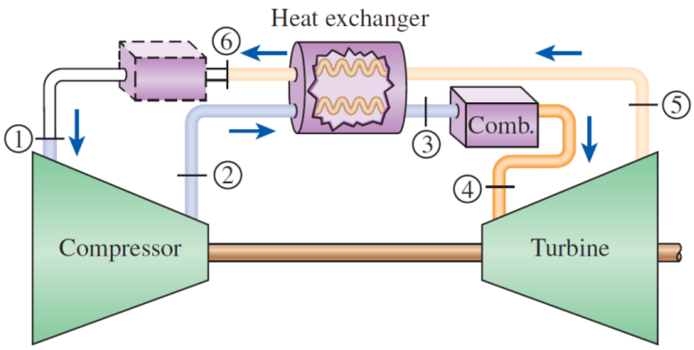
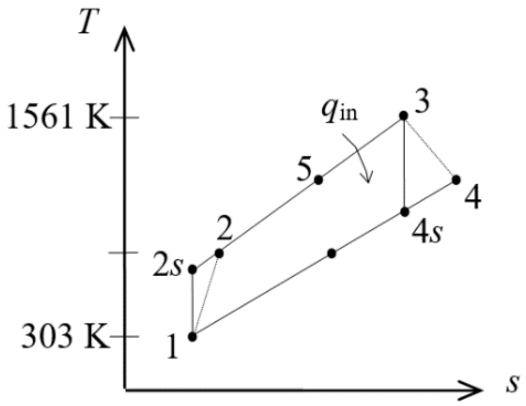
Actual Gas-Turbine Cycle with Regeneration

7. The 7FA gas turbine manufactured by General Electric is reported to have an efficiency of 35.9 percent in the simple-cycle mode and to produce 159 MW of net power. The pressure ratio is 14.7 and the turbine inlet temperature is 1288°C. The mass flow rate through the turbine is 1,536,000 kg/h. Taking the ambient conditions to be 30°C and 100 kPa, determine the isentropic efficiency of the turbine and the compressor. Also, determine the thermal efficiency of this gas turbine if a regenerator with an effectiveness of 65 percent is added.

   [$\eta_{turbine}$ = 83.8 %, $\eta_{compressor}$ = 94.9 %, η_{th} = 44.1 %]





A solution that allows different isentropic efficiencies for the compressor and turbine is to be developed and the effect of the isentropic efficiencies on net work done and the heat supplied to the cycle is to be studied. Also, the *T-s* diagram for the cycle is to be plotted.

EES Code:

```
"Input data"
T[3] = 1288 [C]
Pratio = 14.7
T[1] = 30 [C]
P[1]= 100 [kPa]
{T[4]=659 [C]}
{W_dot_net=159 [MW] }"We omit the information about the cycle net work"
m_dot = 1536000 [kg/h]*Convert(kg/h,kg/s)
{Eta_th_noreg=0.359} "We omit the information about the cycle efficiency."
Eta_reg = 0.65
Eta_c = 0.84 "Compressor isentropic efficiency"
Eta_t = 0.95 "Turbien isentropic efficiency"
```

```
"Isentropic Compressor anaysis"
s[1]=ENTROPY(Air,T=T[1],P=P[1])
s_s[2]=s[1] "For the ideal case the entropies are constant across the compressor"
P[2] = Pratio*P[1]
s_s[2]=ENTROPY(Air,T=T_s[2],P=P[2])
"T_s[2] is the isentropic value of T[2] at compressor exit"
Eta_c = W_dot_compisen/W_dot_comp
"compressor adiabatic efficiency, W_dot_comp > W_dot_compisen"

"Conservation of energy for the compressor for the isentropic case:
E_dot_in - E_dot_out = DELTAE_dot=0 for steady-flow"
m_dot*h[1] + W_dot_compisen = m_dot*h_s[2]
h[1]=ENTHALPY(Air,T=T[1])
h_s[2]=ENTHALPY(Air,T=T_s[2])

"Actual compressor analysis:"
m_dot*h[1] + W_dot_comp = m_dot*h[2]
h[2]=ENTHALPY(Air,T=T[2])
s[2]=ENTROPY(Air,T=T[2], P=P[2])

"External heat exchanger analysis"
"SSSF First Law for the heat exchanger, assuming W=0, ke=pe=0
E_dot_in - E_dot_out =DELTAE_dot_cv =0 for steady flow"
m_dot*h[2] + Q_dot_in_noreg = m_dot*h[3]
q_in_noreg=Q_dot_in_noreg/m_dot
h[3]=ENTHALPY(Air,T=T[3])
P[3]=P[2]"process 2-3 is SSSF constant pressure"

"Turbine analysis"
s[3]=ENTROPY(Air,T=T[3],P=P[3])
s_s[4]=s[3] "For the ideal case the entropies are constant across the turbine"
P[4] = P[3] /Pratio
s_s[4]=ENTROPY(Air,T=T_s[4],P=P[4])"T_s[4] is the isentropic value of T[4] at turbine exit"
Eta_t = W_dot_turb /W_dot_turbisen "turbine adiabatic efficiency, W_dot_turbisen > W_dot_turb"

"SSSF First Law for the isentropic turbine, assuming: adiabatic, ke=pe=0
E_dot_in -E_dot_out = DELTAE_dot_cv = 0 for steady-flow"
m_dot*h[3] = W_dot_turbisen + m_dot*h_s[4]
h_s[4]=ENTHALPY(Air,T=T_s[4])

"Actual Turbine analysis:"
m_dot*h[3] = W_dot_turb + m_dot*h[4]
h[4]=ENTHALPY(Air,T=T[4])
s[4]=ENTROPY(Air,T=T[4], P=P[4])

"Cycle analysis"
"Using the definition of the net cycle work and 1 MW = 1000 kW:"
W_dot_net*1000=W_dot_turb-W_dot_comp "kJ/s"
Eta_th_noreg=W_dot_net*1000/Q_dot_in_noreg"Cycle thermal efficiency"
Bwr=W_dot_comp/W_dot_turb"Back work ratio"

"With the regenerator the heat added in the external heat exchanger is"
m_dot*h[5] + Q_dot_in_withreg = m_dot*h[3]
q_in_withreg=Q_dot_in_withreg/m_dot

h[5]=ENTHALPY(Air, T=T[5])
```

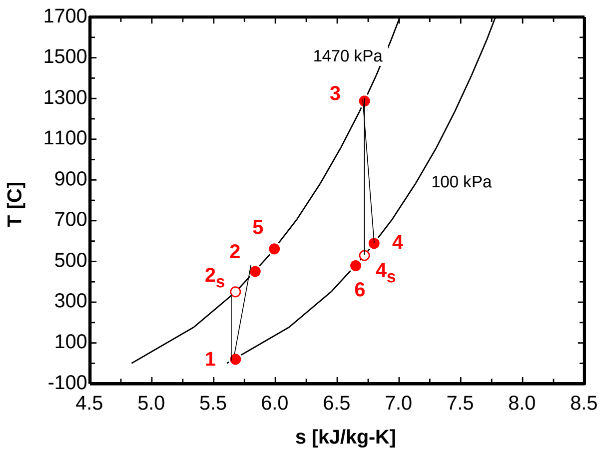
s[5]=ENTROPY(Air,T=T[5], P=P[5])
P[5]=P[2]

"The regenerator effectiveness gives h[5] and thus T[5] as:"
Eta_reg = (h[5]-h[2])/(h[4]-h[2])
"Energy balance on regenerator gives h[6] and thus T[6] as:"
m_dot*h[2] + m_dot*h[4]=m_dot*h[5] + m_dot*h[6]
h[6]=ENTHALPY(Air, T=T[6])
s[6]=ENTROPY(Air,T=T[6], P=P[6])
P[6]=P[4]

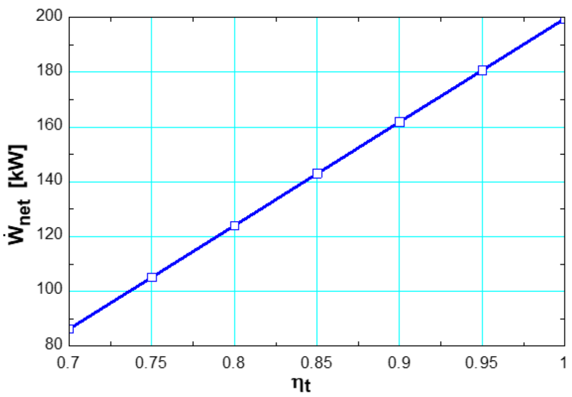
"Cycle thermal efficiency with regenerator"
Eta_th_withreg=W_dot_net*1000/Q_dot_in_withreg

"The following data is used to complete the Array Table for plotting purposes."
s_s[1]=s[1]
T_s[1]=T[1]
s_s[3]=s[3]
T_s[3]=T[3]
s_s[5]=ENTROPY(Air,T=T[5],P=P[5])
T_s[5]=T[5]
s_s[6]=s[6]
T_s[6]=T[6]

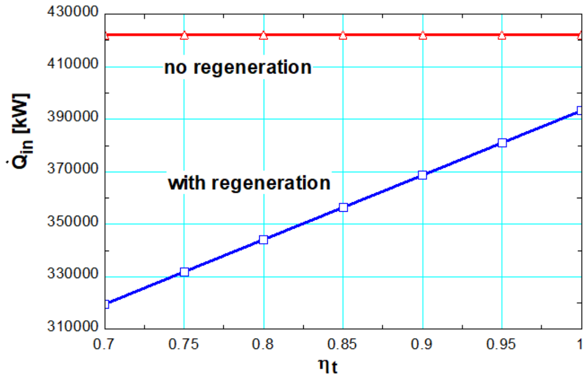
η_t	η_c	$\eta_{th,noreg}$	$\eta_{th,withreg}$	$Q_{innoreg}$ [kW]	$Q_{inwithreg}$ [kW]	W_{net} [kW]
0.7	0.84	0.2044	0.27	422152	319582	86.3
0.75	0.84	0.2491	0.3169	422152	331856	105.2
0.8	0.84	0.2939	0.3605	422152	344129	124.1
0.85	0.84	0.3386	0.4011	422152	356403	142.9
0.9	0.84	0.3833	0.4389	422152	368676	161.8
0.95	0.84	0.4281	0.4744	422152	380950	180.7
1	0.84	0.4728	0.5076	422152	393223	199.6



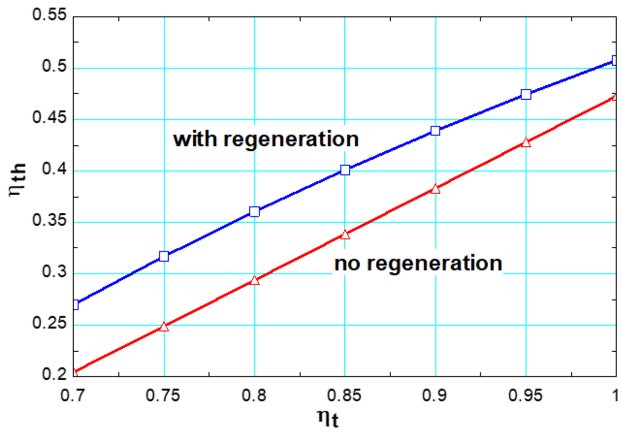
T-s diagram for gas turbine with regeneration



The effect of turbine efficiency on net work out in two cases



The effect of turbine efficiency on heat input to the cycle in two cases



The effect of turbine efficiency on thermal efficiency of the cycle in two cases

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4.4. Real gas turbine cycle and stage calculation

Introducing imperfection, which takes place in real gas turbine cycles, such as component efficiencies less than 100%, pressure losses, change of mass flow between compressor and turbine due to fuel flow to the combustion chamber, as well as change of c_p, as function of temperature, one can note that efficiency as well as specific power will drop considerably. However, the trends are the same, i.e. both efficiency and specific power increases with increasing pressure ration and turbine inlet temperature (TIT) T_3. It can also be observed that TIT has the largest impact, and therefore gas turbine development during the past decades have been focused on increasing the TIT. Given the fact that there is a temperature limit imposed by the material, the art of material cooling has been developed towards perfection, to maintain the material temperature below the limit, although the hot gas temperature (TIT) has been increased.

In the following sections component modeling and analysis for industrial gas turbine are explained, using the concept of repeating stages, which means the velocity triangle at inlet of one stage is the same as the velocity triangle at the outlet of the previous stage. Since the properties and velocities are changing from the root to the tip of the blade, all presented analysis are for the mean diameter $(D_t+D_r)/2$.

4.4.1. Axial compressor

The air compressor increases the air pressure, using the power input via the shaft to accelerate the air stream and convert the kinetic energy to pressure through the diffuser shaped blade channels. The figure below illustrates the difference between compressor (diffuser) and turbine (nozzle) blading.

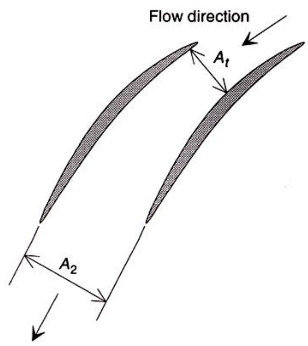


Figure 31: Outlet area of the blade passage is larger than the inlet area, resulting in velocity reduction

Using the continuity equation, assuming constant density for simplicity, one can see that the increasing are (diffuser effect) results in velocity reduction, so that kinetic energy is converted to pressure. $\dot{m} = \rho \dot{v} = \rho \cdot c_1 \cdot A_1 = \rho \cdot c_2 \cdot A_2$

Assuming constant density, we can write: $\frac{A_2}{A_1} = \frac{c_1}{c_2}$

Since the flow moves towards higher pressure, which is the opposite to the rule of nature, there is always a risk for flow reversal and instable operation, called surge. Growing boundary layer around the blades results in reduction of flow passage area, which can result in flow separation, called rotating stall, which can develop to full surge. Large forces are released during surge that can cause serious damage to the gas turbine.

As we have seen in the previous chapter on steam turbine, the change of properties of the working fluid can be explained using the velocity triangles at inlet and outlet of the rotor and stator blades. The figure below shows the connection between velocity and various thermodynamic properties of the flow, such as absolute and static enthalpy, pressure, etc. As can see in the figure, the absolute velocity is increased between inlet and outlet of the rotor blades (flow is accelerated), due to energy input via compressor shaft. This results in increased total enthalpy over the rotor. Over the stator, there is no work input so h_0 remains constant. Same reasoning applies to the total pressure too, however, due to friction losses over the stator blades certain pressure drop takes place as shown in the figure. The static pressure is increased in both rotor and stator passage, due to diffuser effect.

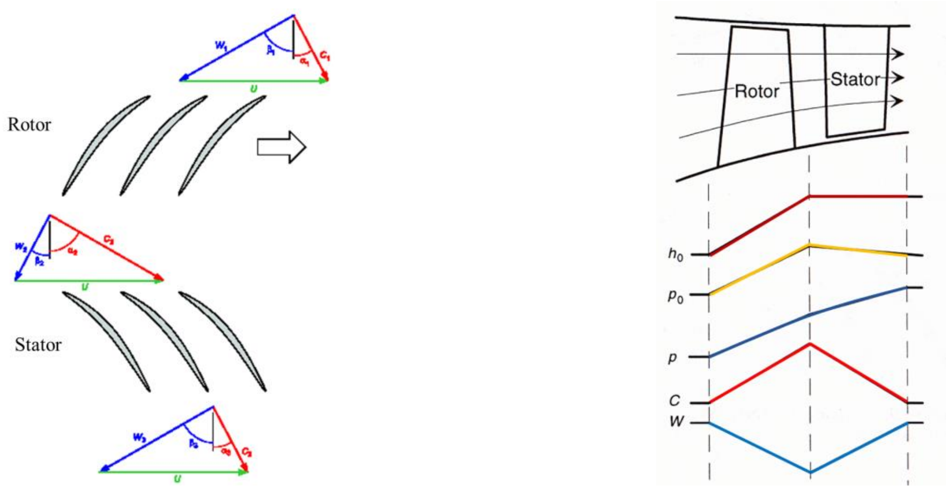


Figure 32: Axial compressor stage

Same analysis can be applied, illustrating the compression in a TS-diagram. Two settings, namely total and static properties are shown in the TS-diagram, where the basic relations presented in the beginning of this chapter can be used to explains the change of properties over the compressor stage. As one can see the difference between total and static properties are the dynamic values, dictated by changes in the flow velocities.

$$T_0 = \frac{h_0}{c_p} = T + \frac{c^2}{2c_p}$$

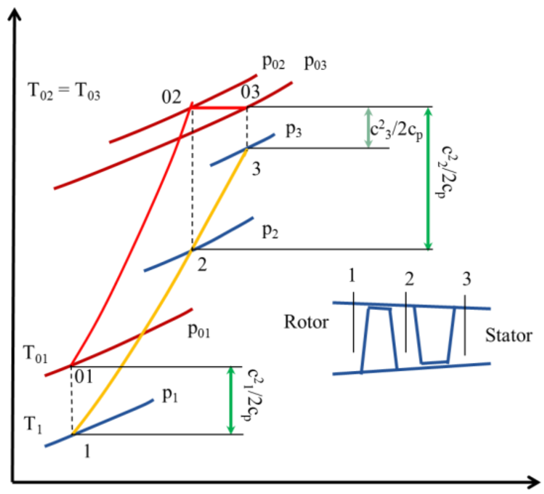


Figure 33: Changes in flow properties over the compressor stage illustrated in a TS-diagram

Stage performance can be analyzed expressing the total temperature change of the stage:

$$\Delta T_{0-stage} = T_{03} - T_{01} = T_{03} - T_{02} = u \cdot c_a(tan\beta_1 - tan\beta_2)$$

Where c_a is the axial component of the absolute velocity, representing the mass flow; u is the blade speed and $(tan\beta_1 - tan\beta_2)$ is the flow deflection, indicating the level of diffusion. To maximize the energy transfer between the blade and the working fluid, one can try to maximize these three parameters. However, there are physical and practical limitation to how much one can increase these values. The blade speed is limited due to mechanical load on the blades, the axial component of the absolute velocity and thereby the mass flow is limited due to size considerations and the turning of the flow, the diffusion, is limited due to stability concerns. For details on mathematical presentation of compressor modeling based on repeated stage theory, the readers are referred to standard gas turbine textbooks.

4.4.1.1. Compressor stability

Compressors are working against the law of nature, i.e. delivering the low pressure working fluid at high pressure at the compressor outlet. Many factors can disturb the flow, resulting in revers flow (following the law of nature, i.e. flow from high to low pressure area) and compressor instability. To have a general understanding of how these instabilities develop within the compressor stage and propagate through the component the short presentation below is provided. For more detail readers are referred to gas turbine text books such as gas turbine theory and introduction to turbomachinery.

There are many parameters, such as mass flow, pressure, temperature, and other properties of the working fluid, which changes between inlet and outlet of the compressor. To provide general analysis tools that help engineers with a general overview over the engine behavior, various dimensionless parameters have been derived. Dimensionless pressure and flow parameters, denoted by ϕ and ψ are useful parameters for analysis of compressor stability.

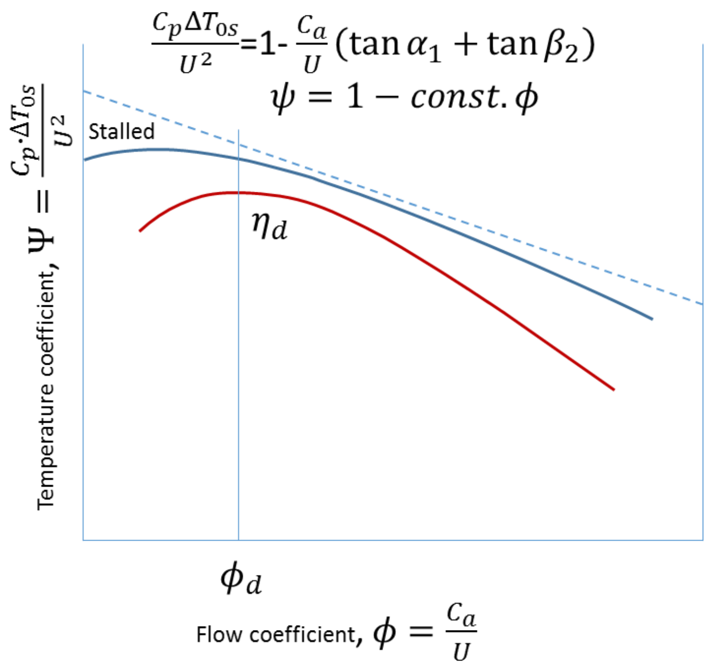


Figure 34: Compressor stability

The ideal relationship between ψ and ϕ over a stage, which is a first degree equation, is represented by a striate line in the figure. However, at higher and lower flows than the design flow deviation between the ideal and real case increases, which is indicated by the blue curve in the figure above. The design cased efficiency η_d, represented by the red curve, which is at its maximum at design flow drops and the risk for instability increases.

4.4.2. Expander (Turbine)

Design of the expander or turbine is a challenging issue, due to high temperatures, high loads, cooling and lifetime of the expensive parts in the turbine. Turbine blade channels are shaped as nozzles, i.e. the outlet area of the channel is smaller than the inlet area. This results in an accelerating flow, and thereby no flow stability issues in the turbine. However, the high temperature of the working fluid in combination with high mechanical load on the turbine blades require implementation of sophisticated cooling technologies, which increases the cost of the engine parts. The stage analysis for the turbine stage is shown in the TS-diagram below. The flow passes through the nozzle blades before entering the rotor blades. The flow is expanded and accelerated in the nozzles from c_1 to c_2, as shown in the figure. The absolute velocity decelerates through the rotor blade passage and the gas expands so that relative velocity increases $w_3 > w_2$.

$$u/_{c_a} = \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3$$

Applying angular momentum to the rotor one can write the stage work per unit mass flow as:

$$w_s = \mathrm{u}(c_{w2} + c_{w3} = \mathrm{u} \cdot c_a(tan\alpha_2 + \tan\alpha_3) = \mathrm{u} \cdot c_a(tan\beta_2 + \tan\beta_3)$$

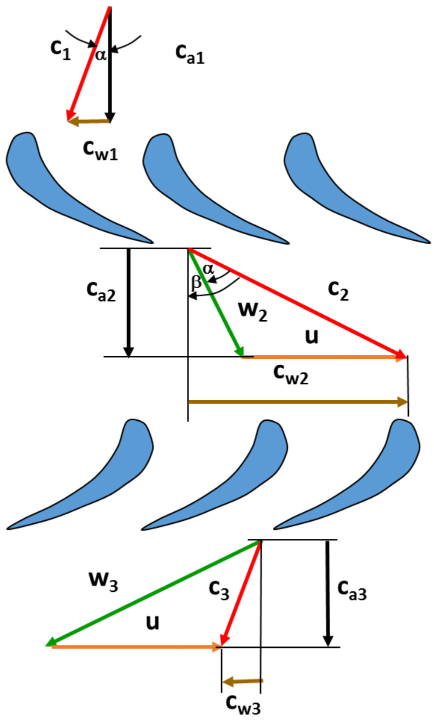


Figure 35: Velocity triangles for the turbine stage

	Stator	**Rotor**
Static pressure	Decrease	Decrease
Total pressure	Small reduct.	Decrease
Static temperature	Decrease	Decrease
Total temperature	Constant	Decrease
Relative velocity	N/A	Increase
Absolute velocity	Increase	Decrease
Entalpy	Constant	Decrease
Density	Decrease	Decrease

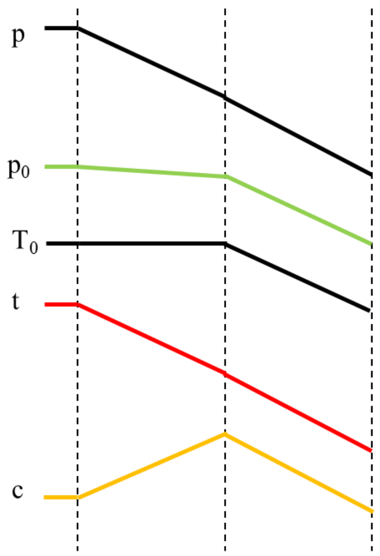


Figure 36: Change of properties of the working fluid

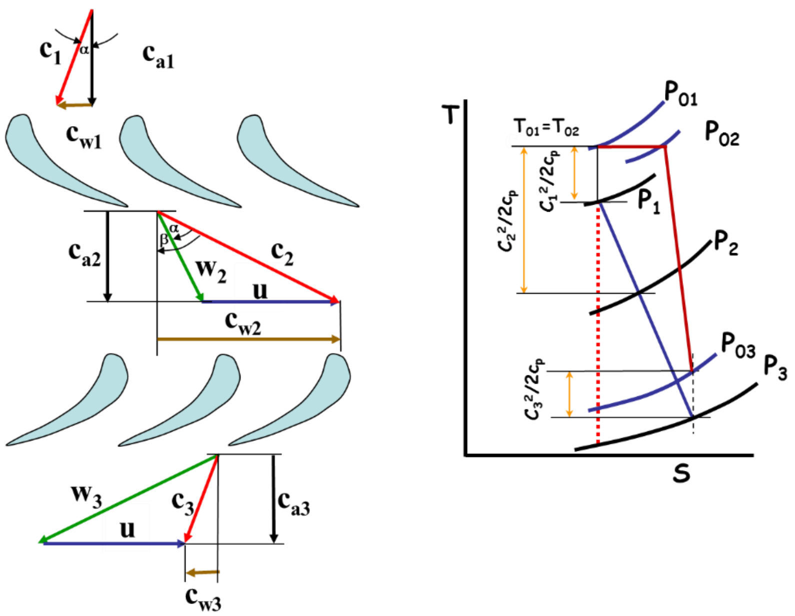
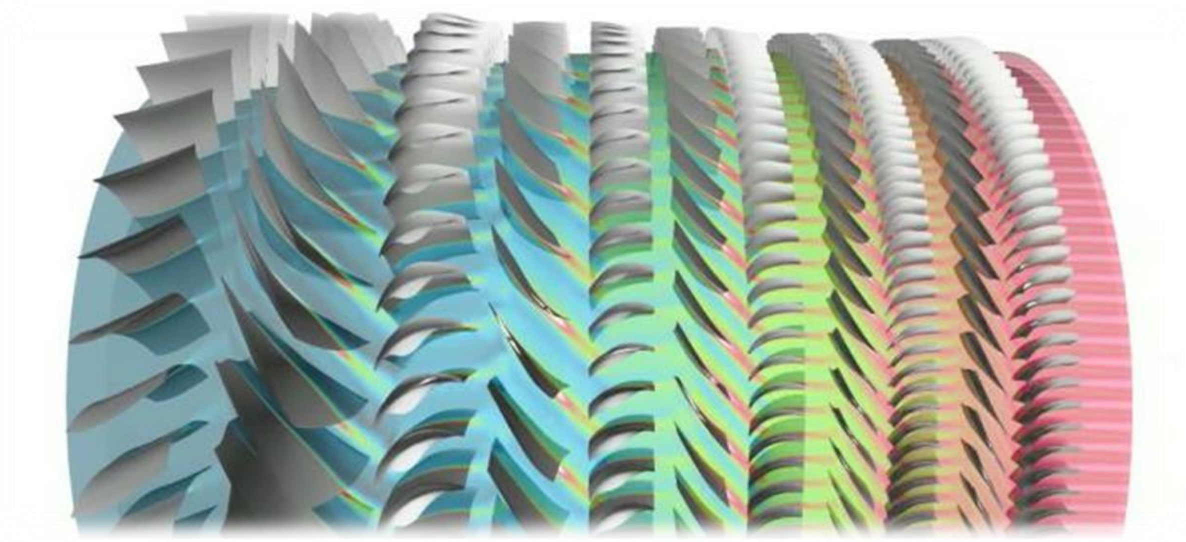


Figure 37: Expansion in TS diagram

4.4.3. Problem: Stage Calculation

Learning objectives:

- Shaft power calculation per mass unit.
- Understanding the concept of velocity triangles.
- Stage temperature change calculation.
- Property change through the stage of a turbomachinery.



I. Example 1 & 2 are concerned about the property change over a stage of a ***compressor*** which is expressed by the help of velocity triangles.
II. Example 3 & 4 are connect to the basic relations regarding to stage calculation of the turbomachinery.
III. Example 5-7 practice the above mentioned learning objectives for a ***turbine***.

****

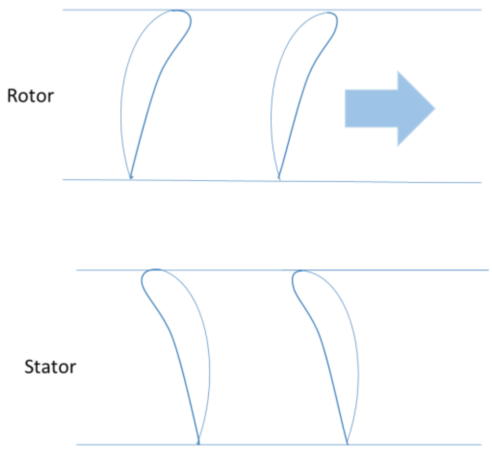
Compressor

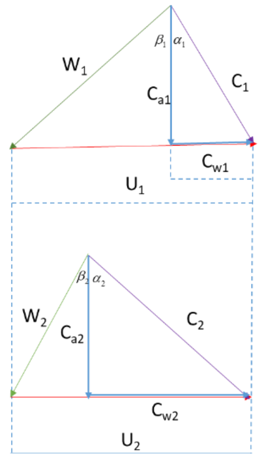
1. An axial flow compressor stage has blade root, mean and tip velocities of 150 m/s, 200 m/s and 250 m/s respectively. The stage is designed for the stagnation temperature rise of 20 K and the axial velocity of 150 m/s, both constant from root to tip. The net work done factor is 0.93. By assuming 50% reaction at the mean radius, please calculate the stage air angles at root, mean and tip and degree of reaction at root and tip for a free vortex design.

[α_1 = 17.07° (=β_2), β_1 = 45.73° (=α_2) at mean radius; α_1 = 13.77°, β_1 = 54.88°, β_2 = 40.23°, α_2 = 39.43° at tip; α_1 = 22.25°, β_1 = 30.60°, β_2 = -20.25°, α_2 = 53.85° at root; Λ = 11.2 per cent at root and 67.4 per cent at tip]

Solution

Objective: The problem aims to show how to analyse the flow velocity through a stage by velocity triangles at root, tip and mean radius of the blade and additionally provides a method about how to calculate the degree of reaction for turbo machinery.





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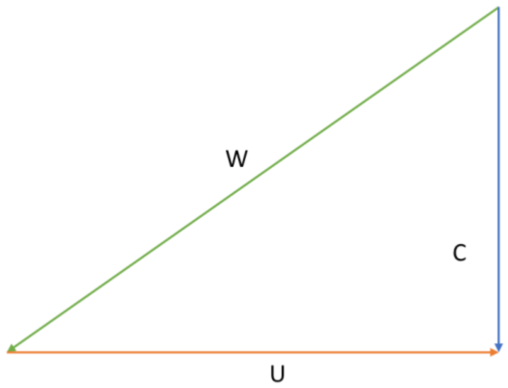
2. The first stage of an axial compressor has no inlet guide vanes. The theoretical speed is 6000 rev/min and the stagnation temperature rise is 20 K. the hub-tip ratio is 0.60, the work done factor is 0.93 and the isentropic efficiency of the stage is 0.89. The inlet velocity is considered 140 m/s, calculate:
   1) The tip radius, if the Mach number relative to tip is limited to 0.95;
   2) The mass flow entering the stage;
   3) The stage stagnation pressure ratio;
   4) The rotor air angles at the root and tip sections;

[(*a*) 0.456 m, 63.95° and 56.40°, (*b*) 65.5 kg/s, (*c*) 1.233, 1317 kW, (*d*) 50.83° and 18.32°]



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3. The conditions of air at the entry of an axial flow compressor stage are P_1= 100 kPa and T_1= 300 K. the air angles are β_1 = 51° and β_2 =10° and $\alpha_1 = \alpha_2$ = 8°. The mean diameter and peripheral speeds are 0.5 m and 150 m/s respectively. The mass flowrate through the stage is set for 30 kg/s and the work done factor is 0.95 and mechanical efficiency is 90%. Assuming an isentropic stage efficiency of 85%, determine:
    1) Blade height at entry.
    2) Stage pressure ratio.
    3) The power required to drive the stage.

[h=0.15 m, R_s = 1.17, P_{stage} = 551 kW]

****

4. The preliminary design of an axial flow compressor is to be based upon a simplified consideration of the mean diameter conditions. Suppose that the characteristics of a repeating stage of such a design are as follows:
   - ✓ Stagnation temperature rise (ΔT) = 30K
   - ✓ Flow number ($\frac{C_a}{U}$) = 0.5
   - ✓ Degree of reaction (R) = 0.6
   - ✓ Blade speed (U) = 300 m/s

Assuming constant axial velocity across the stage and equal absolute velocities at inlet and outlet, determine the blade angles of the rotor for a shock free flow ($C_{p,\,air}$ = 1005 J/kg. K)

[β_1 =56.92°, β_2= 40.86°]

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Turbine:

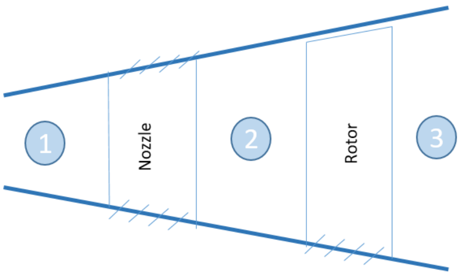
5. A mean-diameter design of a turbine stage having equal inlet and outlet velocities leads to the following data:

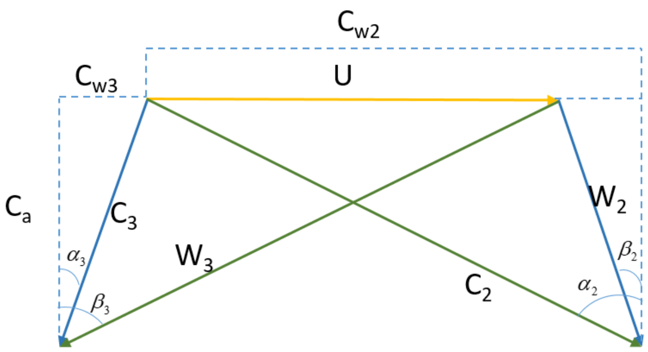
- ✓ Mass flow *m*= 20 kg/s
- ✓ Inlet temperature T_{01} =1000 K
- ✓ Inlet pressure P_{01} =4.0 bar
- ✓ Axial velocity (constant through stage) C_a =260 m/s
- ✓ Blade speed U =360 m/s
- ✓ Nozzle efflux angle (α_2) =65°
- ✓ Stage exit swirl (α_3) = 10°

Determine the rotor blade gas angles, degree of reaction, temperature drop coefficient ($2c_p\Delta T_{0s}/U^2$) and power output.

Assuming a nozzle loss coefficient λ_N of 0.05, calculate the nozzle throat area required (ignoring the effect of friction on the critical conditions).

[37.15°, 57.37°, 0.29, 3.35, 4340 kW, 0.0398 m^2]





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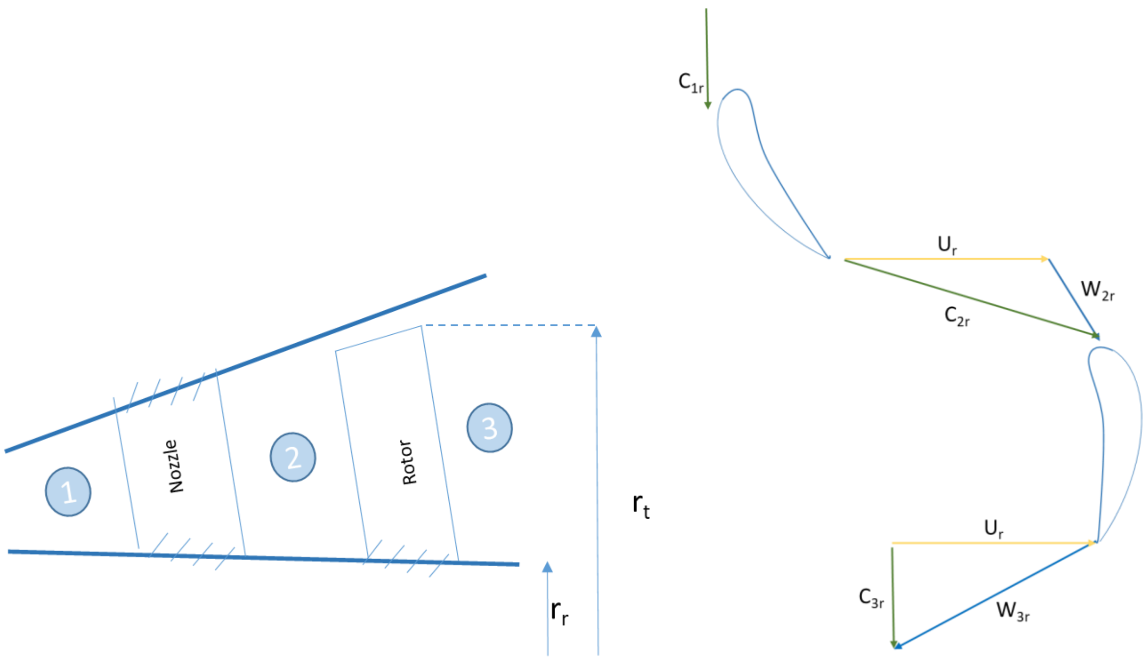
6. The following particulars relate to a single-stage turbine of free vortex design.
    - ✓ Inlet temperature T_{01} = 1050 K
    - ✓ Inlet pressure p_{01} = 3.8 bar
    - ✓ Pressure ratio P_{01}/P_{03} = 2.0
    - ✓ Outlet velocity C_3 = 275 m/s
    - ✓ Blade speed at root radius U= 300 m/s
    - ✓ Turbine isentropic efficiency η= 0.88

    The turbine is designed for zero reaction (Λ= 0) at the root radius, and the velocities at inlet and outlet (C_1 and C_3) are both equal and axial. Calculate the nozzle efflux angle α_2 and blade inlet gas angle β_2 at the root radius. If the tip/root radius ratio of the annulus at exit from the nozzle blades is 1.4, determine the nozzle efflux angle and degree of reaction at the tip radius.

    [61.15°, 40.23°, 52.25°, 0.46; 1.72 and 1.64 bar]



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7. The following data apply to a single-stage turbine designed on free vortex theory.

- ✓ Mass flow, 36 kg/s
- ✓ Inlet temperature T_{01}, 1200 K
- ✓ Inlet pressure P_{01}, 8.0 bar
- ✓ Temperature drop ΔT_{0s}
- ✓ Isentropic efficiency η_t, 0.90
- ✓ Mean blade speed U_m, 320 m/s
- ✓ Rotational speed N, 250 rev/s
- ✓ Outlet velocity C_3, 400 m/s

The outlet velocity is axial. Calculate the blade height and radius ratio of the annulus from the outlet conditions.
The turbine is designed with a constant annulus area through the stage, i.e. with no flare. Assuming a nozzle loss coefficient λ_N of 0.07, show that continuity is satisfied when the axial velocity at exit from the nozzles is 346 m/s. Thence calculate the inlet Mach number relative to the rotor blade at the root radius.

[0.0591 m, 1.34; 0.81]

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4.4.3.1. Turbine cooling

Gas turbine efficiency is a strong function of the turbine inlet temperature (TIT) and a weaker function of pressure ratio (PR). However, there is optimum combination of TIT and PR for various applications and design.

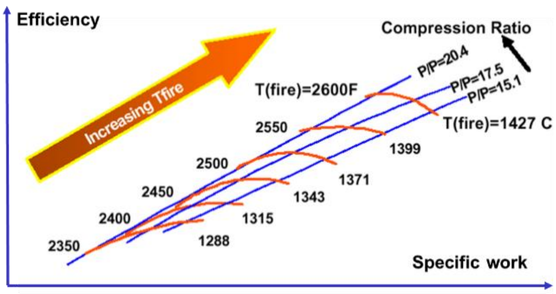


Figure 38: optimized combination of TIT and PR for combined cycle

The high temperature alloys available for gas turbines cannot withstand higher material temperatures than ca. 1000 C. To allow for TIT of about 1500 C, which is the current technology level, one needs to implement sophisticated material cooling technologies so that despite the high gas temperatures at the turbine inlet, the material temperature doesn't exceeds the maximum allowable temperature.

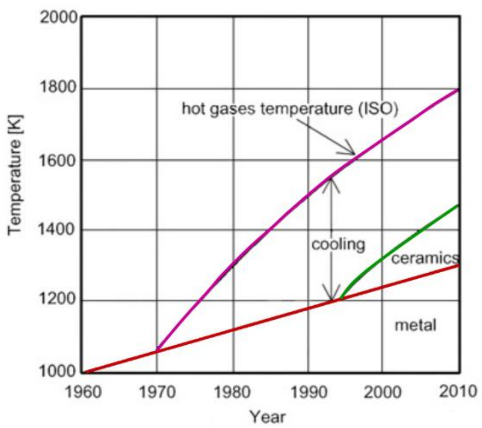


Figure 39: Development of TIT and impact of various technologies

As shown in the figure above, increased TIT is effect of combination of different technologies. Both material development, metal substrates and ceramic thermal barrier coatings (TBC), and most importantly material cooling technologies have contributed to continuous increasing TIT by time.

Using advanced cooling techniques results in complex geometries and thereby expensive components. To maintain safe operation different monitoring techniques are employed. The figure below illustrates this.

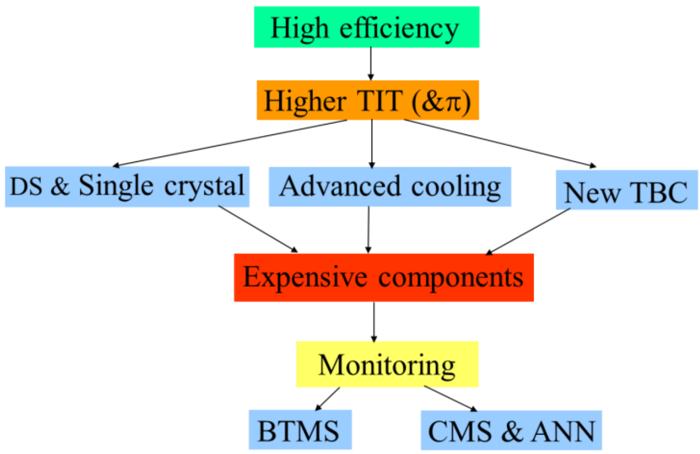


Figure 40: Reaching high efficiency and safe operation require optimized combination of TIT and PR, material cooling and monitoring techniques

There are different definitions for TIT used by various actors. The figure below illustrates different TIT definitions. The lower temperature along the hot gas flow at various sections is due the fact that the hot gas expands and mixes with cooling air.

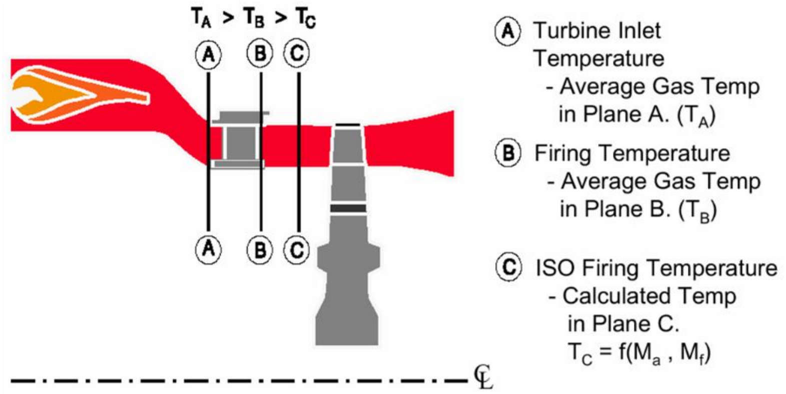


Figure 41: Various TIT definitions

The figure below illustrates the various cooling techniques:

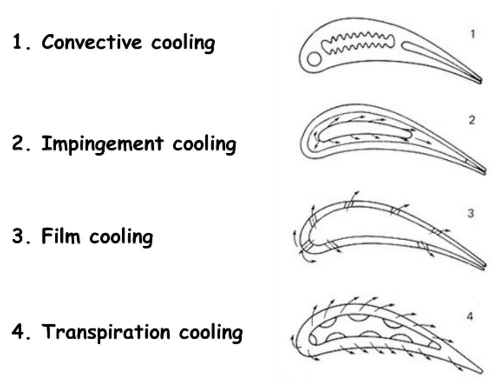


Figure 42: material cooling techniques

In practical applications combination of the three first techniques, i.e. convective cooling (the cooling air is cooling the material during passage through the cooling channels. Growth of the boundary layer between material and cooling flow, due to friction, reduces the heat transfer), impingement cooling (the cooling air flows through small holes building jets that hits the metal surface to break up the boundary later and thereby improve the heat transfer),and film cooling (the cooling air flows through small holes of the blade surface, building thin layer of cooling flow on the blade surface, isolating the blade from direct contact with the hot gas), are applied to achieve the TIT levels of modern gas turbines.

The figure below illustrates the complexity of the blade cooling in modern gas turbines:

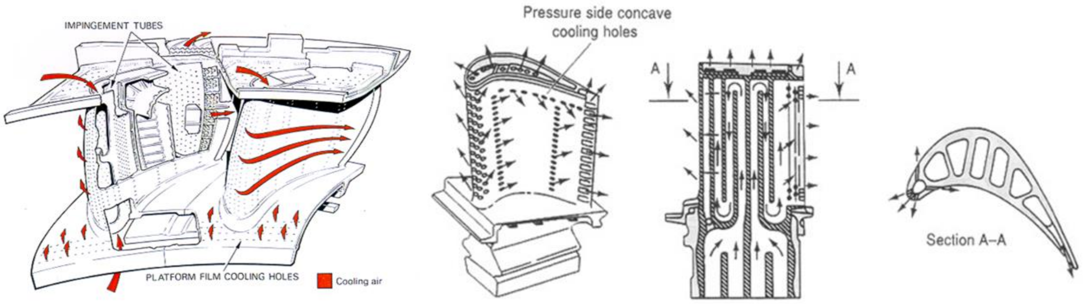


Figure 43: Illustration of nozzle and blade cooling combining various cooling techniques

4.4.4. Combustion chamber

Combustion releases the chemically bound energy in the fuel generating a hot gas flow. Combustion in gas turbines takes place at elevated pressure and rather limited space, which intensifies the heat release and thereby the demand for material cooling. The figure below illustrates air and fuel flow, as well as exhaust gas flow.

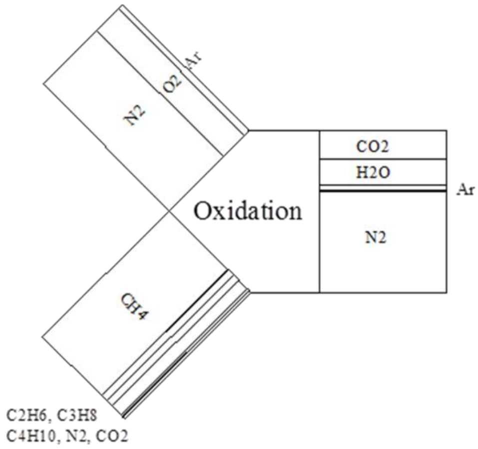


Figure 44: Combustion process

The emissions from gas turbines are formed during combustion. Nitrogen oxides (NO_x), carbon monoxide (CO) and unburned hydrocarbons (C_xH_y) are among the emissions from the combustion process. The main part of NO_x (thermal NO_x) is formed at high combustion temperature and CO and unburned hydrocarbons at low combustion temperature. Therefore there is an optimum operations window, where all the emissions are minimized.

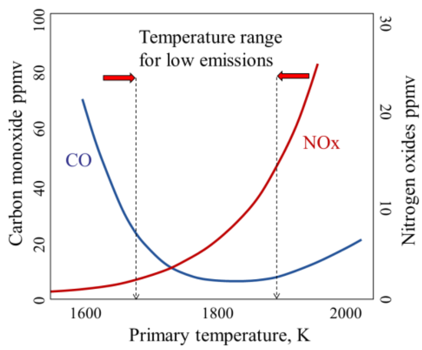


Figure 45: Emissions formation as function of temperature

Controlling the flame temperature is therefore essential for emission levels. In a diffusion flame (left side in the figure below) oxygen is diffusing to the flame front to maintain the flame. The flame temperature is then decided by the adiabatic flame temperature, which results in high NO_x formation. The main advantage of diffusion flame it its stability. To control the flame temperature, one can premix air and fuel before combustion, so called premix combustion. A low fuel to air ratio (lean mixture) results in lower flame temperature and thereby lower NO_x formation. However, lean premixed flame is instable causing vibration problems (humming), which requires sophisticated burner design and control mechanisms.

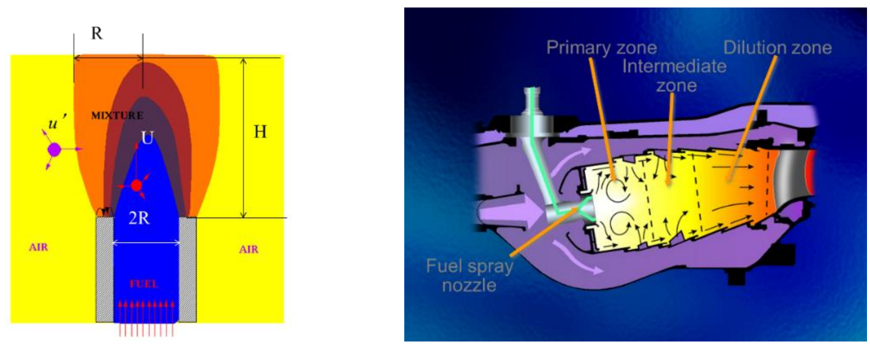


Figure 46: Diffusion flame (left) and premixed flame (right)

One of the key elements in emission control as well as profiling the outlet flow is the air split in the combustion chamber. The figure below shows a typical air split.

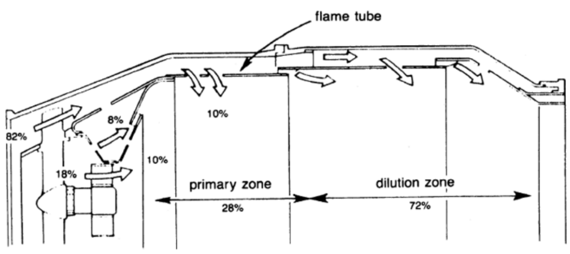


Figure 47: Typical air split

Cooling the flame tube (the metal body which contains the flame) is a challenging task. The air flow in a gas turbine is about three times more than what is required for the combustion. The excess air is mainly used for cooling. Advanced cooling techniques and thermal barrier coatings (TBC) have been developed during the last decades to minimize the cooling air requirement, while maintaining the material temperature below the allowable maximum value. Reducing the cooling air requirement makes it possible to increase the TIT without risk for increased emissions. The figure below shows various cooling techniques for material cooling and the relative cooling air needed for various techniques.

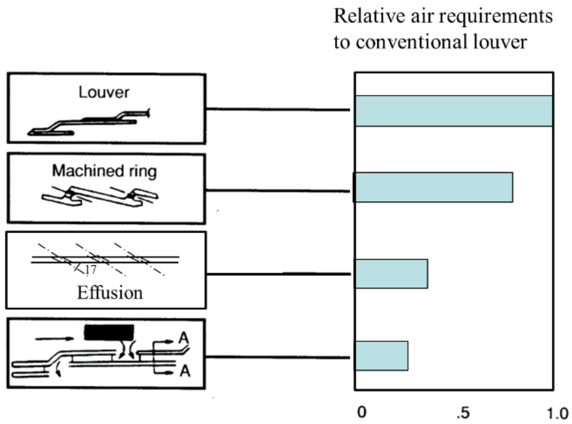
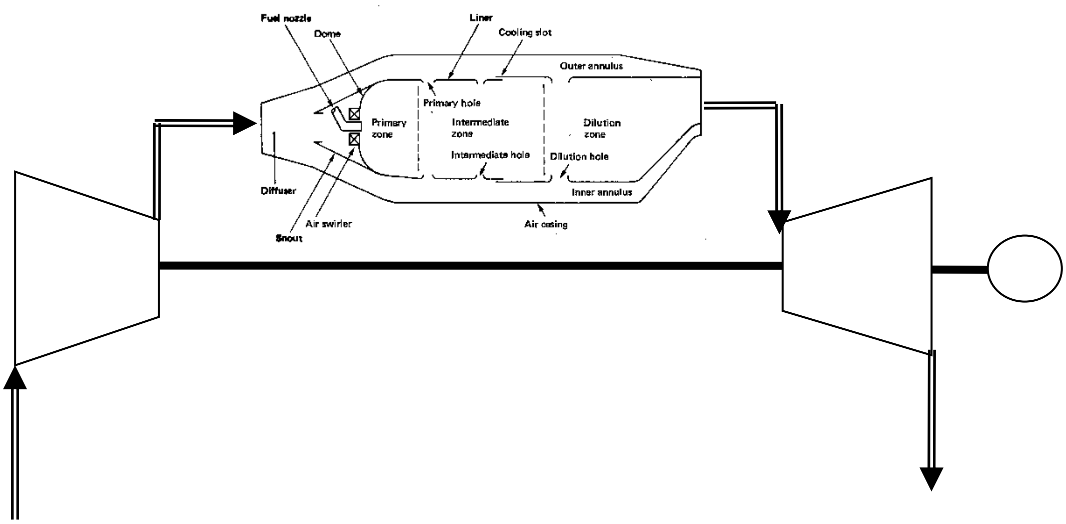


Figure 48: Relative cooling air for various cooling concepts

4.4.5. Combustion process in Gas turbine combustion chamber



Nomenclature:

Notation		Index	
M:	Molar weight	Ar:	argon
V	Molar volume	O2:	Oxygen
N:	Substance amount	N2:	Nitrogen
m:	mass	CO2:	Carbon dioxide
n:	Molar share	L:	Air
g:	Mass share	r:	Exhaust gas
x:	Volume share	br:	Fuel
R:	Gas constant		

Combustion process, the theory

Equations:

$$n_i = \frac{\frac{x_i}{V_i}}{\sum_i \frac{x_i}{V_i}} = \frac{\frac{g_i}{M_i}}{\sum_i \frac{g_i}{M_i}} \qquad (1)$$

$$g_i = \frac{n_i \cdot M_i}{\sum_i n_i \cdot M_i} \qquad (2)$$

Reaction formula for an arbitrary hydrocarbon can be written as:

$$C_x H_y O_z + \left[x + \frac{y}{4} - \frac{z}{2} \right] \cdot O_2 \Rightarrow x \cdot CO_2 + \frac{y}{2} \cdot H_2O \qquad (3)$$

To illustrate the calculation procedure North Sea gas is chosen here as a fuel with the following composition.

	Volume share	Molar share (1)
CH4	0.90982	0.908647
C2H6	0.04730	0.047607
C3H8	0.01732	0.017637
C4H10	0.01456	0.015108
N2	0.00598	0.005961
CO2	0.00502	0.005040

All fuel components, except N2 and CO2, are oxidized during combustion. The oxygen needed for stoichiometric combustion of 1 kmol North Sea gas is calculated as follows:

$$N_{O_2} = \sum_i n_i \left(x + \frac{y}{4} \right)_i$$

Where "i" denotes fuel components (e.g. CH_4,C_2H_6, etc.).

N_{O2} = 0.908647 (1+ 4/4) + 0.047607 (2+ 6/4) + 0.017637 (3+ 8/4) + 0.015108 (4+ 10/4)

N_{O2} = 2.170306 kmol

For calculation of the amount of air needed for combustion, atmospheric air composition is needed. The table below indicates the composition of dry air:

	g_i	M_i	g_i/M_i	n_i
N2	0.75518	28.0134	0.02696	0.780833
O2	0.23142	31.9988	0.00723	0.209479
CO2	0.00049	44.0098	1.1134E-5	0.000322
Ar	0.01291	39.9480	3.2317E-4	0.009361

If we assume that the dry air contains 0.209479 kmol oxygen per kmol of dry air, the quantity of air is calculated according to the following equation:

$$m_L = \frac{N_{O_2}}{n_{O_2}} \sum_i M_i \ n_i$$

where "i" denotes air components (e.g. N_2, O_2, CO_2 and Ar).

m_L = (2.170306/0.209479) (28.0134 * 0.780833 + 31.9988 * 0.209479 + 44.0098 * 0.000322

+ 39.948 * 0.009361)

m_L = 300.0916 kg

Fuel quantity can then be calculated by the following equation:

$$m_{br} = \sum_i M_i \, n_i$$

Where "i" denotes fuel components (e.g. CH4, C2H6, etc.).

m_{br} = 16.0426 * 0.908647 + 30.0694 * 0.047607 + 44.0962 * 0.017637 + 58.123 * 0.015108

+ 28.0134 * 0.005961 + 44.0098 * 0.005040

m_{br} = 18.0532 kg

To calculate the exhaust gas components' mass- and molar share one must add the contributions of combustion air, fuel and oxidation products:
The mass of CO2 in flue gas is calculated according to the following equation:

$$m_{CO_2} = M_{CO_2}\left(N_{O2}\frac{n_{CO_2\,Air}}{n_{O_2\,Air}} + n_{CO_2\,Fuel} + \left(\sum_i x_i n_i\right)_{Combustion}\right)$$

m_{CO2} = 44.0098 [2.170306 * (0.000322/0.209479) + 0.005040 + (1*0.908678 + 2*0.047607

+ 3*0.01763 + 4*0.015108)]

m_{CO2} = 49.537 kg

Similarly, the amount of N2, H2O and Ar in the flue gas are calculated. N2 in the exhaust gas originate partly from the combustion air and partly from the fuel, H2O is formed during combustion and Ar is carried over by the combustion air.

m_{N2} = 226.7644 kg m_{H2O} = 37.9437 kg m_{Ar} =3.8743 kg

Thus the mass of flue gas and mass share of exhaust gas components are calculated as follows:

$$m_r = \sum_i m_i$$

m_r = 49.537 + 226.7644 + 37.9437 + 3.8743 = 318.1194 kg/kmol fuel

g_{CO2} = 49.537/318.1194 = 0.15572 g_{CO2} = 15.572%

g_{N2} = 71.283% g_{H2O} = 11.928% g_{Ar} = 1.217%

The gas constant for exhaust gas mix is calculated as follows:

$$R_r = \sum_i g_i R_i$$

where $R_i = \dfrac{8314}{M_i} \quad [J / kmol, K]$

putting in the calculated values gives:

R_r = 298.557 J/kmol, K

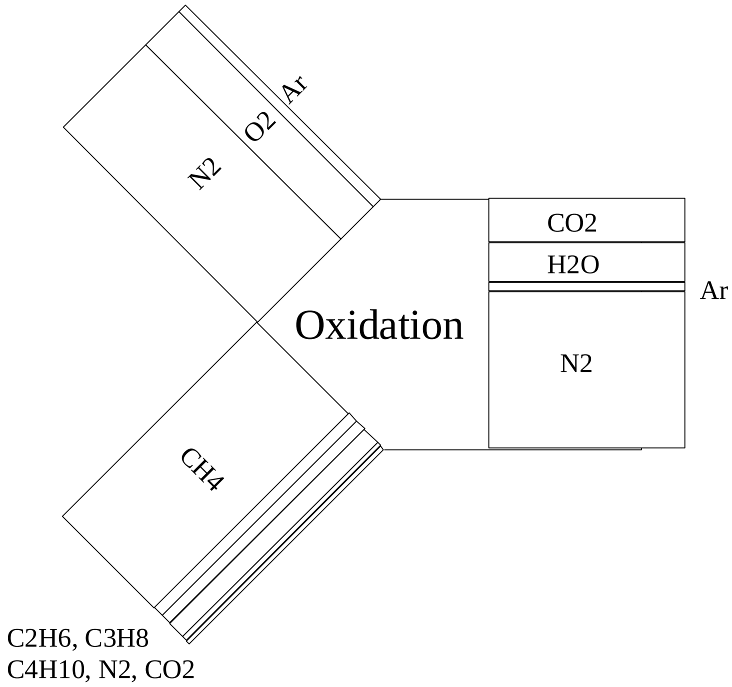
Molar weight for the gas mixture is:

$$M_r = \sum_i M_i n_i$$

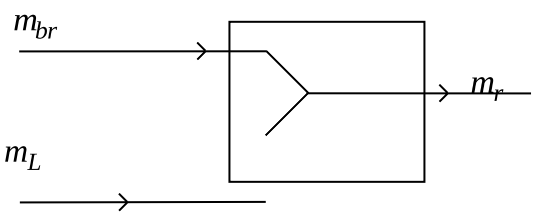
M_r = 27.8473 kg/kmol

Energy balance for the combustion chamber.

Here we will calculate the amount of fuel m_{br} that is to be consumed to generate exhaust gases at temperature of T_{03} , $m_r = m_L + m_{br}$ for a given amount of air m_L at a temperature T_{02}.



Study the schematic gas turbine combustion chamber shown in the figure below.



The fuel to air ratio is defined by:

$$f = \frac{m_{br}}{m_L}$$

The energy balance for the combustion chamber can be written as:

$m_r h_r = m_L h_L + m_{br} h_{br}$

or $(1 + f)h_r = h_L + fh_{br}$

By utilizing integrated mean values of c_p for the studied temperature interval the energy balance can be rewritten as:

$$(1+f)\cdot c_{p,r}\cdot\left(T_{03}-T_{ref}\right)= f\cdot\Delta H_{reac}+c_{p,L}\cdot\left(T_{02}-T_{ref}\right)+f\cdot c_{p,br}\cdot\left(T_{br}-T_{ref}\right)$$

where ΔH_{reac} is equal to the lower heating value, if H_2O in the flue gas is in the gas phase.

For most cases, the last term, the sensible energy supplied with fuel, is neglected.

Integral average value of the heat capacity is defined by:

$$c_{p,m} = \frac{\int_{T_1}^{T_2} c_p(T)dT}{T_2 - T_1}$$

and is often given in tabular form for various gas components. At the end of this booklet there is such a table attached, where $T_1 = T_{ref}$, which means that $c_{p,m}$, for an interval $(T_2 - T_{ref})$ can be directly read out as $c_{p,m}(T_2)$ and then used to calculate the enthalpy of a gas mixture when the composition is known.

Since the flue gas composition is determined by f the solution becomes iterative. According to the above, the mass amount of flue gas is the sum of flue gas components' masses according to:

$$\mathrm{m_r} = \sum_{\mathrm{i}} \mathrm{m_i}$$

The left leg of the energy balance can be written as:

$$m_r \cdot c_{p,r} \cdot \left(T_{03} - T_{ref}\right) = \sum_i m_i c_{p,i} \cdot \left(T_{03} - T_{ref}\right)$$

A possible calculation procedure could then be summarized as follows:

Given T_{02}, T_{03}, T_{br}, ΔH_{reac} and the air composition:

- Guess f.
- Calculate the exhaust gas composition.
- Find c_p-value for exhaust gas components using the table for integral mean values.
- Calculate T_{03} using the energy balance and compare it to the desired value.
- Repeat if necessary until convergence.

Alternatively, if *f* is known, the final exhaust gas temperature T_{03} is to be calculated. In that case one should start by guessing a value of the T_{03}.

4.4.6. Problem: Combustion

Learning Objectives:

- Stoichiometry Calculation, mass balance and heat balance.
- An overview of fuels and combustion.
- Defining the parameters used in combustion analysis, such as air–fuel ratio, percent and theoretical air.
- Applying energy balances to reacting systems for both steady-flow control volumes and fixed mass systems.
- Determining the turbine inlet temperature for given composition of the fuel.

❖ Three questions are provided to cover the learning objectives of abovementioned issues based on given fuel composition in mole, mass and volume fraction for understanding the stoichiometry and the corresponding calculations.

****

Given Data:

Molar mass, molar volume, density and gas constants.

		Molar mass M kg/kmol	Molar volume V_n m^3/kmol	Density ρ_n kg/m^3	Gas constant R kJ/(kgK)
Coal	C	12,011	-	-	-
Sulfur	S	32,060	-	-	-
Nitrogen	N_2	28,0134	22,403	1,2504	0,29666
Oxygen	O_2	31,9988	22,329	1,4290	0,25958
Coal monoxide	CO	28,0104	22,400	1,2505	0,29665
Coal dioxide	CO_2	44,0098	22,261	1,9770	0,18763
Water vapor	H_2O	18,0152	22,41	0,8039	0,46144
Sulfur dioxide	SO_2	64,0588	21,856	2,9309	0,12656
Hydrogen	H_2	2,0158	22,428	0,0899	4,12723
Argon	Ar	39,948	22,392	1,7840	0,20793
Neon	Ne	20,179	22,425	0,8998	0,41224

Molar mass and molar volume for some hydro carbons.

		Molar mass M kg/kmol	Molar volume V_n m^3/kmol
Methane	CH_4	16,0426	22,3600
Ethane	C_2H_6	30,0694	22,1874
Propane	C_3H_8	44,0962	21,9297
Butane	C_4H_{10}	58,1230	21,5205

Mean molar heat capacities of gases at constant pressure [kJ/(kmol · K)]

K	H_2	N_2	CO	Air	O_2	HCl	Cl_2	H_2O	CO_2	SO_2	SO_3	CH_4	C_2H_4	C_2H_6	NH_3
298	28.84	29.13	29.18	29.18	29.39	29.13	33.99	33.57	37.17	40.14	50.69	35.79	43.74	52.87	35.50
400	29.01	29.21	29.32	29.32	29.75	29.19	34.57	33.94	39.23	41.68	54.77	38.31	48.85	59.31	37.21
500	29.15	29.35	29.45	29.45	30.18	29.24	35.16	34.40	40.07	43.32	58.42	41.03	53.71	65.56	38.76
600	29.19	29.53	29.67	29.69	30.65	29.32	35.58	34.93	42.72	44.81	61.63	43.84	58.29	71.56	40.19
700	29.24	29.76	29.98	30.02	31.14	29.43	35.86	35.35	44.15	46.11	64.40	46.69	62.46	77.26	41.55
800	29.31	30.31	30.27	30.34	31.59	29.60	36.10	35.90	45.43	47.23	66.80	49.46	66.26	83.30	42.89
900	29.36	30.34	30.61	30.64	32.00	29.78	36.30	36.49	46.54	48.17	68.85	52.23	69.73	87.39	44.23
1000	29.46	30.64	30.93	30.94	32.37	30.00	36.45	37.08	47.56	49.01	70.27	54.66	72.91	91.93	45.56
1100	29.57	30.93	31.24	31.27	32.70	30.25	36.59	37.68	48.50	49.75	72.27	57.06	75.91	96.20	46.86
1200	29.69	31.22	31.56	31.56	33.02	30.49	36.74	38.29	49.35	50.43	73.72	59.34	78.66	100.15	48.16
1300	29.89	31.50	31.84	31.93	33.32	30.75	36.85	38.89	50.10	51.01	75.11	61.49	81.15	103.09	49.44
1400	30.07	31.77	32.09	32.13	33.60	31.00	36.94	39.45	50.82	51.53	76.36	63.47	83.51	107.17	50.71
1500	30.23	32.04	32.34	32.38	33.84	...	...	40.01	51.43						
1600	30.39	32.25	32.58	32.61	34.05	...	...	40.56	51.99						
1700	30.56	32.46	32.79	32.79	34.23	...	...	41.03	52.54						
1800	30.75	32.67	33.00	33.01	34.40	...	...	41.47	53.18						
1900	30.97	32.86	33.20	33.24	34.66	...	...	41.84	53.43						
2000	31.12	33.03	33.37	33.41	34.83	...	...	42.53	53.74						
2100	31.32	33.20	33.52	33.56	34.98	...	...	42.98	54.25						
2200	31.48	33.35	33.66	33.70	35.12	...	...	43.41	54.56						

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1. The gas that is driven off when low-grade coal is burned with insufficient air for complete combustion is known as producer gas. A particular producer gas has been analysed and it has the molar composition summarized in the following table:

Components	Molar share
CH_4	0.038
C_2H_6	0.001
CO_2	0.048
H_2	0.117
O_2	0.006
CO	0.232
N_2	0.558

The producer gas is combusted with 150% stoichiometric dry air. Determine:

1) The stoichiometric air / fuel ratio.
2) The stoichiometric composition of the exhaust gas in mass parts.
3) The molar mass of the stoichiometric exhaust gas, M.
4) The gas constant for the stoichiometric exhaust gas, R.

[f=1.346, $m_{exhaust}$ =59.38, $M_{exhaust}$ =29.58, $R_{exhaust}$=280.99]

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2. A standard liquid fuel for gas turbine (oil distillate) has the following composition in mass shares:
   Coal, C =0.8443
   Hydrogen, H_2 =0.1355
   Sulphur, S =0.0050
   Oxygen, O_2 = 0.0132
   Nitrogen, N_2 =0.0020
   Determine:
   1. The stoichiometric air / fuel ratio.
   2. The stoichiometric composition of the exhaust gas in mass parts.
   3. The molar mass of the stoichiometric exhaust gas, M.
   4. The gas constant for the stoichiometric exhaust gas, R.

   [*f*=14.23, $m_{exhaust}$ =15.24, $M_{exhaust}$ =28.88, $R_{exhaust}$=287.86, TIT= 1160 K]

****

3. The low heating value gas from a gasification plant has the following composition in volume share:

   Carbon monoxide, CO =0.180
   Hydrogen, H_2 =0.070
   Methane, CH_4 =0.051
   Carbon dioxide, CO_2 = 0.150
   Nitrogen, N_2 =0.469
   Water, H_2O= 0.080

   Determine:
   1. The stoichiometric air / fuel ratio.
   2. The stoichiometric composition of the exhaust gas in mass parts.
   3. The molar mass of the stoichiometric exhaust gas, M.
   4. The gas constant for the stoichiometric exhaust gas, R.

   [*f*=1.61, $m_{exhaust}$ =3.04, $M_{exhaust}$ =29.57, $R_{exhaust}$=281.13]

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4.5. Combined cycle

Exhaust gases leaving the gas turbine contain considerable amount of thermal energy. Gas turbine outlet temperature ranges between 450 and 650 C. To enhance the electrical efficiency of the gas turbine cycles, these are integrated with a steam bottoming cycle where the thermal energy in the exhaust gas is recovered in a heat recovery steam generator (HRSG) to generate steam, which is expanded in a steam turbine to produce additional power. The integrated gas and steam cycle is called combined cycle. The figure below illustrates the simple combined cycle.

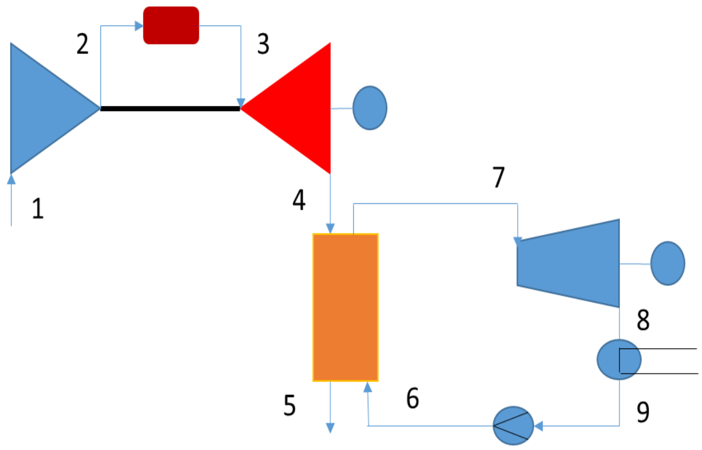


Figure 49: Schematic simple combined cycle
Main components: 1-2: compressor, 2-3: combustion chamber, 3-4: expander, 4-5 & 6-7: HRSG, 7-8: steam turbine, 8-9: condenser, 9-6: feed water pump

Given the fact that usually no combustion takes place in the HRSG, feed water pre-heating doesn't improve the cycle efficiency, and on the contrary it reduces the efficiency since the outlet exhaust gas temperature would increase by feed water preheating. Only a dearator is used to remove the non-condensable gases from the steam flow. However, in some modern combined cycle plants non-condensable gases are also removed in the condenser, avoiding use of dearator.

Design of the HRSG has impact on plant efficiency, operability and footprint. Concerning efficiency, the HRSG can be designed with single or multiple pressure levels. Generally, efficiency improvement when moving from single to two pressure levels is about one percentage point and from two to three pressure levels is about a half percentage point. The steam circuits at various pressure levels can be placed in serial or parallel arrangements in the HRSG. The figures below illustrate arrangements of different pressure levels in the HRSG and the heat transfer between exhaust gas and the steam circuits in TQ-diagrams.

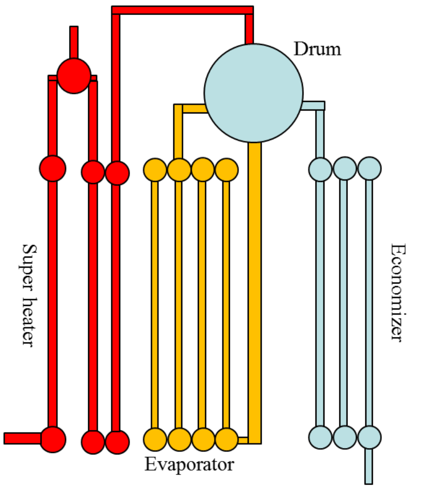


Figure 50: Heat transfer surfaces in single pressure HRSG

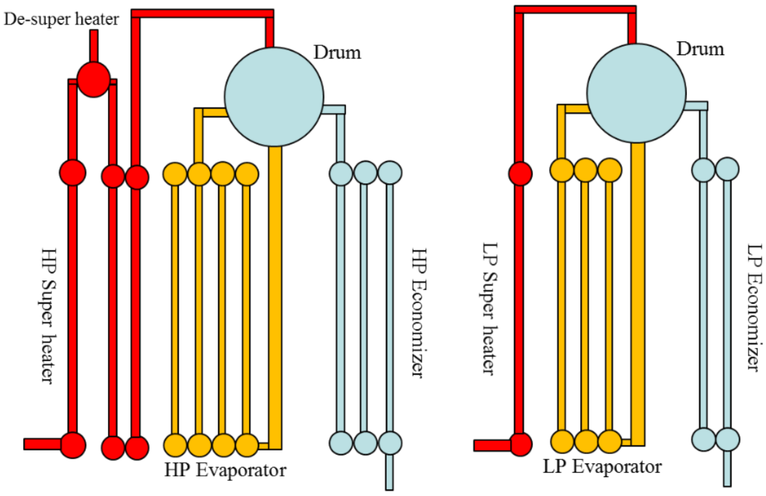


Figure 51: Heat transfer surfaces in two pressure HRSG

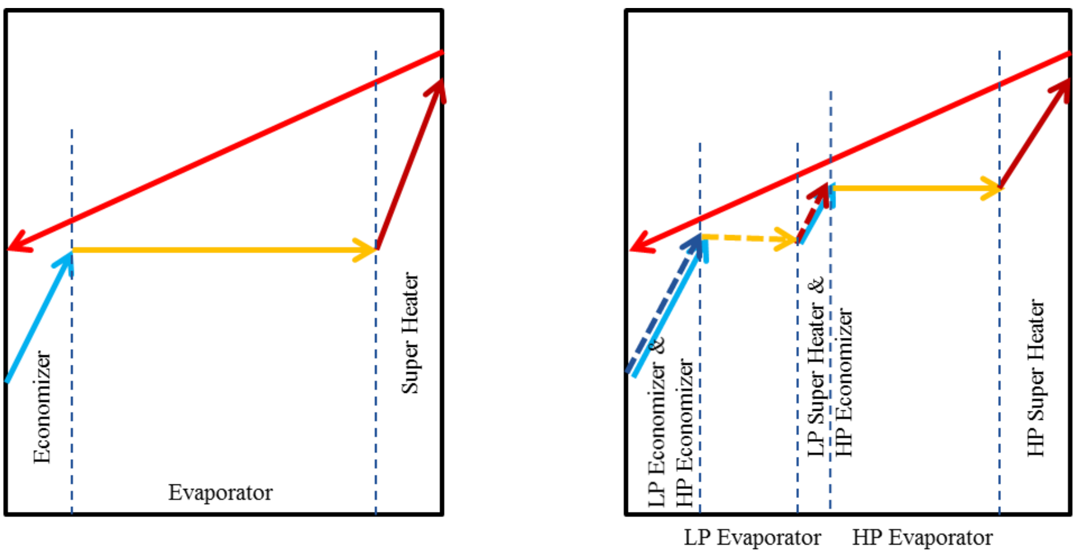


Figure 52: Schematic heat transfer for one and two pressure level HRSG in TQ-diagram

Concerning operability and the footprint, HRSGs can be designed as horizontal or vertical, which is decided by the flow direction of the exhaust gas through the heat recovery surfaces, i.e. economizer, evaporator and super-heater. Generally speaking, the water/steam flow direction is perpendicular to the exhaust gas direction as indicated in the figure below. As the figure indicates, horizontal HRSG has larger footprint but lower height, which makes it more stable (chimney height). Vertical HRSG has smaller footprint but since the chimney is on top of the HRSG it becomes much higher than horizontal one and the weight of the whole package (chimney, heat transfer surfaces, etc.) needs to be carried by the smaller fundament the HRSG is placed on. Concerning the operability, horizontal HRSG needs more careful design of the drainage systems, since there is higher risk for blockage of the pipes by the condensate, which would result in uneven temperature distribution and consequently damage to the pipes and the welding, when the plant is operated with frequent start-stops.

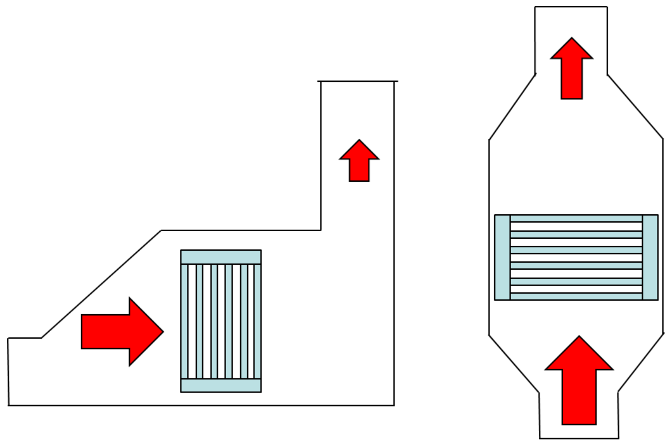


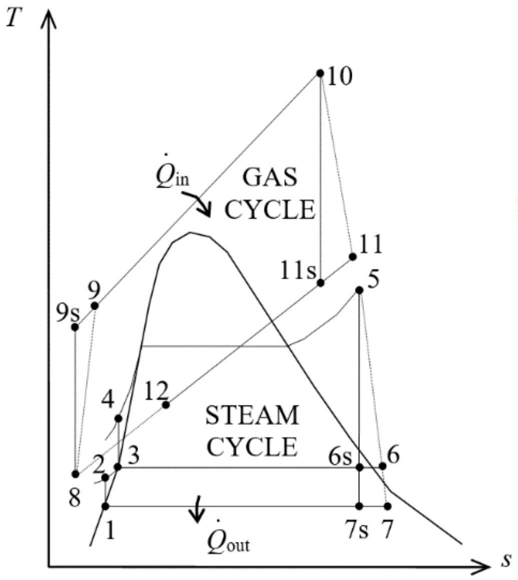
Figure 53: Schematic illustration of horizontal (left) and vertical (right) HRSG

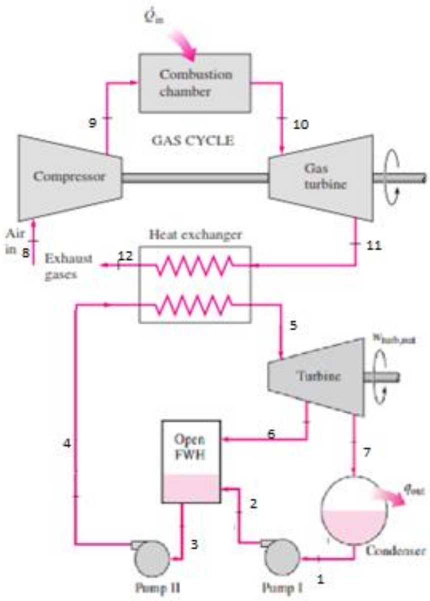
Modern combined cycles of today can reach above 62% electrical efficiency. However, due to increased share of intermittent renewable energy in the electrical grid, flexibility of the plants has become an important issue that requires more research and development work.

4.5.1 Problem: Combined power cycle

1. Consider a combined gas-steam power plant that has a net power output of 280 MW. The pressure ratio of the gas turbine cycle is 11. Air enters the compressor at 300 K and the turbine at 1100 K. The combustion gases leaving the gas turbine are used to heat the steam at 5 MPa to 350°C in a heat exchanger. The combustion gases leave the heat exchanger at 420 K. An open feedwater heater incorporated with the steam cycle operates at a pressure of 0.8 MPa. The condenser pressure is 10 kPa. Assuming isentropic efficiencies of 100 percent for the pump, 82 percent for the compressor, and 86 percent for the gas and steam turbines, determine (a) the mass flow rate ratio of air to steam, (b) the required rate of heat input in the combustion chamber, and (c) the thermal efficiency of the combined cycle.

[m_{air}/m_{steam} = 1341.7, Q_{in} = 671300 kW, η_{th} = 41.7 %]





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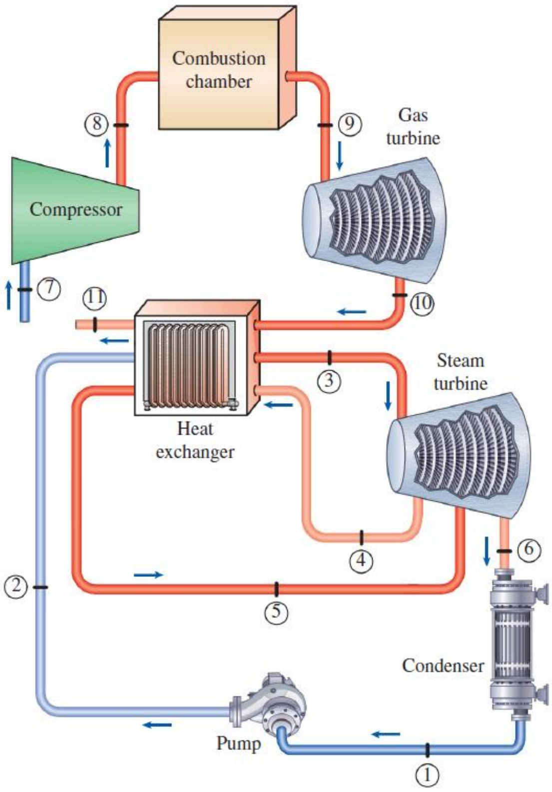
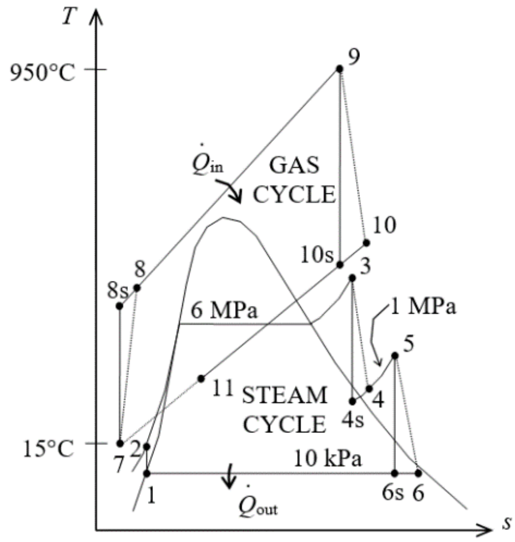
2. Consider a combined gas-steam power cycle. The topping cycle is a simple Brayton cycle that has a pressure ratio of 7. Air enters the compressor at 15°C at a rate of 10 kg/s and the gas turbine at 950°C. The bottoming cycle is a reheat Rankine cycle between the pressure limits of 6 MPa and 10 kPa. Steam is heated in a heat exchanger at a rate of 1.15 kg/s by the exhaust gases leaving the gas turbine, and the exhaust gases leave the heat exchanger at 200°C. Steam leaves the high-pressure turbine at 1.0 MPa and is reheated to 400°C in the heat exchanger before it expands in the low-pressure turbine. Assuming 80 percent isentropic efficiency for all pumps and turbines, determine (a) the moisture content at the exit of the low-pressure turbine, (b) the steam temperature at the inlet of the high pressure turbine, (c) the net power output and the thermal efficiency of the combined plant.

[Moisture = 1.6 %, T_3 = 468 °C, W_{out} = 2897 kW, η_{th} = 38.8 %]





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