

Formulas:

Change of variables for double integrals:

$$\iint_R f(x, y) dx dy = \iint_S f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du dv.$$

Line integral of a function f along a curve \mathcal{C} : $\mathbf{r} = \mathbf{r}(t)$, $a \leq t \leq b$:

$$\int_{\mathcal{C}} f ds = \int_a^b f(\mathbf{r}(t)) \left| \frac{d\mathbf{r}}{dt} \right| dt.$$

Line integral of a vector field $\mathbf{F} = F_1 \mathbf{i} + F_2 \mathbf{j} + F_3 \mathbf{k}$, along a curve \mathcal{C} : $\mathbf{r} = \mathbf{r}(t)$, $a \leq t \leq b$:

$$\int_{\mathcal{C}} \mathbf{F} \cdot \hat{\mathbf{T}} ds = \int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = \int_{\mathcal{C}} F_1 dx + F_2 dy + F_3 dz = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \frac{d\mathbf{r}}{dt} dt = \int_a^b (F_1 \frac{dx}{dt} + F_2 \frac{dy}{dt} + F_3 \frac{dz}{dt}) dt.$$

Integral of a function f over a surface \mathcal{S} : $z = g(x, y)$, parametrised by $(x, y) \in R$:

$$\iint_{\mathcal{S}} f dS = \iint_R f \sqrt{1 + \left(\frac{\partial g}{\partial x} \right)^2 + \left(\frac{\partial g}{\partial y} \right)^2} dx dy.$$

Integral of a function f over a surface \mathcal{S} : $G(x, y, z) = c$, parametrised by $(x, y) \in R$:

$$\iint_{\mathcal{S}} f dS = \iint_R f \frac{|\nabla G|}{\left| \frac{\partial G}{\partial z} \right|} dx dy.$$

Flux of a vector field \mathbf{F} through a surface \mathcal{S} : $z = g(x, y)$, parametrised by $(x, y) \in R$:

$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} = \iint_{\mathcal{S}} \mathbf{F} \cdot \hat{\mathbf{N}} dS = \iint_R \mathbf{F} \cdot \pm \left(-\frac{\partial g}{\partial x} \mathbf{i} - \frac{\partial g}{\partial y} \mathbf{j} + \mathbf{k} \right) dx dy.$$

Flux of a vector field \mathbf{F} through a surface \mathcal{S} : $G(x, y, z) = c$, parametrised by $(x, y) \in R$:

$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} = \iint_{\mathcal{S}} \mathbf{F} \cdot \hat{\mathbf{N}} dS = \iint_R \mathbf{F} \cdot \frac{\pm \nabla G}{\frac{\partial G}{\partial z}} dx dy.$$

Divergence theorem:

$$\iiint_D \nabla \cdot \mathbf{F} dV = \oiint_{\mathcal{S}} \mathbf{F} \cdot \hat{\mathbf{N}} dS.$$

Stokes' theorem:

$$\iint_{\mathcal{S}} (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{N}} dS = \oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}.$$

Formulas involving $\nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k}$:

$$\text{grad } f = \nabla f, \quad \text{div } \mathbf{F} = \nabla \cdot \mathbf{F}, \quad \text{curl } \mathbf{F} = \nabla \times \mathbf{F}.$$

Cylindrical coordinates: $(r \cos \theta, r \sin \theta, z) = (x, y, z)$.

Spherical coordinates: $(R \sin \phi \cos \theta, R \sin \phi \sin \theta, R \cos \phi) = (x, y, z)$.

Trigonometric formulas: $\sin 2\theta = 2 \sin \theta \cos \theta$, $\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$.