MAT300 VEKTORANALYSE – NOTES ON EXAM

AUTUMN 2017, UNIVERSITY OF STAVANGER

This document contains information about the various tasks required to fully solve each exercise, together with a summary of common mistakes I observed while grading students' responses.

1. Details of Exercises and Common Errors

1.1. **Exercise 1(a).** Required tasks:

- Compute derivative of a parametrised curve.
- Evaluate derivative at $t = \pi$.
- Compute length of velocity vector.
- Divide velocity vector by its length to get a unit tangent vector.

Remarks and common errors:

- Many students are still unable to perform simple algebraic manipulations without error. For example, in this question, many students incorrectly wrote $(\cos t + 1)^2 = \cos^2 t + 1$ instead of $\cos^2 t + 2\cos t + 1$. For students studying in an engineering, physics, or mathematics program, this sort of error is not really excusable. If you are struggling with these sorts of algebraic manipulations I strongly recommend you spend as much time as necessary to review and practice until you are able to reliably manipulate such expressions without making any errors. I can try to provide some links to resources if you need assistance.
- Some students drop the unit vectors **i**, **j**, and **k**, and gave a scalar quantity as answer.

1.2. **Exercise 1(b).** Required tasks:

- Compute speed of a parametrised curve.
- Write integrand as a function of the parameter t.
- Write the correct expression for the line integral of a scalar field.
- Evaluate the integral using substitution.

Remarks and common errors:

- Many students incorrectly assumed that the speed was constant and equal to the length of the velocity vector at $t = \pi$, as computed in Exercise 1(a).
- Some students made errors when substituting the expressions for x, y, and z into the integrand, so as to write everything in terms of t. Again, by this stage in their education all students should be able to perform simple substitutions and algebraic manipulations reliably without error. If you find that you consistently make such errors during exams, I strongly recommend you concentrate on developing your exam technique to eliminate this unnecessary loss of marks. You are welcome to discuss this with me for advice, or you might like to read the discussions at the following links.

  `http://www-users.york.ac.uk/~dajp1/Exam_Hints/Exams.html`

Date: December 20, 2017.

https://www.physicsforums.com/threads/keep-making-silly-mistakes-in-exams.860938/

1.3. **Exercise 1(c).** Required tasks:

- Obtain the potential function for a conservative vector field.

Remarks and common errors:

- This question was very well done, and I was very happy to see most students checking their final result by taking the gradient and ensuring they indeed obtained the given vector field.

1.4. **Exercise 1(d).** Required tasks:

- Understand that the line integral of a conservative vector field equals the change in potential.
- Find the endpoints of the curve.
- Compute the change in potential.

Remarks and common errors:

- This question was quite well done, but several students appeared to be unaware that the line integral of a conservative vector field equals the change in potential, and attempted to compute the line integral directly using the parametrisation of the curve.
- Many students made errors when evaluating the potential function at the endpoint of the curve and hence obtained an incorrect final result.

1.5. **Exercise 2(a).** Required tasks:

- Sketch the given curves and identify the region in the xy-plane.
- Rewrite each curve in terms of the u and v variables.
- Draw the curves and identify the region in the uv-plane.

Remarks and common errors:

- Many students had difficulty writing the equation of the y-axis ($x = 0$) in terms of the u and v variables. The idea here is to first set $x = 0$ in the two equations for u and v, obtaining $u = y$ and $v = -y$. It then remains to combine these two equations into a single equation involving u and v only, namely, $u + v = 0$.

1.6. **Exercise 2(b).** Required tasks:

- Compute the two Jacobian determinants.

Remarks and common errors:

- This question was very well done.
- A few students mistakenly took the absolute value here.

1.7. **Exercise 2(c).** Required tasks:

- Obtain an expression for dA (taking the absolute value of the Jacobian determinant).
- Use change of variables to write the integral over the region S in the uv-plane.
- Simplify the integrand and write in terms of the u and v variables.
- Write down the limits of integration for u and v over the region S.
- Compute the double integral.

Remarks and common errors:

- Some students forgot to take the absolute value of the Jacobian determinant in the expression for dA.
- Some students got stuck trying to rewrite the integrand in terms of u and v before cancelling the term $2x + 1$ with the expression for dA. Experience in solving a similar problem in Assignment 1 was probably useful here.

1.8. **Exercise 3(a).** Required tasks:

- Recognise symmetry of the domain under the exchange of x and z variables.
- Determine an equation for the plane through the three given points.
- Recognise that the integral can be computed by integrating x^2 and multiplying the final result by 2.
- Obtain the correct limits of integration for the variables x, y, and z.
- Compute the triple integral.

Remarks and common errors:

- Many students did not understand how to simplify the problem using the given fact that the integrals of x^2 and z^2 were equal, and attempted to directly compute the integral of $x^2 + z^2$. Typically this led to very unwieldy calculations.
- Many students were unable to obtain the correct limits describing the given region, and some students used constant limits for all three variables (thus mistakenly describing a rectangular box).
- Some students had difficulty obtaining an equation for the plane through the three given points.
- Many students made computational errors when performing the triple integral.
- The triple integral in this question is relatively straight-forward to compute (with some care) provided that the hint is used AND a suitable order of iteration of the variables is chosen. The ability to make a suitable choice of order of iteration (and to quickly notice if an unsuitable choice has been made) is a skill that cannot be taught directly and can only be acquired through the experience of solving exercises during semester.

1.9. **Exercise 3(b).** Required tasks:

- Obtain an equation describing the given plane (same as in Exercise 3(a)).
- Determine the projection of the surface onto the xy-plane.
- Obtain an expression for dS.
- Eliminate z from the integrand using the equation of the surface.
- Compute the double integral over the triangular region R.

Remarks and common errors:

- Some students were unable to find the correct limits of integration for x and y to describe the triangular region R in the xy-plane, and gave constant limits for both x and y (mistakenly describing a rectangular region).
- Some students made an error when substituting the defining equation of the surface into the integrand.

1.10. **Exercise 4(a).** Required tasks:

- Compute the divergence and curl of the given vector field.

Remarks and common errors:

- This question was answered very well in general.

- Some students incorrectly gave the divergence as a vector quantity and/or the curl as a scalar quantity.

1.11. **Exercise 4(b).** Required tasks:

- Recognise that the given cone is not a closed surface and that the flux through the top disc needs to be considered in order to use the divergence theorem.
- Choose the correct normal vector on the disc (pointing out of the cone, namely, upwards).
- Evaluate the flux integral over the disc (optionally using symmetry to simplify the computation).
- Describe the cone in cylindrical coordinates.
- Compute the integral of the divergence over the solid cone.
- Combine the divergence integral and the disc flux to get the flux through the conical surface.

Remarks and common errors:

- Many students did not recognise that the given cone was not a closed surface, and that it is necessary to have a closed surface in order to use the divergence theorem. Possibly many of the students who *did* realise this had encountered similar situations in exercises during the semester and in Assignment 2.
- Many students gave incorrect limits of integration when attempting to describe the solid cone. Typically students wrote $0 \le z \le r$ instead of $r \le z \le 1$. Some students gave constant limits for all three variables (mistakenly describing a solid cylinder).
- Some students did not understand the relationship between the given orientation on the conical surface (downward pointing normal vector) and the fact that the divergence theorem requires the choice of the outward pointing normal vector to a region.
- Some students attempted to use spherical coordinates to describe the solid cone. This would be a good idea if the top of the cone were a rounded piece of a sphere (centred at the origin), but is a bad choice when the top of the cone is a flat disc.
- Some students used the equation of the cone $z^2 = x^2+y^2$ to simplify the integrand in the triple integral over the solid cone, and got a result of 0. This is incorrect because this equation only holds on the *surface* of the cone.

1.12. **Exercise 4(c).** Required tasks:

- Derive the given formula for the curl of $z\mathbf{G}$.
- Compute the curl of $z\mathbf{F}$ using the given formula, and notice that it equals the vector field in the given flux integral.
- Apply Stokes' theorem to write the flux integral as a curve integral of $z\mathbf{F}$ around the circle of radius 1 in the plane $z = 1$.
- Obtain the correct orientation on this circular path.
- Use the fact that $z = 1$ on the circle.
- Compute the path integral, either directly by parametrising the circle (difficult), or by using Stokes' theorem again to convert to a disc integral, with correct choice of normal vector (and optionally using symmetry of the disc to simplify the final integral).

Remarks and common errors:

- This question was the most difficult problem in the exam. Those students who obtained full marks for this problem did so by demonstrating a high degree of mathematical competency and a thorough understanding of the relevant topics in the syllabus.