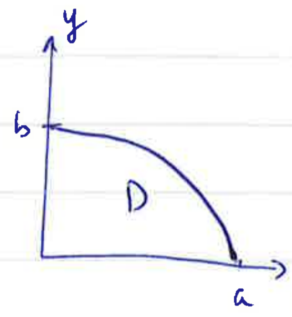
14.4 30. Volume below $z = 1 - x^2/a^2 - y^2/b^2$ in first octant.

$z = 0 \Rightarrow x^2/a^2 + y^2/b^2 = 1$
ellipse



$$V = \iint_D 1 - x^2/a^2 - y^2/b^2 \; dA$$

Let $x = au$, $y = bv$. Then S: $\frac{(au)^2}{a^2} + \frac{(bv)^2}{b^2} = 1$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} = ab$$

$u^2 + v^2 = 1$ circle in uv plane.

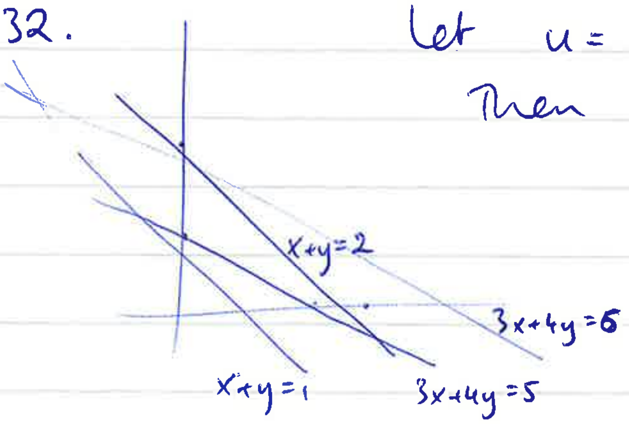
$$V = \iint_B (1 - u^2 - v^2)\, ab \, du\, dv$$

Now use polar coords $u = r\cos\theta$ $v = r\sin\theta$

$$= ab \int_0^{\pi/2}\int_0^1 (1 - r^2)\, r\, dr\, d\theta = \frac{\pi ab}{2}\left[\frac{r^2}{2} - \frac{r^4}{4}\right]_0^1$$

$$= \frac{\pi ab}{8}.$$

32.



Let $u = x+y$, $v = 3x+4y$.

Then $u = 1$ to 2, $v = 5$ to 6

$(v - 4u) = -x$

$v - 3u = y$

$\Rightarrow x^2 + y^2 = (v-4u)^2 + (v-3u)^2$

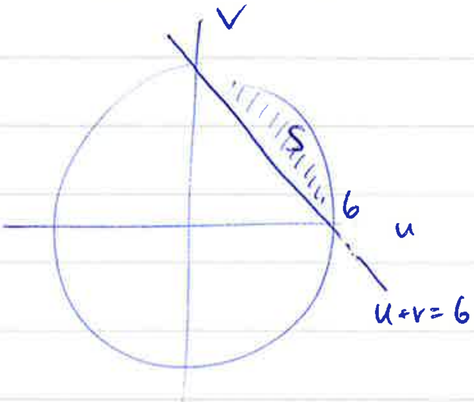
$= 2v^2 - 14uv + 25u^2$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 4 - 3 = 1 \Rightarrow \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = 1.$$

$$\int_5^6 \int_1^2 (2v^2 - 14uv + 25u^2)\, du\, dv = 7/2$$

36. Area D inside $4x^2+9y^2=36$, above $2x+3y=6$.

Let $u=2x$, $v=3y$, $u^2+v^2=36$, $u+v=6$.



$$\text{Area } D = \iint_S dA = \iint_S \tfrac{1}{6}\, du\, dv = \tfrac{1}{6} \text{ Area of } S$$

dA: $dx\,dy$

in u-v plane

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6 \Rightarrow \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \tfrac{1}{6}$$

$$\text{Area } S = \tfrac{1}{4}\pi\cdot 6^2 - \tfrac{1}{2}\,6\cdot 6 = \left(\tfrac{\pi}{4}-\tfrac{1}{2}\right)\cdot 36$$

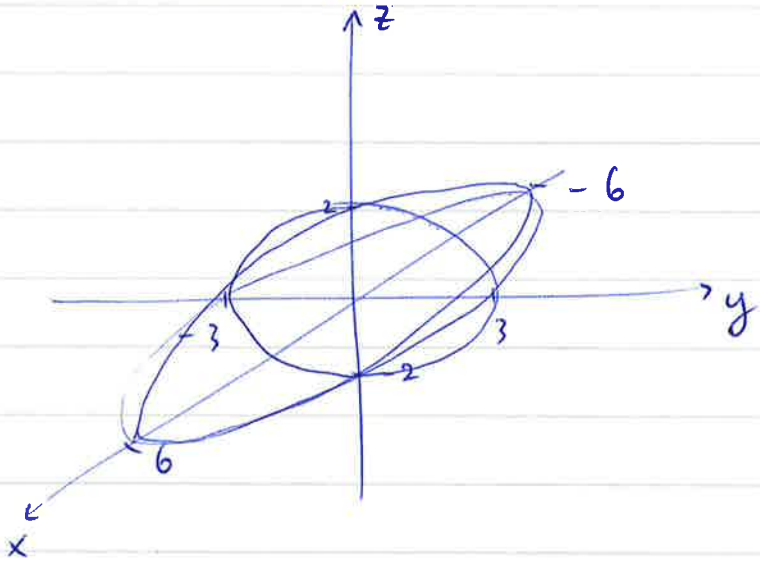
$$= 9\pi - 18$$

$$\therefore \quad \text{Area } D = \tfrac{3}{2}\pi - 3.$$

10.5

1. $x^2+4y^2+9z^2=36$

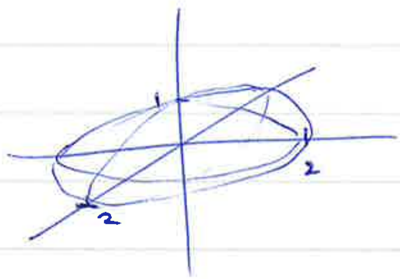
$$\left(\frac{x}{6}\right)^2+\left(\frac{y}{3}\right)^2+\left(\frac{z}{2}\right)^2=1 \qquad \text{ellipsoid.}$$



2. $x^2+y^2+4z^2=4$

$$\left(\frac{x}{2}\right)^2+\left(\frac{y}{2}\right)^2+z^2=1$$

ellipsoid



(3)

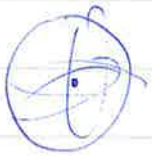
3. $2x^2 + 2y^2 + 2z^2 - 4x + 8y - 12z + 27 = 0.$

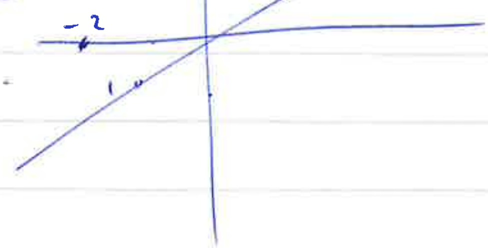
$$2(x-1)^2 - 2 + 2(y+2)^2 - 8 + 2(z-3)^2 - 18 + 27 = 0$$

$$2(x-1)^2 + 2(y+2)^2 + 2(z-3)^2 = 1$$

$$(x-1)^2 + (y+2)^2 + (z-3)^2 = 1/2$$

sphere radius $\frac{1}{\sqrt{2}}$ centred at $(1, -2, 3)$.





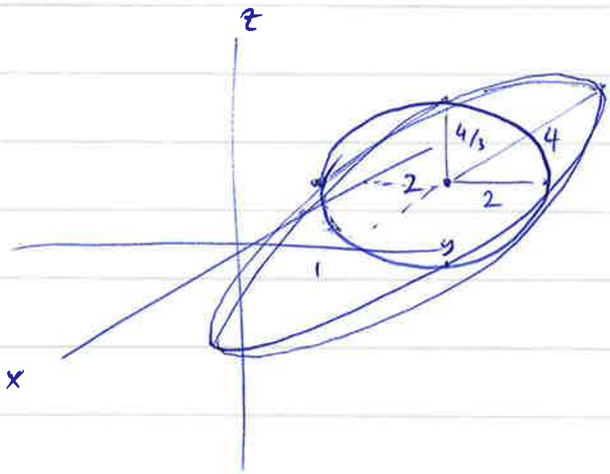
4. $x^2 + 4y^2 + 9z^2 + 4x - 8y = 8$

$$(x+2)^2 - 4 + 4(y-1)^2 - 4 + 9z^2 = 8$$

$$(x+2)^2 + 4(y-1)^2 + 9z^2 = 16$$

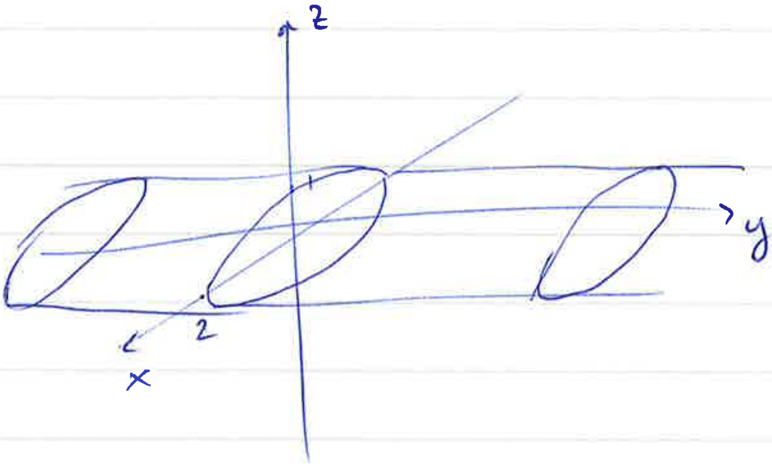
$$\left(\frac{x+2}{4}\right)^2 + \left(\frac{y-1}{2}\right)^2 + \left(\frac{z}{4/3}\right)^2 = 1$$

ellipsoid centre $(-2, 1, 0)$.

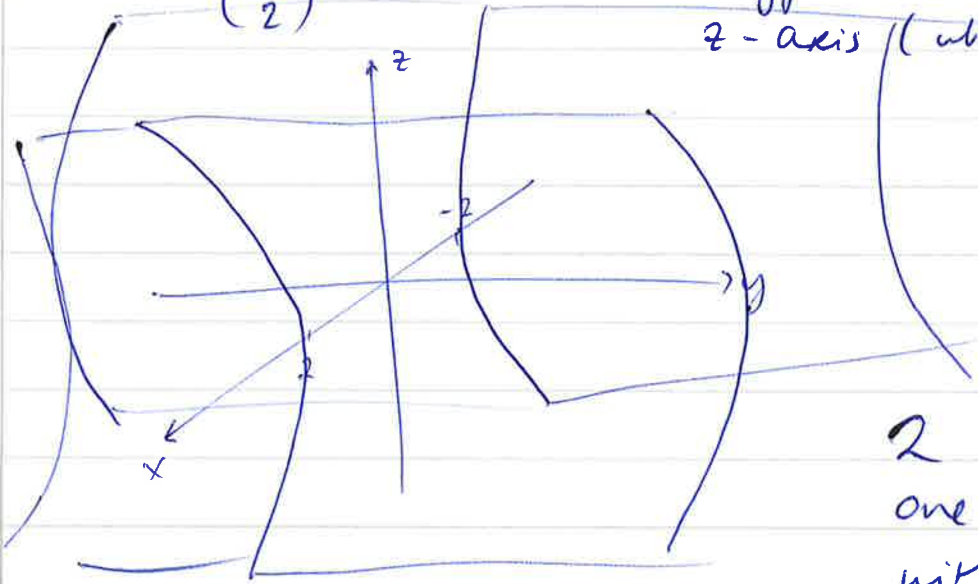


10. $x^2 + 4z^2 = 4$ no y variable $\Rightarrow$ cylinder along y axis.

in xz plane this is an ellipse
$\left(\frac{x}{2}\right)^2 + z^2 = 1$.

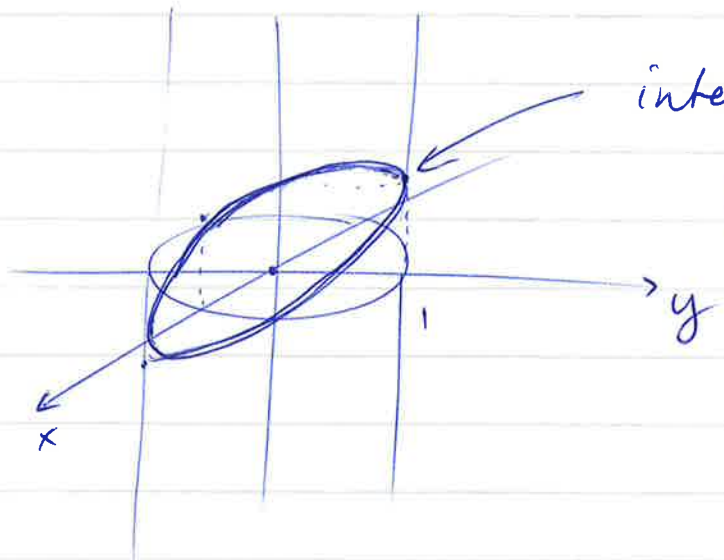


11. $x^2 - 4z^2 = 4$ again, cylinder along y-axis.
$\left(\frac{x}{2}\right)^2 - z^2 = 1$ hyperbola. Does not intersect z-axis (when $x=0$).



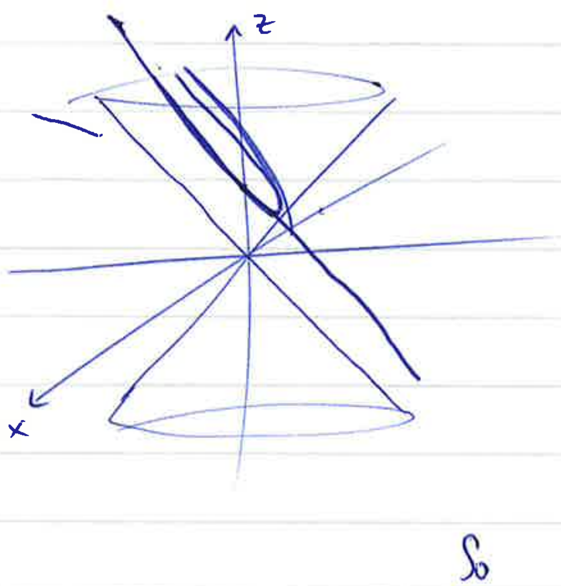
2 separate sheets, one with $x > 0$, other with $x < 0$.

18. $\begin{cases} x^2+y^2=1 \\ z = x+y \end{cases}$ circular cylinder along z-axis
plane through origin.

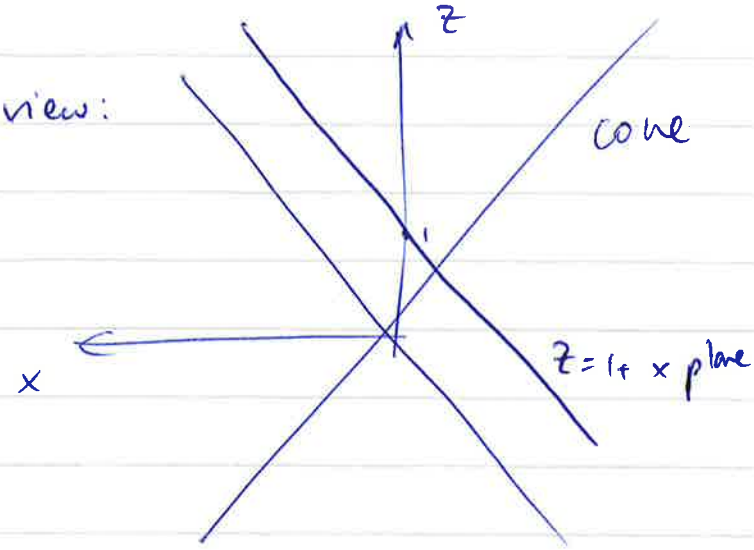


intersection of plane and cylinder.
(an ellipse).

19. $\begin{cases} z^2 = x^2+y^2 \\ z = 1+x \end{cases}$ cone
plane



Side view:



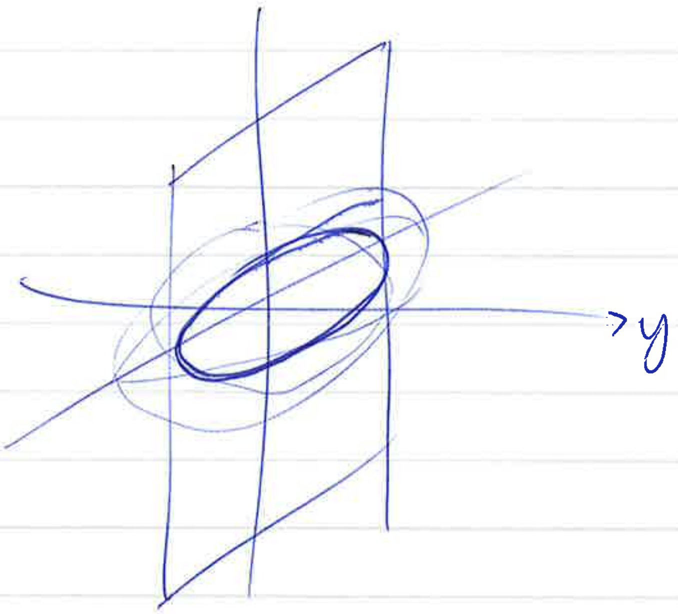
So the intersection is a parabola, as the plane $z=1+x$ is parallel to the edge of the cone.

20. $\begin{cases} x^2 + 2y^2 + 3z^2 = 6 & \text{ellipse} \\ y = 1 & \text{vertical plane.} \end{cases}$

substitute: $x^2 + 2 + 3z^2 = 6$

$x^2 + 3z^2 = 4$

ellipse in plane $y=1$.



14.5

1. $\iiint_R (1 + 2x - 3y)\,dV, \quad -a \le x \le a, -b \le y \le b, -c \le z \le c.$

$= \int_{-c}^{c} dz \int_{-b}^{b} dy \int_{-a}^{a} (1 + 2x - 3y)\,dx = \int_{-c}^{c} dz \int_{-b}^{b} dy \, (x + x^2 - 3yx)\Big|_{-a}^{a}$

$= \int_{-c}^{c} dz \int_{-b}^{b} dy \, (2a - 6ya) = \int_{-c}^{c} dz \, (2ay - 3y^2 a)\Big|_{-b}^{b}$

$= \int_{-c}^{c} dz \, (4ab) = 4ab \, z\Big|_{-c}^{c} = 8abc.$

2. $\iiint_B xyz\, dV \qquad 0 \le x \le 1, \quad -2 \le y \le 0, \quad 1 \le z \le 4$

$$\int_1^4 \int_{-2}^0 \int_0^1 xyz\, dx\, dy\, dz = \int_1^4 \int_{-2}^0 \left(\tfrac{1}{2} x^2 yz \Big|_0^1 \right) dy\, dz$$

$$= \int_1^4 \tfrac{1}{4} y^2 z \Big|_{-2}^0 dz = -\int_1^4 z\, dz = -\tfrac{1}{2} z^2 \Big|_1^4 = \frac{-15}{2}$$

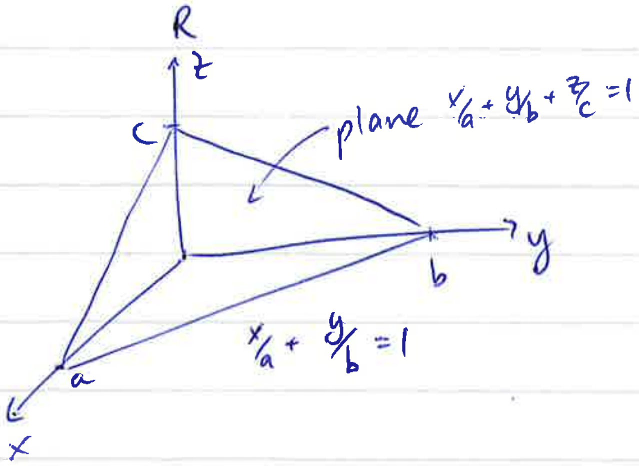
3. $\iiint_D (3 + 2xy)\, dV \quad , \quad D : x^2 + y^2 + z^2 \le 4, \quad z \ge 0.$

radius 2

The function xy is odd under reflection $x \mapsto -x$, but D is symmetric under this reflection, so $\iint_D 2xy\, dV = 0$. $\iiint_D 3\, dV = 3 \times$ volume D

$= 3 \cdot \tfrac{1}{2} \cdot \tfrac{4}{3} \cdot \pi\, 2^3 = 16\pi.$

4. $\iiint_R x\, dV$



$x = 0$ to a

$y = 0$ to $b - \frac{b}{a} x$

$z = 0$ to $c - \frac{c}{a} x - \frac{c}{b} y.$

$$\int_0^a dx \int_0^{b - \frac{b}{a}x} dy \int_0^{c - \frac{c}{a}x - \frac{c}{b}y} x\, dz$$

$$= \int_0^a dx \int_0^{b - \frac{b}{a}x} dy \left(xc - \frac{c}{a} x^2 - \frac{c}{b} xy \right)$$

$$\int_0^a dx \left(xyc - \frac{c}{a}x^2y - \frac{c}{2b}xy^2 \right)\Big|_0^{b-\frac{b}{a}x}$$

$$= \int_0^a dx \left(xbc - \frac{cb}{a}x^2 - \frac{c}{a}x^2b + \frac{cb}{a^2}x^3 - \frac{c}{2b}xb^2 + \frac{cb}{a}x^2 - \frac{cb}{2a^2}x^3 \right)$$

$$= \left(\frac{1}{2}a^2bc - \frac{1}{3}a^2bc - \frac{1}{3}a^2bc + \frac{1}{4}a^2bc - \frac{cba^2}{4} + \frac{1}{3}cba^2 - \frac{cba^2}{8} \right)$$

$$= a^2bc \left(\frac{1}{2} - \frac{1}{3} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{3}} - \frac{1}{8} \right) = \frac{a^2bc}{24}$$