15.4 (5) $\underset{\sim}{F}(x,y,z) = yz\underset{\sim}{i} + xz\underset{\sim}{j} + xy\underset{\sim}{k}$

from $(-1,0,0)$ to $(1,0,0)$ along intersection of $x^2+y^2=1$ and $z=y$.

Let $x = \cos\theta \Rightarrow y = \sin\theta \quad (x^2+y^2=1)$

$\Rightarrow z = y = \sin\theta$.

$\underset{\sim}{r}(\theta) = \cos\theta\,\underset{\sim}{i} + \sin\theta\,\underset{\sim}{j} + \sin\theta\,\underset{\sim}{k} \qquad \theta \in [\pi, 2\pi]$

$\underset{\sim}{r}(\pi) = -\underset{\sim}{i} = (-1,0,0)$, $\underset{\sim}{r}(2\pi) = \underset{\sim}{i} = (1,0,0)$.

$\underset{\sim}{F}(\underset{\sim}{r}(\theta)) = \sin^2\theta\,\underset{\sim}{i} + \cos\theta\sin\theta\,\underset{\sim}{j} + \cos\theta\sin\theta\,\underset{\sim}{k}$

$d\underset{\sim}{r} = (-\sin\theta\,\underset{\sim}{i} + \cos\theta\,\underset{\sim}{j} + \cos\theta\,\underset{\sim}{k})\,d\theta$

$\underset{\sim}{F} \circ d\underset{\sim}{r} = (-\sin^3\theta + \cos^2\theta\sin\theta + \cos^2\theta\sin\theta)\,d\theta$

$= \sin\theta(2\cos^2\theta - \sin^2\theta)\,d\theta = \sin\theta(2\cos^2\theta - (1-\cos^2\theta))\,d\theta$

$= \sin\theta(3\cos^2\theta - 1)\,d\theta = (3\sin\theta\cos^2\theta - \sin\theta)\,d\theta$

$$\int_C \underset{\sim}{F}\cdot d\underset{\sim}{r} = \int_{\pi}^{2\pi} 3\sin\theta\cos^2\theta - \sin\theta \quad d\theta$$

$$= -\cos^3\theta + \cos\theta \Big|_{\pi}^{2\pi} = -1 + 1 - (1-1)$$

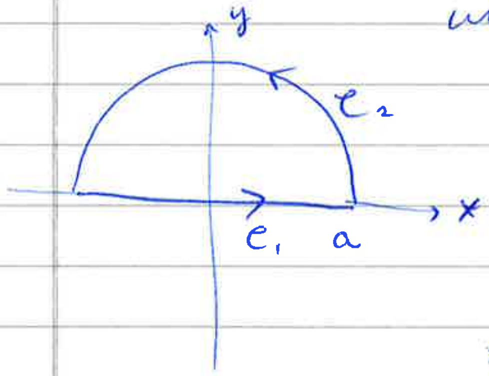
$$= 0.$$

Alternatively, note $\underset{\sim}{F} = \underset{\sim}{\nabla}\phi$ where $\phi(x,y,z) = xyz$.

So $\int_C \underset{\sim}{F}\cdot d\underset{\sim}{r} = \phi(1,0,0) - \phi(-1,0,0) = 0 - 0 = 0$.

17. Find $\oint_C x\,dy$, $\oint_C y\,dx$ when

C is boundary of $x^2+y^2 \le a^2$, $y \ge 0$, with positive orientation.



Let $C = C_1 + C_2$ as shown

On C_1, $y=0 \Rightarrow dy = 0. \Rightarrow$

$$\Rightarrow \int_{C_1} x\,dy = 0, \qquad \int_{C_1} y\,dx = 0.$$

On C_2, $x = a\cos\theta$, $y = a\sin\theta$, $\theta = 0$ to π.
$dx = -a\sin\theta\,d\theta$, $dy = a\cos\theta\,d\theta$.

$$\int_{C_2} x\,dy = \int_0^\pi a\cos\theta\, a\cos\theta\,d\theta = a^2\int_0^\pi \cos^2\theta\,d\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \tfrac{1}{2}(\cos 2\theta + 1)$$

$$\int_{C_2} x\,dy = \frac{a^2}{2}\int_0^\pi (\cos 2\theta + 1)\,d\theta = \frac{a^2}{2}\left[\frac{1}{2}\sin 2\theta + \theta\right]_0^\pi$$

$$= \frac{a^2\pi}{2}.$$

$\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow$
$\sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)$

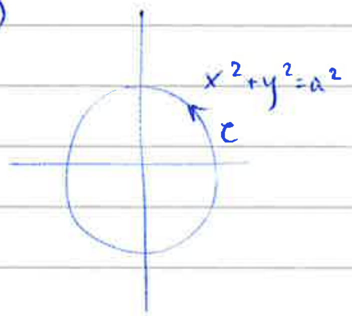
$$\int_{C_2} y\,dx = -a^2\int_0^\pi \sin^2\theta\,d\theta = -\frac{a^2}{2}\int_0^\pi (1 - \cos 2\theta)\,d\theta$$

$$= -\frac{a^2}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^\pi = -\frac{\pi a^2}{2}.$$

$$\Rightarrow \oint_C x\,dy = 0 + \frac{a^2\pi}{2} = \frac{a^2\pi}{2}, \qquad \oint_C y\,dx = 0 - \frac{\pi a^2}{2} = -\frac{\pi a^2}{2}.$$

(22) $\dfrac{1}{2\pi}\oint_{\mathcal{C}} \dfrac{-y\,dx + x\,dy}{x^2+y^2} = I$

(a)



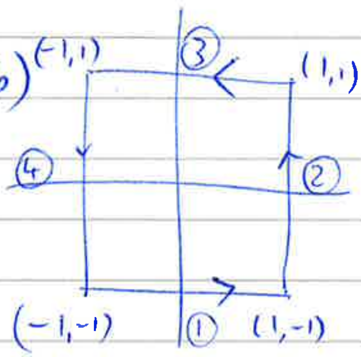
$x = a\cos\theta \qquad y = a\sin\theta \qquad \theta = 0 \text{ to } 2\pi.$

$dx = -a\sin\theta\,d\theta \qquad dy = a\cos\theta\,d\theta$

$$I = \frac{1}{2\pi}\int_0^{2\pi} \frac{a^2\sin^2\theta + a^2\cos^2\theta}{a^2}\,d\theta$$

$$= 1$$

(b)



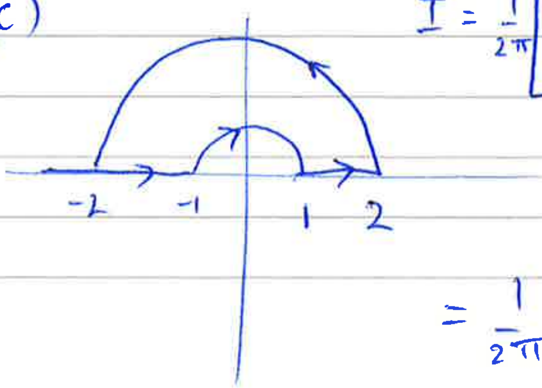
$$I = \frac{1}{2\pi}\left[\int_{-1}^{1}\frac{dx}{x^2+1} + \int_{-1}^{1}\frac{dy}{1+y^2} - \int_{1}^{-1}\frac{dx}{x^2+1} - \int_{1}^{-1}\frac{dy}{1+y^2}\right]$$

$$= \frac{4}{2\pi}\int_{-1}^{1}\frac{dx}{x^2+1}.$$

Now $\displaystyle\int_{-1}^{1}\frac{1}{x^2+1}\,dx = \arctan x\Big|_{-1}^{1} = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

$\Rightarrow \quad I = \dfrac{4}{2\pi}\cdot\dfrac{\pi}{2} = 1.$

(c)



$$I = \frac{1}{2\pi}\left[\int_0^{\pi}\frac{2^2\sin^2\theta + 2^2\cos^2\theta}{2^2}\,d\theta + \int_{-2}^{-1}\frac{0\;dx}{1+0^2} + \int_{\pi}^{0}\frac{1^2\sin^2\theta + 1^2\cos^2\theta}{1^2}\,d\theta + \int_1^2 0\,dx\right]$$

$$= \frac{1}{2\pi}\left[\pi - \pi\right] = 0.$$

(15.5) 3. $Ax + By + Cz = D$ inside $x^2/a^2 + y^2/b^2 = 1$.

Need to find dS on plane.
Let $f(x,y,z) = Ax + By + Cz$, then the given plane is the level surface $f(x,y,z) = D$.

Assume that $C \neq 0$, so that the normal vector is not parallel to xy-plane (otherwise area of intersection is infinite).
Then, since the surface has 1-1 projection onto xy-plane,

$$dS = \left| \frac{\nabla f(x,y,z)}{\frac{\partial f}{\partial z}(x,y,z)} \right| dx\,dy.$$

We have $\nabla f = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$

$$\frac{\partial f}{\partial z} = C$$

$$\Rightarrow \quad dS = \frac{\sqrt{A^2 + B^2 + C^2}}{|C|}\, dx\,dy.$$

The projection of the intersection of the plane and cylinder onto xy-plane is the solid ellipse $x^2/a^2 + y^2/b^2 \leq 1$. So the area of the surface:

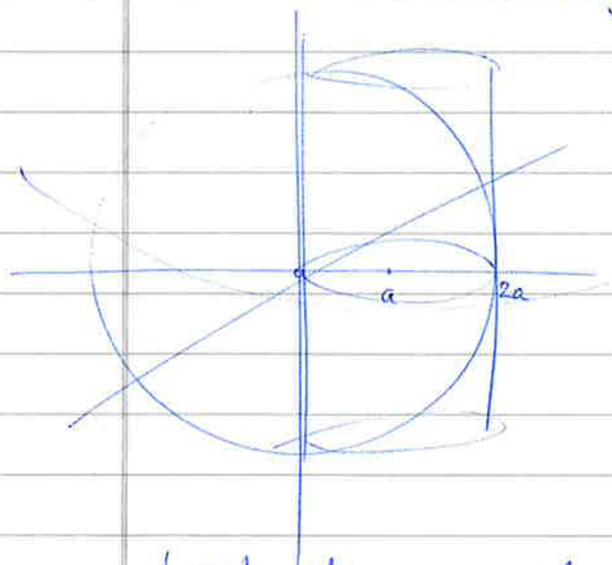
$$\text{Area} = \iint_{x^2/a^2 + y^2/b^2 \leq 1} \frac{\sqrt{A^2 + B^2 + C^2}}{|C|}\, dx\,dy.$$

$$= \frac{\sqrt{A^2+B^2+C^2}}{|C|} \cdot \text{Area of ellipse}$$

$$= \frac{\sqrt{A^2+B^2+C^2}}{|C|} \cdot \pi a b$$

(4) Find area of S: piece of $x^2+y^2+z^2 = 4a^2$ inside $x^2+y^2 = 2ay$.

$$\Rightarrow x^2 + (y-a)^2 = a^2$$

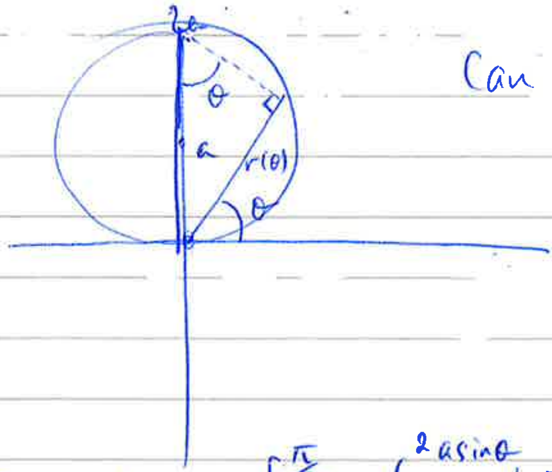


Let $f(x,y,z) = x^2+y^2+z^2$, then S: $f(x,y,z) = 4a^2$.

$$\underset{\sim}{\nabla} f = 2x\hat{i} + 2y\hat{j} + 2z\underline{k} \Rightarrow |\underset{\sim}{\nabla} f| = 2\cdot 2a = 4a$$

$$\partial f/\partial z = 2z.$$

Look at piece of S with $z \geq 0, x \geq 0$. Projection onto xy-plane is $x^2+(y-a)^2 \leq a^2$



Can parametrise by $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq r \leq 2a\sin\theta$.

$$\text{So Area} = \int_0^{\frac{\pi}{2}} d\theta \int_0^{2a\sin\theta} \left| \frac{\underset{\sim}{\nabla} f}{\frac{\partial f}{\partial z}} \right| r\,dr = \int_0^{\frac{\pi}{2}} d\theta \int_0^{2a\sin\theta} \frac{2a}{|z|}\, r\,dr$$

Now $z = \sqrt{4a^2 - x^2 - y^2}$

$= \sqrt{4a^2 - r^2}$

$$\Rightarrow A = \int_0^{\frac{\pi}{2}} d\theta \int_0^{2a\sin\theta} \frac{2ar}{\sqrt{4a^2-r^2}}\, dr$$

$$= \int_0^{\frac{\pi}{2}} d\theta\, (-2a)(4a^2-r^2)^{\frac{1}{2}} \Big|_0^{2a\sin\theta}$$

$$= -2a \int_0^{\frac{\pi}{2}} \underbrace{(4a^2 - 4a^2\sin^2\theta)^{\frac{1}{2}}}_{2a\cos\theta} - 2a \; d\theta$$

$$= -4a^2 \int_0^{\pi/2} \cos\theta - 1 \; d\theta$$

$$= -4a^2 (\sin\theta - \theta)\Big|_0^{\pi/2} = -4a^2\left(1 - \frac{\pi}{2}\right)$$

$$= 4a^2(\pi/2 - 1)$$

That was for the top half, $z \geq 0$, $x \geq 0$

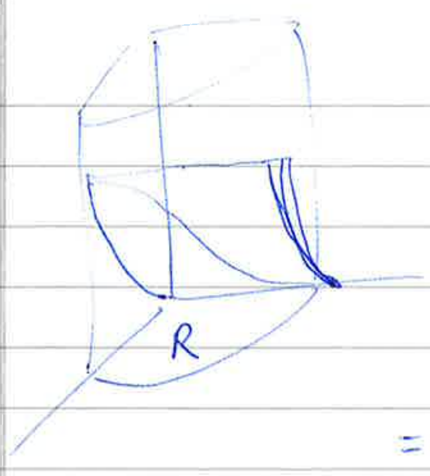
By symmetry, total area $= 16a^2(\pi/2 - 1)$

$= 8a^2(\pi - 2)$

(7) $\iint_S x \, dS$ over $z = \frac{x^2}{2}$ in 1^{st} octant of $x^2+y^2=1$.

For a surface that is a graph $z = \frac{x^2}{2} = f(x,y)$ we know that

$dS = \sqrt{1 + (\partial f/\partial x)^2 + (\partial f/\partial y)^2} = \sqrt{1+x^2}$ in our case.



So $\mathbb{I} = \iint\limits_R x\sqrt{1+x^2}\,dx\,dy$

$$= \int_0^1 dx \int_0^{\sqrt{1-x^2}} x\sqrt{1+x^2}\,dy$$

$$= \int_0^1 dx\,\left(x\sqrt{1-x^2}\sqrt{1+x^2}\right)$$

$$= \int_0^1 x\sqrt{1-x^4}\,dx \qquad \text{Let} \quad u = x^2, \quad du = 2x\,dx$$

$$= \frac{1}{2}\int_0^1 \sqrt{1-u^2}\,du \qquad \text{let} \quad u = \sin\theta, \quad du = \cos\theta\,d\theta \qquad u=0 \Rightarrow \theta=0, \quad u=1 \Rightarrow \theta = \pi/2$$

$$= \frac{1}{2}\int_0^{\pi/2} \cos\theta \cdot \cos\theta\,d\theta = \frac{1}{2}\int_0^{\pi/2} \cos^2\theta\,d\theta$$

use $\cos 2\theta = 2\cos^2\theta - 1$

$$= \frac{1}{4}\int_0^{\pi/2} \cos 2\theta + 1\,d\theta = \frac{1}{4}\left(\frac{1}{2}\sin 2\theta + \theta\right)\Bigg|_0^{\pi/2}$$

$$= \frac{1}{4}\left[\frac{\pi}{2}\right] = \pi/8.$$

(8) Area of $z^2 = x^2 + y^2$ inside $x^2 + y^2 = 2ay$.

Consider part with $z \geq 0$, $x \geq 0$ first.

Let $z = f(x,y) = \sqrt{x^2+y^2}$.

We have $2z \dfrac{\partial z}{\partial x} = 2x \Rightarrow \dfrac{\partial z}{\partial x} = \partial f/\partial x = x/z$

$2z \dfrac{\partial z}{\partial y} = 2y \Rightarrow \partial z/\partial y = \partial f/\partial y = y/z$.

So $dS = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\,dy$

$= \sqrt{1 + x^2/z^2 + \frac{y^2}{z^2}}\, dx\,dy = \sqrt{\frac{z^2 + \overbrace{x^2+y^2}^{z^2}}{z^2}}\, dx\,dy$

$= \sqrt{2}\, dx\,dy$.

Thus $A = \int_0^{\pi/2} d\theta \int_0^{2a\sin\theta} \sqrt{2}\, r\, dr$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

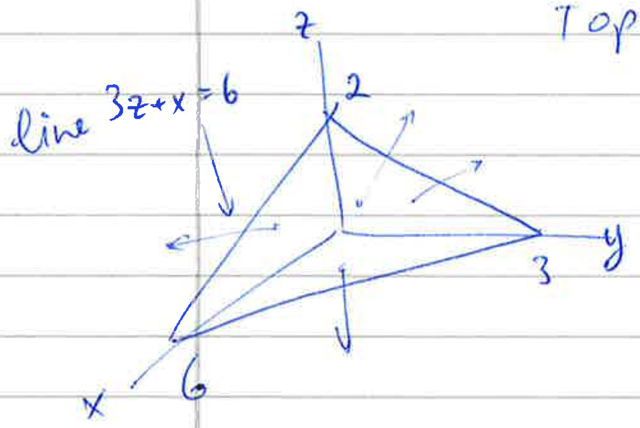
$= \int_0^{\pi/2} d\theta \left(\frac{1}{\sqrt{2}}\, 4a^2 \sin^2\theta\right)$

$= \frac{\cancel{2}\sqrt{2}\, a^2}{\cancel{2}} \left(\theta - \frac{1}{2}\sin 2\theta\right)_0^{\pi/2}$

$= \sqrt{2} a^2 (\pi/2)$

total area $= 4A = 2\sqrt{2}\, a^2 \pi$

15.6 (1) $\underset{\sim}{F} = x\underset{\sim}{i} + z\underset{\sim}{j}$, $x + 2y + 3z = 6$
and coord planes.



Top surface:

$\underset{\sim}{n} = \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, $|\underset{\sim}{n}| = \sqrt{14}$

$$\hat{\underset{\sim}{N}} = \frac{\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}}{\sqrt{14}}$$

$$\underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = \frac{x + 2z}{\sqrt{14}}.$$

Using projection onto xz-plane, $dS = \left|\frac{\nabla f}{\partial f/\partial y}\right| dx\,dz$

where $f = x + 2y + 3z \Rightarrow dS = \frac{\sqrt{14}}{2}\, dx\,dz.$

$$\text{So} \iint_{\text{top}} \underset{\sim}{F} \cdot \hat{\underset{\sim}{N}}\, dS = \frac{1}{2} \iint_{\substack{\text{proj.} \\ \text{onto} \\ xz\text{-plane}}} x + 2z \, dx\,dz = \frac{1}{2} \int_0^2 dz \int_0^{6-3z} x + 2z \, dx$$

$$= \frac{1}{2} \int_0^2 \frac{(6-3z)^2}{2} + 2z(6-3z)\, dz$$

$$= \frac{1}{4} \int_0^2 36 - 36z + 9z^2 + 24z - 12z^2 \, dz$$

$$= \frac{1}{4} \left(36z - 6z^2 - z^3 \right)_0^2 = 10.$$

bottom surface: $\hat{\underset{\sim}{N}} = -\underset{\sim}{k}$, $\underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = 0 \Rightarrow$ no flux

back surface: $\hat{\underset{\sim}{N}} = -\underset{\sim}{i}$, $\underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = -x = 0$ as $x = 0$.

left surface: $\hat{\underset{\sim}{N}} = -\underset{\sim}{j}$, $\underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = -z$, $dS = dx\,dz$.

$$\text{Flux} = -\int_0^2 dz \int_0^{6-3z} z\, dx = -\int_0^2 z(6-3z)\, dz$$

$$= -\left(3z^2 - z^3\right)_0^2 = -4$$

total flux $= 10 - 4 = 6$.

(2) $\underset{\sim}{F} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$, outward through $x^2 + y^2 + z^2 = a^2$.

$$\hat{\underset{\sim}{N}} = \hat{\underset{\sim}{r}} = \frac{\underset{\sim}{r}}{|\underset{\sim}{r}|} = \frac{x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}}{a}$$

$$\underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = \frac{1}{a}\left(x^2 + y^2 + z^2\right) = a$$

$$\text{flux} = \iint_S \underset{\sim}{F} \cdot \hat{\underset{\sim}{N}}\, dS = a \iint_S dS = a \cdot 4\pi a^2$$
$$= 4\pi a^3$$

(3) $\underset{\sim}{F} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$0 \le x \le a, \quad 0 \le y \le b, \quad 0 \le z \le c.$

top: $\hat{\underset{\sim}{N}} = \underset{\sim}{k}, \quad \underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = z = c$

bottom: $\hat{\underset{\sim}{N}} = -\underset{\sim}{k}, \quad \underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = -z = 0$

left: $\hat{\underset{\sim}{N}} = -\underset{\sim}{j}, \quad \underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = -y = 0$

right $\hat{\underset{\sim}{N}} = \underset{\sim}{j}, \quad \underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = y = b$

front: $\hat{\underset{\sim}{N}} = \underset{\sim}{i}, \quad \underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = x = a$

back: $\hat{\underset{\sim}{N}} = -\underset{\sim}{i}, \quad \underset{\sim}{F} \cdot \hat{\underset{\sim}{N}} = -x = 0.$

total flux $= \underset{\substack{\text{top} \\ \text{(area)}}}{\iint} c \, dS + \underset{\substack{\text{right} \\ \text{(area)}}}{\iint} b \, dS + \underset{\substack{\text{front} \\ \text{(area)}}}{\iint} a \, dS$

$= c\,ab + b\,ac + abc$

$= 3abc.$

(6) $\underset{\sim}{F} = x\underset{\sim}{i} + x\underset{\sim}{j} + \underset{\sim}{k}$ upward through $z = x^2 - y^2$
$= f(x,y)$

inside cylinder $x^2 + y^2 = a^2$.

$\underset{\sim}{n} = -\partial f/\partial x \, \underset{\sim}{i} - \partial f/\partial y \, \underset{\sim}{j} + \underset{\sim}{k}$

$= -2x\underset{\sim}{i} + 2y\underset{\sim}{j} + \underset{\sim}{k}$

And then: $d\underset{\sim}{S} = \underset{\sim}{n} \, dx\,dy$

$\Rightarrow \underset{\sim}{F} \cdot d\underset{\sim}{S} = -2x^2 + 2xy + 1$

integrates to zero by symmetry.

$$\text{Flux} = \iint_{x^2+y^2 \le a^2} -2x^2 + 2xy + 1 \; dx\,dy$$

$$= \int_0^{2\pi} \int_0^a (-2r^2\cos^2\theta + 1)\, r\,dr\,d\theta$$

$$= \int_0^{2\pi} -\tfrac{1}{2}a^4\cos^2\theta + \tfrac{1}{2}a^2 \; d\theta$$

$$= -\tfrac{1}{2}a^4\left[\tfrac{1}{2}\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{2\pi} + \pi a^2$$

$$= \pi a^2\left(1 - \frac{a^2}{2}\right)$$

(7) $\underset{\sim}{F} = y^3\underset{\sim}{i} + z^2\underset{\sim}{j} + x\underset{\sim}{k}$ downward through $z = 4 - x^2 - y^2$ above $z = 2x+1$.

Find intersection: $2x+1 = z = 4 - x^2 - y^2$

$$\Rightarrow \quad x^2 + 2x + 1 + y^2 = 4$$
$$(x+1)^2 + y^2 = 2^2.$$

circle radius 2 centred at $(-1, 0)$.

$z = f(x,y) = 4 - x^2 - y^2 \Rightarrow$

$\underset{\sim}{n} = 2x\underset{\sim}{i} + 2y\underset{\sim}{j} + \underset{\sim}{k}$ but we need downward

so take $\underset{\sim}{n} = -2x\underset{\sim}{i} - 2y\underset{\sim}{j} - \underset{\sim}{k}$.

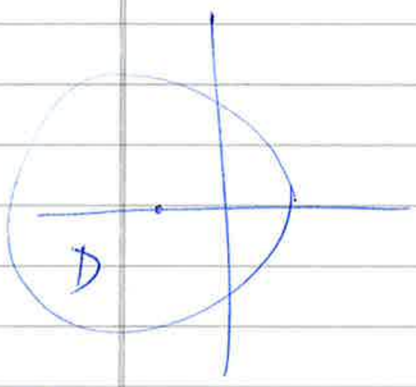
Then $d\underset{\sim}{S} = \underset{\sim}{n}\, dx\, dy = (-2x\underline{i} - 2y\underline{j} - \underline{k})\, dx\, dy$

$\underset{\sim}{F} \cdot d\underset{\sim}{S} = \left(-2xy^3 + 2y(4-x^2-y^2)^2 - x\right) dx\, dy$

let $D = \{ (x+1)^2 + y^2 \le 2^2 \}$, then

Flux $= -\iint_D \left(+2xy^3 + 2y(4-x^2-y^2)^2 + x\right) dx\, dy$

odd in y, so integrate to 0.



D symmetric about x axis ($y \mapsto -y$).

Flux $= -\iint_D x \, dx\, dy$

$= 4\pi.$

(9) $\underset{\sim}{F} = x\underset{\sim}{i} + y\underset{\sim}{j}$ upward through $z = 2 - x^2 - 2y^2$
above xy plane.

intersects xy plane when $x^2 + 2y^2 = 2$

$\left(\frac{x}{\sqrt{2}}\right)^2 + y^2 = 1$ ellipse. E

$u^2 + v^2 = 1$

$z = f(x,y) = 2 - x^2 - 2y^2 \Rightarrow \underset{\sim}{n} = +2x\underset{\sim}{i} + 4y\underset{\sim}{j} + \underset{\sim}{k}$

$d\underset{\sim}{S} = \underset{\sim}{n}\, dx\, dy = (+2x\underset{\sim}{i} + 4y\underset{\sim}{j} + \underset{\sim}{k})\, dx\, dy$

$\underset{\sim}{F} \cdot d\underset{\sim}{S} = (+2x^2 + 4y^2)\, dx\, dy$

flux $= \iint_E 2x^2 + 4y^2 \, dx\, dy.$ let $u = \frac{x}{\sqrt{2}}$, $v = y$,

$dx = \sqrt{2}\, du$ $dv = dy$

$= \iint_{\substack{\text{disc} \\ \text{radius } 1}} (4u^2 + 4v^2)\sqrt{2}\, du\, dv$

$= 4\sqrt{2} \int_0^{2\pi} \int_0^1 r^2 \cdot r\, dr\, d\theta$

$= \frac{4\sqrt{2} \cdot 2\pi}{4} = 2\sqrt{2}\pi.$

(11) $\underset{\sim}{F} = x\underset{\sim}{i} + y\underset{\sim}{j} + z^2\underset{\sim}{k}$ upward through $\underset{\sim}{r} = u\cos v\,\underset{\sim}{i} + u\sin v\,\underset{\sim}{j} + u\underset{\sim}{k}$

$0 \le u \le 2$, $0 \le v \le \pi$.

$$\underset{\sim}{n} = \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} = (\cos v\,\underset{\sim}{i} + \sin v\,\underset{\sim}{j} + \underset{\sim}{k}) \times (-u\sin v\,\underset{\sim}{i} + u\cos v\,\underset{\sim}{j})$$

$$= \begin{vmatrix} i & j & k \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = \underset{\sim}{i}(-u\cos v) + \underset{\sim}{j}(-u\sin v) + \underset{\sim}{k}(u)$$

$$d\underset{\sim}{S} = \pm \underset{\sim}{n}\,du\,dv = -u\cos v\,\underset{\sim}{i} - u\sin v\,\underset{\sim}{j} + u\underset{\sim}{k}$$ (pointing up).

$$\underset{\sim}{F}(\underset{\sim}{r}(u,v)) = u\cos v\,\underset{\sim}{i} + u\sin v\,\underset{\sim}{j} + u^2\underset{\sim}{k}$$

$$\underset{\sim}{F} \cdot d\underset{\sim}{S} = -u^2\cos^2 v - u^2\sin^2 v + u^3$$

$$= u^3 - u^2.$$

$$\text{Flux} = \int_0^\pi dv \int_0^2 u^3 - u^2\,du$$

$$= \pi \cdot \left[\frac{u^4}{4} - \frac{u^3}{3}\right]_0^2 = \pi\left(4 - \frac{8}{3}\right)$$

$$= \frac{4\pi}{3}.$$