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University of
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FACULTY OF SCIENCE AND TECHNOLOGY

DATE: August 28, 2018

SUBJECT: PET 565 – Core scale modeling and interpretation

TIME: 4 hours

AID: No printed or written means allowed. Definite basic calculator allowed.

THE EXAM CONSISTS OF 5 PROBLEMS ON 4 PAGES

REMARKS: You may answer in English or Norwegian. Exercises 1 - 2 (part A) and exercises 3 - 5 (part B) are given equal weight.

Problem 1.

✓ (a) Consider the conservation law

$$(*) \qquad u_t + f(u)_x = 0, \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x, t = 0) = \phi(x).$$

- We assume that $f'' > 0$. The expression

$$(***) \qquad u(x,t) = \phi\Big(x - f'(u(x,t))t\Big)$$

is then a general solution of (*) with (**), subject to the condition

$$1 + \phi'(x_0) f''(\phi(x_0))t \neq 0.$$

Verify this by performing a direct calculation (i.e. putting (***) into (*) and (**)).

✓ (b) Based on (***) compute an expression for u_x.
- Explain the concept of breaking time for the equation (*). Can you say something about the behavior of the characteristics at breaking time?

✓ (c) Consider a shock wave moving to the left with speed $s = -\frac{\Delta x}{\Delta t}$. Given that u_L and u_R are the solution values to the left and the right of the shock, respectively, the Rankine-Hugoniot condition states that

$$s = \frac{f(u_R) - f(u_L)}{u_R - u_L}.$$

- Show how to obtain this expression by integrating equation (*) over the region $R = [x_1, x_1 + \Delta x] \times [t_1, t_1 + \Delta t]$.

✓ (d) Consider (*) with $f(u) = \frac{1}{3}u^2$ and

$$\phi(x) = \begin{cases} 2, & 0 \leq x < 2; \\ 0, & \text{otherwise} \end{cases}$$

- Consider the two Riemann problems, one at $x = 0$ and the other at $x = 2$, respectively. Compute solutions in terms of shock waves and/or rarefaction waves.
- Compute the time T_c when the two waves will start to interact.

(e) Utilize the Rankine-Hugoniot condition in order to compute the solution for $t > T_c$, for the problem discussed in (d).

$2^{-0,5} = 0,707$

$(f')^{-1} = \frac{3}{2} u = \frac{3}{2} \frac{x}{t}$

Problem 2.

(a) Mass conservation of water and oil in a 1D reservoir is represented by the following equations:

$$\frac{\partial}{\partial x}\Big[\frac{kk_{rl}}{\mu_l}(\frac{\partial p}{\partial x}+\gamma_l)\Big]=\phi\frac{\partial S_l}{\partial t}, \qquad l=w,o$$

where $\gamma_l=\rho_l g\sin(\alpha)$ accounts for the gravity force.

- list some of the main assumptions for deriving the Buckley-Leverett (BL) model and define the different quantities
- introduce mobility functions λ_l and demonstrate how we can find the following expression for the pressure gradient

$$\frac{\partial p}{\partial x}=-\frac{u_T+\lambda_w\gamma_w+\lambda_o\gamma_o}{\lambda_T}, \qquad \lambda_T=\lambda_w+\lambda_o.$$

- what is u_T and can you express this in terms of some other available variables?

(b) Explain how to obtain the BL formulation

$$\phi\frac{\partial}{\partial t}S+u_T\frac{\partial F(S)}{\partial x}=0, \qquad S=S_w$$

Find the expression for the fractional flow function $F(S)$ when you let $f(S)=\frac{\lambda_w(S)}{\lambda_T(S)}$.

(c) Now, we consider a horizontal reservoir with fractional flow function $f(S)$ as shown in Fig. 1. The BL model takes the following form in dimensionless variables x_D and t_D

$$\frac{\partial S}{\partial t_D}+\frac{\partial f(S)}{\partial x_D}=0.$$

Based on Fig. 1 (next page), compute the solution (saturation distribution) after a time $T=0.5$.

(d) Use the plot of $f'(S)$ and give a sketch of the unphysical solution after a time $T=0.5$ based on the method of characteristics.

- Use the principle of mass conservation and derive the general mathematical expression for the front height S^* satisfied by the physical correct solution.

(e) For the flux function $f(S)$ shown in Fig. 1, we now want to include gravity. It is assumed that $\rho_w>\rho_o$. In particular, consider the following two cases:

(i) upwards dip ($\sin(\alpha)>0$);

(ii) downward dip ($\sin(\alpha)<0$).

- Explain (by sketching an approximate solution) how the solution will change compared to the one computed in (c). Back up your explanation by referring to a sketch of the corresponding fractional flow function for (i) and (ii).

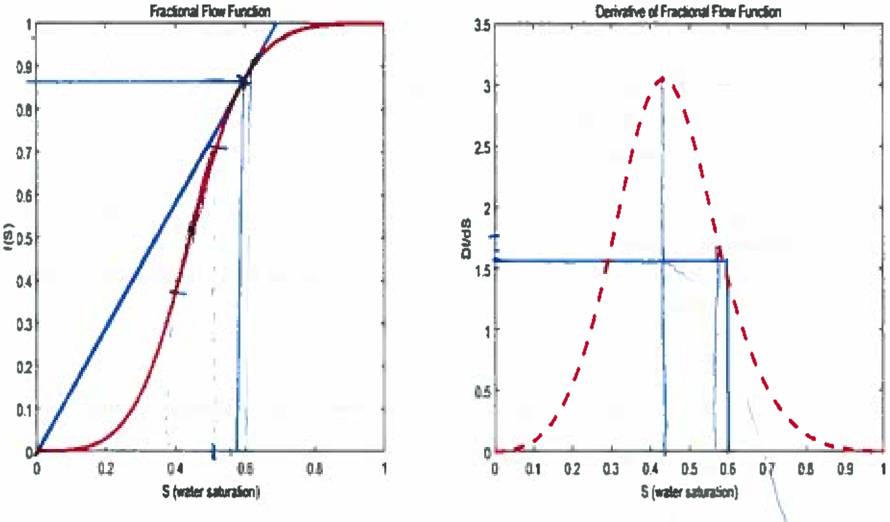

FIGURE 1. Left: $f(S)$. Right $f'(S)$.

Problem 3.

a). Consider a brine containing 0.1 mol/L $CaCl_2$. Calculate

- the concentrations of Ca, Cl (1p),
- the ionic strength of the brine (2p).
- the activity coefficients of the ion using Davies equation (2p)

b). Further,

- Mention two other ways to calculate activity coefficients than Davies equation. (2p)
- What are the main differences of the three approaches? (3p)

c). Dolomite mineral dissolves according to the reaction

$$CaMg(CO_3)_2 \text{ (s)} \rightleftharpoons Ca^{2+} \text{ (aq)} + Mg^{2+} \text{ (aq)} + 2CO_3^{2-} \text{ (aq)} \tag{1}$$

where the solubility constant is $K_d = 10^{-17}$. Assuming dolomite is dissolved in the 0.1 mol/L $CaCl_2$ brine:

- Write the law of mass balance for the dissolution reaction. (3p)
- Using stoichiometry of the reaction, express the relation between ion concentrations m_{ca}, m_{mg}, m_{co3} and the concentration of dissolved dolomite m_d. Why is $m_{ca} \neq m_d$? (4p)
- Using appropriate assumptions, which must be stated, calculate the concentrations $m_d, m_{ca}, m_{mg}, m_{co3}$. (4p)

Problem 4. Consider injection of a species with concentration c that is controlled by advection, and adsorption described by an isotherm $q(c)$. Let the initial concentration be 0 mol/L, the injected concentration is 5 mol/L and $q(c) = 0.05c^3$ mol/L.

a). Calculate the retardation factor R as function of concentration and plot R from $c = 0$ to 5 mol/L. What class of concentration front will be observed and why? (5p)

b). How many pore volumes must be injected to reach steady state (defined by having the injected concentration value at any point in the system)? (5p)

Problem 5. Consider a system where 0.1 mol/L $MgCl_2$ brine is injected into chalk. Two processes take place: a) ion exchange between Ca and Mg on the surface

$$CaX_2 + Mg^{2+} \rightleftharpoons Ca^{2+} + MgX_2 \tag{2}$$

and also b) precipitation of MgCO3 and dissolution of calcite CaCO3 which occurs in a substitution-like process as:

$$CaCO_3 + Mg^{2+} \rightleftharpoons Ca^{2+} + MgCO_3 \tag{3}$$

The surface originally contains only Ca and the initial brine is distilled water (no ions).

a). Present partial differential equations for the ions Ca, Mg and Cl and the minerals $CaCO_3$ and $MgCO_3$. Explain the terms, the mechanisms they represent and how the source terms are related. (5p)

b). Plot effluent concentrations of Ca, Mg and Cl as function of pore volumes injected. Explain the trends and what determines the absolute values. (5p)

c). Assume the injection rate is doubled. What would be different in the previous plot and why? (5p)

Formulas. Ionic strength:

$$I = 1/2 \sum_i m_i z_i^2 \tag{4}$$

Davies equation:

$$\log \gamma_i = -A(T) z_i^2 \Big(\frac{\sqrt{I}}{1+\sqrt{I}} - 0.3I \Big), \qquad A(25°C) \approx 0.51 \tag{5}$$

pH:

$$pH = -log([H^+]) \tag{6}$$

Retardation:

$$R_c = 1 + dq/dc, \quad R_f = 1 + \Delta q/\Delta c \tag{7}$$