Solution: MPE 340 Reservoir simulation, introduction
DATE: Dec 19, 2008

Problem 1

a) See course material for assumptions (Hans' part/Differential equations (text), p 34).
Water phase consists of water component only.
Oil phase consist of oil component and part of the gas component (dissolved gas).
Gas phase consists of part of the gas component (free gas).

b) See course material (Hans' part/Differential equations (text), p 36).

c) Mass balance equations

$$\nabla \circ \left[\frac{\bar{K} k_{rw}}{\mu_w B_w} (\nabla p_w - \gamma_w \nabla d) \right] + Q_w = \frac{\partial}{\partial t}\left(\frac{\varphi S_w}{B_w} \right)$$

$$\nabla \circ \left[\frac{\bar{K} k_{ro}}{\mu_o B_o} (\nabla p_o - \gamma_o \nabla d) \right] + Q_o = \frac{\partial}{\partial t}\left(\frac{\varphi S_o}{B_o} \right)$$

$$\nabla \circ \left[\frac{\bar{K} k_{rg}}{\mu_g B_g} (\nabla p_g - \gamma_g \nabla d) \right] + \nabla \circ \left[\frac{\bar{K} R_{so} k_{ro}}{\mu_o B_o} (\nabla p_o - \gamma_o \nabla d) \right] + \nabla \circ \left[\frac{\bar{K} R_{sw} k_{rw}}{\mu_w B_w} (\nabla p_w - \gamma_w \nabla d) \right]$$
$$+ Q_g = \frac{\partial}{\partial t}\left(\frac{\varphi S_g}{B_g} + \frac{\varphi R_{so} S_o}{B_o} + \frac{\varphi R_{sw} S_w}{B_w} \right)$$

where R_{so} and R_{sw} denote the solution ratios of gas in oil and gas in water.

Problem 2

a) See course material (Hans' part/Project Buckley-Leverett Base/Theory B-L base (text), p 1)

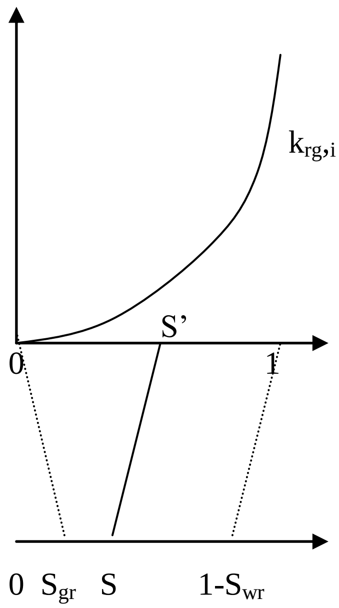
b) See course material (Hans' part/Project Buckley-Leverett Base/Theory B-L base (text), pp 1 - 3).

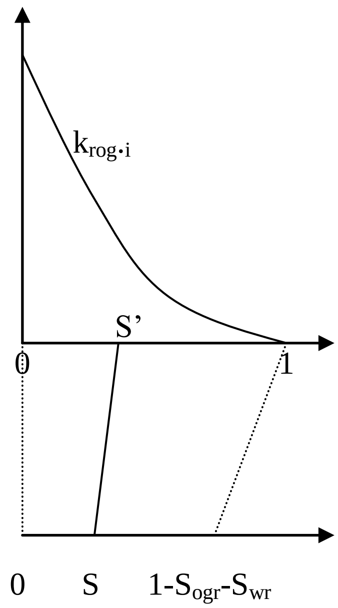
c) Water saturation as function of position and time is computed. The parameters involved in the fractional flow function are relative permeabilities and viscosities.

Problem 3

a) See course material (Hans' part/Input data (text), p 26).

b) Solution if input curves $k_{rg,i}, k_{rog,i}$ are normalized.





S' divides interval from 0 to 1 in the same proportion as S divides interval from S_{gr} to 1-S_{wr}

$$S' = \frac{S - S_{gr}}{1 - S_{gr} - S_{wr}}$$

$$k_{rg}(S) = KRG * k_{rg,i}(S')$$

S' divides interval from 0 to 1 in the same proportion as S divides interval from 0 to 1- S_{ogr} - S_{wr}

$$S' = \frac{S}{1 - S_{ogr} - S_{wr}}$$

$$k_{rog}(S) = KRO * k_{rog,i}(S')$$

KRG and *KRO* are parameters for scaling relative permeability values.

Problem 4

a) See course material (Hans' part/Initialization (text), p 124).

b) See course material. (Hans' part/Initialization (text), pp 124 - 126).

Problem 5

a) Explicit formula for computing the solution:

$$u_i^{n+1} = \frac{1}{4}(u_{i-1}^n + 3u_i^n) .$$

	1	2	3	4	5
t = 0	**0**	**0.5**	**1**	**1**	**1**
t = 1/16	**0**	**0.375**	**0.875**	**1**	**1**
t = 1/8	**0**	**0.281**	**0.750**	**0.969**	**1**

b) Implicit method is always stable (unconditionally stable). Explicit method is not always stable and requires restriction on time step length for stability to be guaranteed.

From the numerical experimentation carried out in class we conclude that the maximal time step length that can be used in a) is $\Delta t = \frac{1}{4}$, i.e. $\frac{\Delta t}{\Delta x} = 1$.