University of Stavanger

**FACULTY OF SCIENCE AND TECHNOLOGY**

**SUBJECT: MPE 340 Reservoir Simulation, Introduction DATE: May 8, 2009**

**TIME: 4 hours**

**AID: No printed or written means are allowed. Definite basic calculator allowed.**

**THE EXAM CONSISTS OF 5 PROBLEMS ON 5 PAGES**

**REMARKS: You may answer in English or Norwegian.**
**All problem parts a), b), ... are given equal weight.**

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## Problem 1

The single-phase continuity equation is given by

$$\nabla \cdot (\rho \vec{v}) = -\frac{\partial}{\partial t}(\phi\rho) + q.$$

a) Define the different quantities (including the definition of $\nabla \cdot = \text{div}$).

- Write down the general Darcy's law in 3D, define the various quantities, and use Darcy's law to rewrite the above continuity equation. Assume that $z$-axis is directed downwards parallel to the vertical direction.

- By imposing various simplifying assumptions on the mass balance equation we get an equation of the form

$$\frac{k}{\mu}\frac{\partial}{\partial x}\left(\rho\frac{\partial p}{\partial x}\right) + q = \phi\frac{\partial \rho}{\partial t}. \tag{1}$$

  List the different assumptions.

b) We assume that the fluid is compressible, i.e., $\rho = \rho(p)$. Let $c$ be the compressibility (constant) and given by

$$c = -\frac{1}{V(p)}\frac{dV(p)}{dp},$$

where $V(p)$ represents volume. Show that $\rho c = \frac{d\rho(p)}{dp}$.

c) Show that $\rho c \nabla p(\rho) = \nabla \rho$ by using the result of b).
Finally, use this result in Eq. (1) and show how to obtain a diffusion equation in terms of the density $\rho$ given by

$$\kappa \frac{\partial^2 \rho}{\partial x^2} + \frac{q}{\phi} = \frac{\partial \rho}{\partial t}.$$

In particular, identify the constant $\kappa$.

**Problem 2**

In this problem the solution of the Buckley-Leverett equation $\partial_t S + \partial_x f(S) = 0$ for a horizontal 1D domain is considered in dimensionless variables $x$ and $t$.

a) Give the main assumptions used in the Buckley-Leverett theory for the study of two-phase water-oil flow in 1D porous media.
Define the fractional flow function $f(S)$ in terms of relative permeability functions and viscosities.

b) A specific fractional flow function $f(S)$ is given in Fig. 1 and the corresponding derivative $f'(S)$ is given in Fig. 2. In the following we shall use this information to construct a good approximation of the solution for a dimensionless time $T = 0.4$. In particular,

- compute the (approximate) water front height $S_f$ and its position $x_f$ at time $T = 0.4$;
- compute (approximate) saturations at dimensionless distances $x_1 = 0.2$ and $x_2 = 0.4$ and make a sketch of the solution.

c) Recovery factor is defined as the amount of produced oil divided by the initial oil in place.

- For the example given in b), find the time $T_b$ when the water front reaches the producer located at $x = 1$.
- What is the expression for the oil recovery for time $T < T_b$? Give a derivation of this expression.
- Let $S^*(T)$ denote the height of the water front at $x = 1$ for a time $T > T_b$. Use $S^*(T)$ and derive the expression $R = S^*(T) + T(1 - f(S^*(T)))$ for the oil recovery for time $T > T_b$.
  For the example given in b), compute oil recovery R for the dimensionless time $T = 1$.

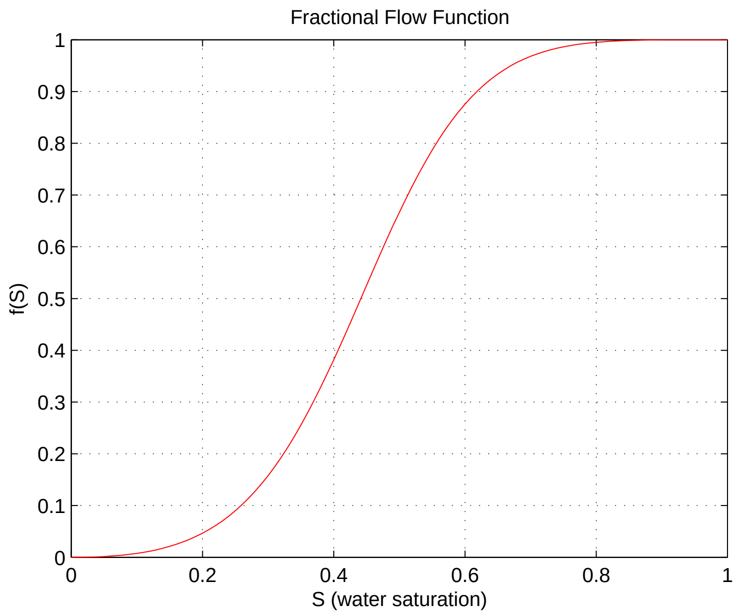


FIGURE 1. $f(S)$

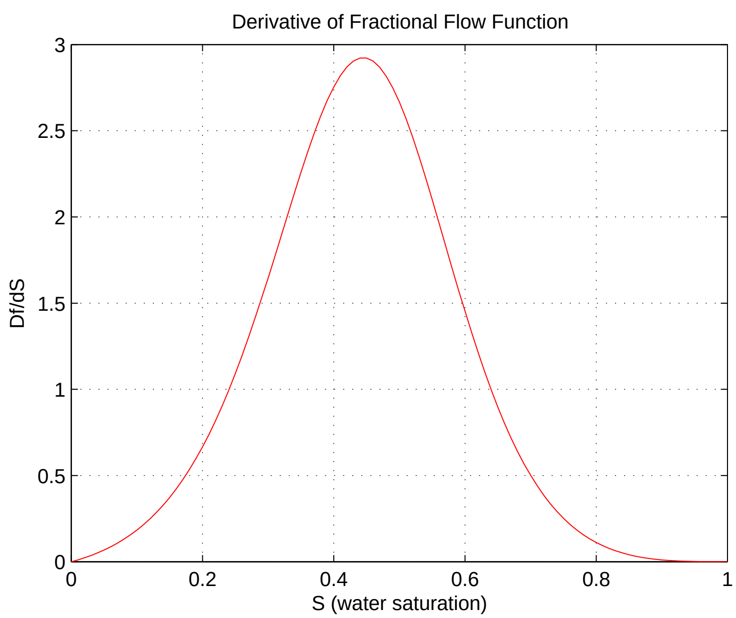


FIGURE 2. $f'(S)$

**Problem 3**

In this problem the standard Black-Oil Model is discussed.

a) What are the main assumptions for the standard Black-Oil Model.

b) Let $\rho_w$, $\rho_o$, $\rho_g$, and $\rho_{dg}$ be the component densities at reservoir conditions. Write down the continuity equations for the water, oil, and gas components. No derivation is required.

c) Introduce densities at standard conditions $\rho_w^s$, $\rho_o^s$, and $\rho_g^s$ (which are constant) and rewrite the continuity equations in b) by making use of volume factors $B_l$ for $l = w, o, g$ and gas-oil solution ratio $R_s$. It's not necessary to introduce Darcy's law.

**Problem 4**

Given Corey normalized relative permeability functions

$$k_{rw}^t(x) = x^3, \qquad k_{row}^t(x) = (1-x)^2, \qquad \text{oil-water system}$$
$$k_{rg}^t(x) = x^2, \qquad k_{rog}^t(x) = (1-x)^3, \qquad \text{oil-gas system.}$$

where $x \in [0,1]$. End point saturations are given as follows:

$$S_{wr} = 0.15, \qquad S_{owr} = 0.1, \qquad S_{gr} = 0.2, \qquad S_{ogr} = 0.25.$$

a) Write down the expressions for the relative permeability curves $k_{rw}^{2p}(S_w)$ and $k_{row}^{2p}(S_w)$, with corresponding intervals for $S_w$, for the water-oil system by using the normalized curves. Do the same for $k_{rg}^{2p}(S_g)$ and $k_{rog}^{2p}(S_g)$ for the two-phase gas-oil system. Let maximum relative permeability value for all cases be 1.

b) Consider a three-phase water-oil-gas system. Explain how to obtain $k_{rw}(S_w)$, $k_{rg}(S_g)$, and $k_{ro}(S_w, S_g)$ for the three-phase system by making use of the results from a).

c) Assume that $S_w = 0.5$ and $S_g = 0.3$.
Compute value of three-phase water relative permeability.
Compute value of three-phase gas relative permeability.
Compute value of three-phase oil relative permeability by using Baker's formula given by

$$k_{ro}(S_w, S_g) = \frac{S_w - S_{wr}}{S_w + S_g - S_{wr}} k_{row}^{2p}(S_w) + \frac{S_g}{S_w + S_g - S_{wr}} k_{rog}^{2p}(S_g).$$

**Problem 5**

For a liquid the diffusivity equation may be approximated to the following form,

$$\frac{Ckb_r}{\mu}\frac{\partial^2 p}{\partial x^2} + q = \phi c_f b_r \frac{\partial p}{\partial t}, \tag{2}$$

where $b_r$ is the formation volume factor (inverse) at a reference pressure $p_r$, $c_f$ the fluid compressibility, $C$ a unit conversion factor, $k$ permeability, $\mu$ viscosity, and $q$ is the surface volume rate per bulk volume.

a)
A one-dimensional reservoir is discretized into $N$ equal gridblocks with length $\Delta x$ and the time is discretized into timesteps of equal length $\Delta t$. Show that Eq. (2) with implicit numerical formulation may be expressed as

$$\begin{aligned}&\frac{Ckb_r}{\mu(\Delta x)^2}[p_{i+1}(t_{n+1}) - p_i(t_{n+1})]\\ &-\frac{Ckb_r}{\mu(\Delta x)^2}[p_i(t_{n+1}) - p_{i-1}(t_{n+1})] + q_i = \phi c_f b_r (p_i(t_{n+1}) - p_i(t_n))/\Delta t.\end{aligned} \tag{3}$$

b)
Show that Eq. (3) may be written in the following form,

$$a_i p_{i-1} - p_i + c_i p_{i+1} = -d_i,$$

where $a, c, d$ are constant coefficients. Assume that the reservoir is closed (i.e. no flow) at left and right boundary. Consequently, $p_x = 0$ at the boundary.