Solution.

MPE 340 Reservoir simulation, introduction   **DATE: May 7, 2010**

Problem 1

a) Input parameters needed: B_o (oil volume factor, both saturated and under saturated properties), R_s (gas in oil solution ratio), $\rho_{o,s}$ and $\rho_{g,s}$ (oil and gas reference densities).

$$\rho_o = \frac{\rho_{o,s}(V_o)_{STC} + \rho_{g,s}(V_{dg})_{STC}}{(V_o)_{RC}} = \frac{\rho_{o,s}}{B_o} + \frac{\rho_{g,s}(V_{dg})_{STC}(V_o)_{STC}}{(V_o)_{RC}(V_o)_{STC}} = \frac{\rho_{o,s}}{B_o} + \frac{\rho_{g,s}R_s}{B_o}$$

b) Rel. perm. input data: four tables of two-phase oil/water and oil/gas properties, usually normalized tables. End point saturations (critical saturations), end point rel. perm. values, one set of parameters for each grid block. Then scaling is used to compute real rel. perm. values.

c) Scaling procedure for gas relative permeability k_{rg}.
    Input: normalized table k_{rg}^t, critical gas saturation S_{gr}, critical water saturation S_{wr}, maximal value of gas rel. perm., *KRG*.
    Given gas saturation S_g, compute

$$S' = \frac{S_g - S_{gr}}{1 - S_{wr} - S_{gr}}, \quad k_{rg}(S_g) = KRG k_{rg}^t(S')$$

Problem 2

a) Black Oil assumptions:
    -   3 phases, water, oil, gas
    -   3 components, water, oil, gas
    -   no mass transfer water/hydrocarbons
    -   gas dissolves in oil
    -   constant temperature

    Component mass balance equations

$$\nabla \cdot \left[\frac{[k]k_{rw}}{\mu_w B_w}(\nabla p_w - \rho_w g \nabla d) \right] + Q_w = \frac{\partial}{\partial t}\left(\varphi \frac{S_w}{B_w}\right)$$

$$\nabla \cdot \left[\frac{[k]k_{ro}}{\mu_o B_o}(\nabla p_o - \rho_o g \nabla d) \right] + Q_o = \frac{\partial}{\partial t}\left(\varphi \frac{S_o}{B_o}\right)$$

$$\nabla \cdot \left[\frac{[k]k_{rg}}{\mu_g B_g}(\nabla p_g - \rho_g g \nabla d) \right] + \nabla \cdot \left[\frac{[k]k_{ro}R_s}{\mu_o B_o}(\nabla p_o - \rho_o g \nabla d) \right] + Q_g = \frac{\partial}{\partial t}\left(\varphi \frac{S_g}{B_g} + \varphi \frac{R_s S_o}{B_o}\right)$$

b) Compositional model assumptions:
  - 3 phases, water, oil, gas
  - Nc hydrocarbon components + water
  - no mass transfer water/hydrocarbons
  - hydrocarbon components exists in both oil and gas phase
  - constant temperature

Compositional mass balance equations

Water:

$$\nabla \cdot [\frac{[k]k_{rw}}{\mu_w} \rho_w (\nabla p_w - \rho_w g \nabla d)] + Q_w = \frac{\partial}{\partial t}(\varphi \rho_w S_w)$$

Each hydrocarbon component:

$$\nabla \cdot [\frac{[k]k_{ro}}{\mu_o} \rho_o x_i (\nabla p_o - \rho_o g \nabla d)] + \nabla \cdot [\frac{[k]k_{rg}}{\mu_g} \rho_g y_i (\nabla p_g - \rho_g g \nabla d)]$$

$$+ Q_i = \frac{\partial}{\partial t}(\varphi \rho_o x_i S_o + \varphi \rho_g y_i S_g), \ i = 1,..., N_c$$

Densities are mole densities and compositions $\{x_i\}$, $\{y_i\}$ are mole fractions.

Problem 3

a) Porosity is computed using the formula
$$\varphi(p) = \varphi_0 [1 + c(p - p_0)],$$
  where
  - φ_0 is a reference porosity at reference pressure p_0
  - c is rock compressibility

  Assumptions for formula above
  - c is constant and small

b) Differences
  - Permeability is anisotropic, porosity is not.
  - Porosity varies with time (pressure dependent), permeability does not.
  - Input of permeability requires 3 numbers per grid block, input of porosity requires one number per grid block (reference), a reference pressure and rock compressibility.
  - Sources of information are the same, but well testing can be used to obtain additional information for permeability.

Problem 4

a) φ and u are respectively the porosity and Darcy velocity Q/A, where Q is volumetric injection rate, A is cross section area of reservoir.

Let $x_D = x/L$ be dimensionless space variable. Then
$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x_D}\frac{\partial x_D}{\partial x} = \frac{\partial f}{\partial x_D}\frac{1}{L}$$
and equation (**) becomes
$$\frac{\varphi L}{u}\frac{\partial S}{\partial t} + \frac{\partial f}{\partial x_D} = 0 .$$

Next, set dimensionless time $t_D = ut/\varphi L$. Then

$$\frac{\partial S}{\partial t} = \frac{\partial S}{\partial t_D}\frac{\partial t_D}{\partial t} = \frac{\partial S}{\partial t_D}\frac{u}{\varphi L}$$
and substitution in the above equation gives
$$\frac{\partial S}{\partial t_D} + \frac{\partial f}{\partial x_D} = 0 .$$

b) Principle: Conservation of mass.
   This leads to the equality
$$\int_0^1 f'(z)T dz = [f(1) - f(0)]T = \int_{S_f}^1 f'(z)T dz + f'(S_f)TS_f$$
   or
$$T = [1 - f(S_f)]T + f'(S_f)TS_f$$

   and the result follows.

c) From Fig. 2 (left) we find that $S_f \approx 0.5$ (front height). This implies that the speed of the front S_f is
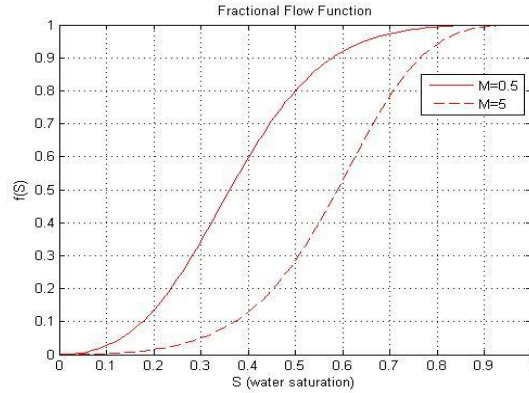$$V = \frac{f(S_f)}{S_f} \approx \frac{0.8}{0.5} \approx 1.6$$
   Position of front after T=0.3: $x_f = VT \approx 1.6 \cdot 0.3 = 0.48$
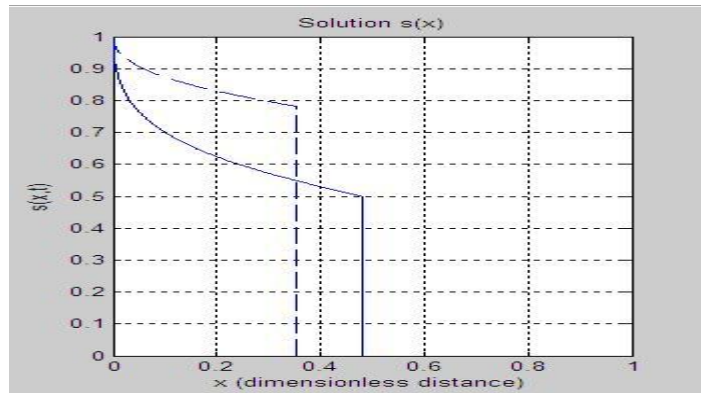   Behind front, using Fig. 2 (right) we see that:
   $S = 0.6 \Rightarrow f'(S) \approx 0.8 \Rightarrow x_S = 0.8 \cdot 0.3 = 0.24$
   $S = 0.7 \Rightarrow f'(S) \approx 0.3 \Rightarrow x_S = 0.3 \cdot 0.3 = 0.09$
   $S = 0.8 \Rightarrow f'(S) \approx 0.1 \Rightarrow x_S = 0.1 \cdot 0.3 = 0.03$

Fractional Flow Function

d) From the Fig. of f(S) that shows fractional flow function for M=0.5 and M=5 it follows that the main difference between the solution in c) and the new one is that (i) the height of the front is larger; (ii) the position of the front is behind.

Solution s(x)

Problem 5

a) Important assumptions:
  - 1D horizontal reservoir with length L and constant cross section area A
  - immiscible flow of two incompressible fluids water and oil
  - homogeneous, incompressible reservoir, i.e. constant porosity φ and constant absolute permeability k
  - capillary pressure is zero
  - reservoir initially filled with oil
  - constant injection rate of water at one end and production at the other end

Fractional flow function: $f = \dfrac{\lambda_w}{\lambda_T} = \dfrac{k_{rw}/\mu_w}{k_{rw}/\mu_w + k_{ro}/\mu_o} = \dfrac{k_{rw}}{k_{rw} + M k_{ro}}$

where M is the viscosity ratio $M = \mu_w/\mu_o$. Normalized Corey type relative permeabilities are specified using expressions $k_{rw}(S) = S^{nw}$, $k_{ro}(S) = (1-S)^{no}$.

c) M=1. In order to obtain that $f(S) = S$ we must then assume that
$k_{rw}(S) = S, \quad k_{ro}(S) = (1 - S)$, i.e. the Corey exponents are 1.

d) Discrete scheme for i=2,..,N:

$$\frac{u_i^n - u_{i-1}^n}{h} = -\frac{u_i^{n+1} - u_i^n}{\Delta t}$$

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{h}(u_i^n - u_{i-1}^n)$$

Boundary and initial conditions:

Initially (time 0), u is specified at all points along x- axis ($u_1^0 = 1, u_2^0 = 0, ..., u_N^0 = 0$).

For all steps n, $u_1^n = 1$.

The ratio $\Delta t / h$ determines stability: CFL condition is $\Delta t / h \leq 1$