SPRING 2011
MPE690: SKETCH OF SOLUTION FOR TASK 2 AND 3

Task 2.

(1) **Water-oil:** We have

$$k_{rw}(s_w) = KRW \cdot k_{rw}^t(s'), \qquad s' = \frac{s_w - s_{wr}}{1 - s_{wr}}, \qquad \text{for } s_{wr} \le s_w \le 1$$

$$k_{row}(s_w) = KROW \cdot k_{row}^t(s'), \qquad s' = \frac{s_w - s_{wr}}{1 - s_{wr} - s_{owr}}, \qquad \text{for } s_{wr} \le s_w \le 1 - s_{owr}$$

Gas-oil: We have

$$k_{rg}(s_w) = KRG \cdot k_{rg}^t(s'), \qquad s' = \frac{s_g - s_{gr}}{1 - s_{wr} - s_{gr}}, \qquad \text{for } s_{gr} \le s_g \le 1 - s_{wr}$$

$$k_{rog}(s_w) = KROG \cdot k_{rog}^t(s'), \qquad s' = \frac{s_g}{1 - s_{wr} - s_{ogr}}, \qquad \text{for } 0 \le s_g \le 1 - s_{wr} - s_{ogr}$$

See course material for relevant figures.

(2)

$$P_{cow}(s_w) = p_o - p_w \qquad \text{(decreasing, positive and negative values)}$$
$$P_{cgo}(s_g) = p_g - p_o \qquad \text{(increasing, positive values)}$$
$$P_c = J(s)\sigma\sqrt{\frac{\phi}{k}}$$

(3)

$$\nabla \cdot (\overline{\rho}_g \overrightarrow{v}_g + \overline{\rho}_{dg} \overrightarrow{v}_o) = -\frac{\partial}{\partial t}(\phi \overline{\rho}_g s_g + \phi \overline{\rho}_{dg} s_o) + q_g$$

(4) Let ρ_l^s be densities at standard conditions, $l = w, o, g$.

$$\overline{\rho}_l = \frac{m_l}{[V_l]_{RC}} = \frac{\rho_l^s}{B_l(p)}, \qquad l = w, o, g$$
$$\overline{\rho}_{dg} = \frac{m_{dg}}{[V_o]_{RC}} = \rho_g^s \frac{[V_{dg}]_{ST}}{[V_o]_{RC}} \cdot \frac{[V_o]_{ST}}{[V_o]_{ST}} = \rho_g^s \frac{R_s(p)}{B_o(p)}$$

where we have used that $\rho_g^s = \frac{m_{dg}}{[V_{dg}]_{ST}}$.
Inserting these relations in the component equations in point (3) gives us the reformulated equations we are searching for.

(5) Using darcy law

$$\overrightarrow{v}_l = -\frac{[k]k_{rl}}{\mu_l}(\nabla p_l - \gamma_l \nabla d),$$

the equations from point (4) will depend on the unknown variables p_w, p_o, p_g and s_w, s_o, s_g (6 variables). Additional (3) equations are:

$$s_w + s_o + s_g = 1, \qquad P_{cow}(s_w) = p_o - p_w, \qquad P_{cgo}(s_g) = p_g - p_o.$$

We can eliminate two pressure by using capillary pressure curves, and one saturation $s_w + s_o + s_g = 1$ in the three mass balance equations such that we get 3 mass balance equations with 3 unknowns.

(6) Starting point:

$$\nabla \cdot (\frac{\overrightarrow{v}_w}{B_w}) = -\frac{\partial}{\partial t}(\phi \frac{s_w}{B_w}) + Q_w$$

- $Q_w = 0$, $s_w = 1$, incompressible fluid $B_w = 1$, 1D horizontal reservoir,
- Darcy law: $v_w = -\frac{k}{\mu} p_x$
- rock compressibility: $c = \frac{1}{\phi}\frac{d\phi}{dp}$, which implies that $\phi = \phi_0 e^{c(p-p_0)}$. This can for small c be approximated by $\phi(p) = \phi_0[1 + c(p - p_0)]$ (Taylor expansion).

Task 3.

(1) From fig 1. (left) we see that front height $s^* \approx 0.5$. $V = f'(s^*) = f'(0.5) \approx 1.6$ which gives
- front position at time $T = 0.3$ is: $x^* = VT = 1.6 \cdot 0.3 = 0.48$.
- behind front: For example, let $s_1 = 0.6$ which gives $V_1 = f'(s_1) \approx 0.8$ which implies $x_1 = 0.8 \cdot 0.3 = 0.24$.

(2) Main difference is (this should be supplemented with some reference to the fraction flow functions)
- lower speed of front
- front height is higher

(3) We have that before breakthrough $R(T) = T$. Thus,

$$R(T_b) = T_b = \frac{1}{f'(s^*)} \approx \frac{1}{9/7} = \frac{7}{9} = 0.78$$

(4) Similarly as above we get

$$R(T_b) = T_b = \frac{1}{f'(s^*)} \approx \frac{1}{1.6} = 0.63.$$