University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: **PET 510 – Computational Reservoir and Well Modeling**

DATE: **March 6**

TIME: **4 hours**

AID: **Basic calculator is allowed**

THE EXAM CONSISTS OF 7 PROBLEMS ON 7 PAGES AND APPENDIX A - D

REMARKS:
You may answer in English or Norwegian. Exercises 1 and 2 (part A) and exercises 3-7 (part B) are given equal weight.

COURSE RESPONSIBLE: Steinar Evje and Kjell-Kåre Fjelde

Problem 1.

(a) Consider the linear transport equation

$$(*) \qquad u_t - xu_x = q, \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x, t = 0) = \phi(x).$$

Set $q = 0$.
- Compute the solution $u(x,t)$ by using the method of characteristics.
- Illustrate the characteristics in a $x - t$ coordinate system.
- Verify that your solution satisfies (*) and (**).

(b) Consider (*) with $q = q(x,u) = \frac{1}{2}x^2u$.
- Compute the solution $u(x,t)$ by using the method of characteristics. Verify that your solution satisfies (*) and (**).

(c) Now, consider the simpler transport equation

$$u_t + u_x = 0, \qquad x \in [0, 1]$$

with initial data

$$u(x, t = 0) = 0,$$

and boundary data

$$u(x = 0, t) = 1.$$

- Describe the characteristics for this model and make a plot of some of them for $x \in [0, 1]$.
- Describe the solution $u(x, t = 0.5)$ and make a sketch of it.

(d) Present a discrete scheme for the model problem given in (c). Present a scheme based, respectively, on upwind discretization and central-based discretization where explicit discretization in time is used. What can be said about stability properties of these two different schemes?

(e) The model in (c) can be understood as flooding of water in an initially oil-filled reservoir where an injector is located at $x = 0$ and a producer at $x = 1$.
- Reformulate the problem such that we obtain a model reflecting that water is injected at $x = 1$ and oil is produced at $x = 0$.
- Formulate a stable discrete scheme for this model problem. X_j^n

Problem 2.

(a) In the following we consider a horizontal 1D reservoir.
- State the single-phase porous media mass balance equation in 1D (without source term) and identify the various variables (rock and fluid).
- Introduce Darcy's law and derive an equation for the pressure where it is assumed that $\phi = \phi(p)$, $\rho = \rho(p)$, and permeability and viscosity are constant.
- Assuming a weakly compressible rock (compressibility c_r is small) we get a linear relation for $\phi(p)$.

$$\phi(p) = \phi_0[1 + c_r(p - p_0)],$$

where p_0 and ϕ_0 are reference pressure and porosity. Use this together with the assumption that the fluid is incompressible and show that we then can obtain a pressure equation of the form

$$(*) \qquad p_t = \kappa p_{xx}, \qquad x \in \mathbb{R} = (-\infty, +\infty),$$

and identify the constant parameter $\kappa > 0$.

(b) We now set $\kappa = 1$ in (*). Verify that

$$(**) \qquad p(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{x}{2\sqrt{t}}} e^{-\theta^2} \, d\theta$$

satisfies (*).
What must be the initial condition corresponding to the solution (**) ?
(**Hint**: use that $\int_{-\infty}^{\infty} e^{-\theta^2} d\theta = \sqrt{\pi}$)

(c) Next, consider (*) with $\kappa = 1$ on the spatial domain $[0, 1]$. Formulate a discrete version of (*) based on an explicit time discretization when we assume the following boundary condition:

$$p(x = 0, t) = p(x = 1, t) = 1.$$

Divide the domain into M cells with points $x_1, x_2, \ldots, x_M$ located at the center of each cell. The cell interface $x_{1/2}$ corresponds to $x = 0$ and $x_{M+1/2}$ to $x = 1$.
What is the stability condition for this scheme?

(d) We consider the pressure like equation

$$u_t = (d(u)u_x)_x, \qquad x \in (0,1), \qquad 0 < t,$$
$$u(x = 0, t) = a(t), \qquad u(x = 1, t) = b(t),$$
$$u(x, t = 0) = u_0(x).$$

In the following we let

$$d(u) = u, \qquad a(t) = t, \qquad b(t) = 1 + t, \qquad u_0(x) = x.$$

- Show (by direct calculations) that $u(x,t) = x + t$ is a solution of this model problem
- Sketch the solution at time $t = 1$ and $t = 2$.

(c) Consider the following discrete scheme:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{1}{\Delta x^2}\left(d_{j+1/2}^n[u_{j+1}^n - u_j^n] - d_{j-1/2}^n[u_j^n - u_{j-1}^n]\right),$$

where

$$d_{j+1/2}^n = \frac{d(u_j^n) + d(u_{j+1}^n)}{2}.$$

Show that $u_j^n = x_j + t^n$ satisfies the discrete scheme. More precisely, show that
(i) $\{u_j^n\}$ satisfies the initial data
(ii) Assume that $u_j^n = x_j + t^n$ is the solution at time t^n. Then show that $u_j^{n+1} = x_j + t^{n+1}$ is a solution at time t^{n+1}.

Exam Part B – Solving Nonlinear Equations & Modelling of Well Flow

There are 13 questions in total. Some formulas, equations and Matlab codes are found in Appendixes. This part constitutes 50 % of exam.

Exercise 3 – Matlab Questions

a) What are the three types of matlab control statement ?

b) Assume that we have discretized a vertical well into 50 boxes. Each box has height dx = 50 meters. The vector p starting at bottom with the value p(1) and ending at top p(51) will contain the pressure at different locations in the well. Note that p(i), will be the pressure at the boundaries between the boxes. Write a matlab script that fills up/initializes this p vector with values when we consider a mudweight of 1.7 sg. Try to make the code flexible such that we easily can change number of boxes and mudweight.

c) The function $z = f(x, y) = x^2 + y^2$ will give a surface in three dimensions. How will you write the matlab code for a function that calculates and returns the z value based on the input values x and y ?

Exercise 4 – Bisection Method

a) We are given the function $f(x) = x^2 - 4{,}7 \cdot x + 3{,}7$. Make a rough sketch of the graph

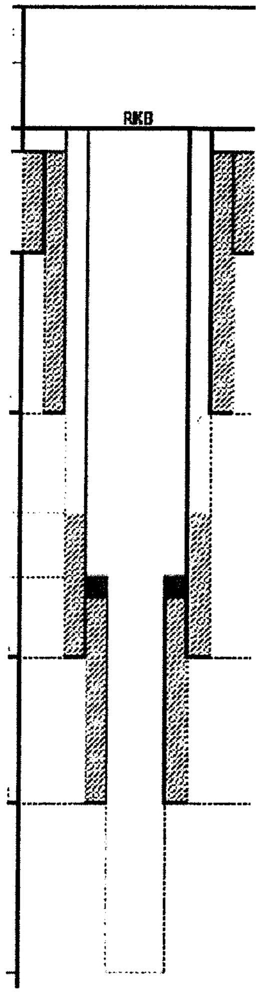
b) We shall now show that we can use the bisection method to pick out the largest of the two roots of this function. The number of iterations shall be determined by the requirement that abs(ftol)=0.02.

Iteration	x1	x2	x3	f(x1)	f(x2)	f(x3)
1						
2						
3						
etc						
etc						

Exercise 5 – Cuttings Transport

a) The figure to the right shows the geometry of a planned casing program for a vertical well. We can assume that a drillpipe with uniform outer diameter will be placed inside this well, but that the lowermost part of it (BHA or bottomhole assembly) has a somewhat larger outer diameter.

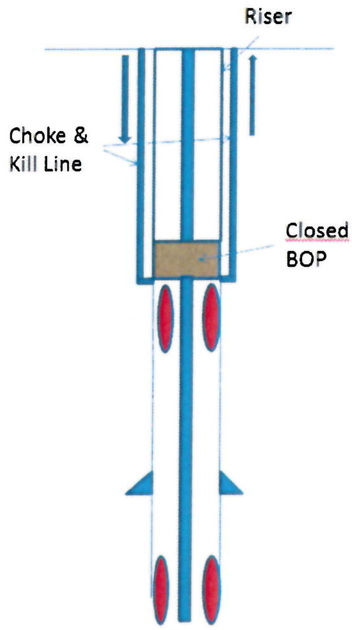
Where will you be most concerned with checking if the suggested flowrate is adequate for proper hole cleaning ?



b) When working with the matlab code for the Larsen model, how did an increase in cuttings size, mudweight and viscosities affect the minimum required fluid velocity to avoid the formation of cuttings bed in inclined wells?

Exercise 6 – Well pressures

We have a 5000 meters TVD (true vertical depth) deep well. The mudweight measured at surface is 1.7 sg. We are cross circulating across the BOP to clean the annular space in the BOP because we want to omit that potential hydrates (hydrocarbon ice) has formed there. This means that we circulate down the kill line, through the BOP and up the chokeline. The well below the BOP is static. The well is pressure dominated and we expect that this will change the effective downhole mudweight by 0.03sg. The depth of the BOP is 1500 meters. The pressure gradient in the choke and kill line was measured to be 0.01 bar/m when circulating with 500 lpm.



a) What will the ECD at the bottom of the well be when performing this type of circulation with 1000 lpm ? (answer shall be provided in sg)
b) What will the pump pressure be in that case ?
c) When the well is static, we take a 2 m^3 kick at bottom of the well. We assume that the pressure at the bottom is 850 bar and the temperature is 150 C. The well is closed in and the kick migrates to the just beneath the closed BOP. Here we assume that the temperature is 50 C. What will the pressure be when the kick has migrated to just beneath the closed BOP ?

d) How long time will it take for the kick to migrate from bottom to the closed BOP ?

Exercise 7 – Conservation laws

a) In a steady state flow situation the mass rate $M = Q \cdot \rho$ is the same all places in the well when considering a compressible fluid. If we have a compressible gas, how will Q and ρ change when moving from the bottom of the well to the top of the well ?

b) Describe the solution technique used for solving the steady state two phase flow conservation equations for a discretized well!

Appendix A – Some Units & Formulas

1 inch =2.54 cm = 0.0254 m

1 feet = 0.3048 m

1 bar = 100000 Pa

1 sg = 1 kg/l (sg - specific gravity)

$M = Q \cdot \rho$ M massrate (kg/s), Q Volumerate (m^3/s), ρ density (kg/m^3)

$Q = v \cdot A$ Q Volumerate (m^3/s), v velocity m/s. A area m^2

$p = \rho \cdot h \cdot 0.0981$ p (bar), ρ density (sg), h – vertical depth (m)

$\frac{P \cdot V}{T} = C$, from Ideal gas law, NB T is in Kelvin and the relation to Celsius is $K = {}^{\circ}C + 273{,}15$

$P \cdot V = C$, Boyles law (temperature is assumed constant)

Appendix B

Main.m

```
% Main program that calls up a routine that uses the bisection
% method to find a solution to the problem f(x) = 0.
% The search intervall [a,b] is specified in the main program.
% The main program calls upon the function bisection which again calls upon
% the function func.

% if error = 1, the search intervall has to be adjusted to ensure
% f(a) x f(b)<0

% Specify search intervall, a and b will be sent into the function
% bisection
 a = 4.0;
 b = 5.0;

% Call upon function bisection which returns the results in the variables
% solution and error.
  [solution,error] = bisection(a,b);

 solution % Write to screen.
 error   % Write to screen.
```

Bisection.m

```
function [solution,error] = bisection(a,b)

% The numerical solver implemented here for solving the equation f(x)= 0
% is called Method of Halving the Interval (Bisection Method)

% You will not find exact match for f(x)= 0. Maybe f(x) = 0.0001 in the
end.
% By using ftol we say that if abs(f(x))<ftol, we are satisfied. We can
% also end the iteration if the search interval [a,b] is satisfactory
small.
% These tolerance values will have to be changed depending on the problem
% to be solved.


 ftol = 0.01;

% Set number of iterations to zero. This number will tell how many
 % iterations are required to find a solution with the specified accuracy.

  noit = 0;

  x1 = a;
  x2 = b;
```

```
  f1 = func(x1);
  f2 = func(x2);


%   First include a check on whether f1xf2<0. If not you must adjust your
%   initial search intervall. If error is 1 and solution is set to zero,
%   then you must adjust the search intervall [a,b].


 if (f1*f2)>=0
     error = 1;
     solution = 0;
 else
 % start iterating, we are now on the track.
     x3 = (x1+x2)/2.0;
     f3 = func(x3);

     while (f3>ftol | f3 < -ftol)
        noit = noit +1 ;

        if (f3*f1) < 0
           x2 = x3;
        else
           x1 = x3;
        end

        x3 = (x1+x2)/2.0;
        f3 = func(x3);
        f1 = func(x1);

     end
     error = 0;
     solution = x3;
     noit % This statement without ; writes out the number of iterations to
the screen.
 end
```

func.m

```
function f = func(x)


  f = x^2-4*x+2;
```

Appendix C

```
% Program where the Larsen Cuttings Transport Model is implemented

%  First specify all input parameters:

do = 8.5; % Outerdiameter (in) ( 1 in = 0.0254 m)
di = 5; % Innerdiameter   (in)
rop = 33 % Rate of Penetration - ROP ft/hr (1 ft = 0.3048m)
pv = 15 % Plastic viscosity (cP)
yp = 16 % Yield point (lbf/100ft2)
dcutt = 0.1 % Cuttings diameter (in) (1 inch = 0.0254 m)
mw = 10.833 % Mudweight (ppg - pounds per gallon) 1 ppg = 119.83 kg/m3.
rpm = 80 % rounds per minute
cdens = 19 % cuttings density (ppg - pounds per gallon)
angstart = 50 % Angle with the vertical

% vcut - Cuttings Transport Velocity (CTF in Larsens paper)
% vcrit - Critical Transport fluid velocity (CTFV) in Larsens paper. This
% is the minimum fluid velocity required to maintain a continously upward
% movement of the cuttings.
% vslip - Equivalent slip velocity (ESV) defined as the velocity difference
% between the cuttings and the drilling fluid
% vcrit = vcut+vslip
% All velocities are in ft/s.
% ua - apparent viscosity

% It should be noted that the problem is nested. Vcrit depends on vslip
% which again depends on an updated/correct value for vcrit. An iterative
% approch on the form  x(n+1) = g(x(n)) will be used.

for i = 1:8

ang(i)=angstart+i*5
vcut = 1/((1-(di/do)^2)*(0.64+18.16/rop));

vslipguess = 3;
vcrit = vcut + vslipguess;

% Find the apparent viscosity (which depends on the "guess" for vcrit)
ua = pv+ (5*yp*(do-di))/vcrit

% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
if (ua <= 53)
 vslip = 0.0051*ua+3.006;
else
 vslip = 0.02554*(ua-53)+3.28;
end

%Now we have two estimates for vslip that can be compared and updated in a
% while loop. The loop will end when the vslip(n+1) and vslip (n) do not
% change much anymore. I.e the iterative solution is found.
n=1;
while (abs(vslip-vslipguess))>0.01
 vslipguess = vslip;
 vcrit = vcut + vslipguess;
% Find the apparent viscosity (which depends on the "guess" for vcrit)
  ua = pv+ (5*yp*(do-di))/vcrit;
```

```
% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
  if (ua <= 53)
   vslip = 0.0051*ua+3.006;
  else
   vslip = 0.02554*(ua-53)+3.28;
  end
  n=n+1;
  vslip  % Take away ; and you will se how vslip converges to a solution
end % End while loop

%
% Cuttings size correction factor: CZ = -1.05D50cut+1.286
 CZ = -1.05*dcutt+1.286
% Mud Weight Correction factor (Buoancy effect)
  if (mw>8.7)
  CMW = 1-0.0333*(mw-8.7)
  else
  CMW = 1.0
  end

% Angle correction factor

CANG = 0.0342*ang(i)-0.000233*ang(i)^2-0.213

vslip = vslip*CZ*CMW*CANG;   % Include correction factors.

% Find final minimum velocity required for cuttings transport (ft/s).

vcrit = vcut + vslip

vcritms = vcrit*0.3048 % Velocity in m/s

Q = 3.14/4*((8.5*0.0254)^2-(5*0.0254)^2)*vcritms % (m3/s)
Q = Q*60*1000  % (lpm)

yrate(i)=Q
end

plot(ang,yrate)
```

Appendix D – Steady State Model for Two Phase Flow

Conservation of liquid mass

$$\frac{\partial}{\partial z}(A\rho_l \alpha_l v_l) = 0$$

Conservation of gas mass

$$\frac{\partial}{\partial z}(A\rho_g \alpha_g v_g) = 0$$

Conservation of momentum.

$$\frac{\partial}{\partial z} p = -(\rho_{mix} g + \frac{\Delta p_{fric}}{\Delta z})$$

Gas slippage model (simple):

$v_g = Kv_{mix} + S$ (K=1.2, S = 0.55)

Liquid density model (simple)

$\rho_l(p) = \rho_{lo} + \dfrac{(p - p_O)}{a_L^2}$, assume water: $\rho_{lo} = 1000$ kg/m^3, $p_O = 100000Pa$, $a_L = 1500$ m/s

Gas density model (simple)

$\rho_g(p) = \dfrac{p}{a_g^2}$, ideal gas: $a_g = 316$ m/s.

Friction model

The friction model presented here is for a Newtonian fluids like water. The general expression for the frictional pressure loss gradient term is given by:

$$\frac{\Delta p_{fric}}{\Delta z} = \frac{2f\rho_{mix} v_{mix} abs(v_{mix})}{(d_{out} - d_{in})} \quad \text{(Pa/m)}$$

A - (m^2)

ρ_i - phase densities (kg/m^3), liquid – > i=l, gas ->i =g

v_i - phase velocities (m/s)

p - pressure (Pa)

g – gravity constant 9.81 m/s^2

α_i - phase volume fractions taking values between 0 and 1. $\alpha_l + \alpha_g = 1$.

$\rho_{mix} = \alpha_l \rho_l + \alpha_g \rho_g$ - mixture density

$v_{mix} = \alpha_l v_l + \alpha_g v_g$ - mixture velocity