SPRING 2012
MPE690: SKETCH OF SOLUTION FOR TASK 2 AND 3

Task 2.

a) We have

$$(\phi s_l \rho_l)_t + (\rho_l u_l)_x = 0, \qquad u_l = -\frac{k k_{rl}}{\mu_l}(p_l)_x = -\lambda_l (p_l)_x, \qquad l = w, o$$

Assumptions:
- 1D, horizontal flow
- Constant k, ϕ, ρ_l, μ_l
- $P_c = p_o - p_w = 0$, that is, $p_w = p_o = p$.
- two-phase: $s_w + s_o = 1$

Summing the two mass balance equations (after using the assumptions) gives:

$$\phi(s_w + s_o)_t + (u_w + u_o)_x = 0$$

i.e.

$$(u_w + u_o)_x = 0, \qquad u_T = u_w + u_o = \text{constant}.$$

That is,

$$u_T = -\lambda_T p_x, \quad \text{which implies that} \quad p_x = -\frac{u_T}{\lambda_T},$$

for $\lambda_T = \lambda_w + \lambda_o$.

b)

$$f(s) = \frac{\lambda_w(s)}{\lambda_T(s)} \qquad \text{(s-shaped function)}$$

Algorithm for computing solution of the water-flooding problem:
- Front height s^* is given as the solution of: $f'(s^*) = f(s^*)/s^*$.
- Position of front after a time T: $x^* = f'(s^*)T$
- Solution behind front is computed as: $x_s = f'(s)T$ for $s \in [s^*, 1]$

c) Mass conservation must be ensured when a front is introduced in the unphysical solution which gives rise to the condition

$$A_I = A_{II} \tag{1}$$

where

$$A_I = \int_0^1 f'(s)T\,ds = T(f(1) - f(0)) = T,$$

and

$$\begin{aligned} A_{II} &= s^* f'(s^*)T + \int_{s^*}^1 f'(s)T\,ds \\ &= s^* f'(s^*)T + T(f(1) - f(s^*)) = s^* f'(s^*)T + T(1 - f(s^*)) \end{aligned}$$

In view of (1) we get

$$1 = s^* f'(s^*) + (1 - f(s^*)), \qquad \text{which implies} \qquad f(s^*) = s^* f'(s^*)$$

d) From fig 1. (left) we see that front height $s^* \approx 0.7$. $V = f'(s^*) = \frac{f(s^*)}{s^*} \approx 0.9/0.7 \approx 1.29 \approx 1.3$ which gives breakthrough time T_b

$$V \cdot T_b = 1, \qquad \text{which implies that} \qquad T_b = 1/V \approx 7/9 \approx 0.78.$$

Oilrecovery $R(T)$ up to breakthrough time is given by $R(T) = T$, that is,

$$R(T_b) = T_b \approx 0.78.$$

e) Similarly as above, we find that speed of front is

$$V^{M=0.5} = f'(s^*) = \frac{f(s^*)}{s^*} \approx 0.8/0.5 = 1.6,$$

which gives breakthrough time $T_b^{M=0.5}$

$V^{M=0.5} \cdot T_b^{M=0.5} = 1, \qquad T_b^{M=0.5} = 1/V^{M=0.5} \approx 1/1.6 \approx 0.63 < 0.78 \approx T_b^{M=2.25}$

Consequently, oil recovery for case with $M = 0.5$ at time $T_b = 0.78$ is given by

$$\begin{aligned} R(T_b = 0.78) &= s^*(T_b) \cdot 1 + \int_{s^*(T_b)}^{1} f's)T_b \, ds \\ &= s^*(T_b) + T_b(1 - f(s^*(T_b))) \\ &\approx 0.53 + 0.78(1 - 0.84) \approx 0.65. \end{aligned}$$

where $s^*(T_b)$ is the saturation which has travelled a distance 1 after time T_b, that is, $f'(s^*(T_b)) \cdot T_b = 1$, that is

$$f'(s^*(T_b)) = 1/0.78 \approx 1.3.$$

From Fig.1 (right) for $M = 0.5$ we see that $s^*(T_b) \approx 0.53$. Moreover, we see from Fig.1 (left) that $f(0.53) \approx 0.84$.

Task 3.

a) Starting point is:

$$\partial_t(\phi\rho) + \partial_x(\rho u) = q.$$

- $q = 0$, incompressible fluid $\rho =$constant
- Darcy law: $u = -\frac{k}{\mu} p_x$ (horizontal reservoir)
- rock compressibility: $c = \frac{1}{\phi}\frac{d\phi}{dp}$, which implies that $\phi = \phi_0 e^{c(p-p_0)}$. This can for small c be approximated by $\phi(p) = \phi_0[1 + c(p - p_0)]$ (Taylor expansion).

b)

$$\frac{p_j^{n+1} - p_j^n}{\Delta t} = \frac{\kappa}{\Delta x}\Big(\partial_x p|_{j+1/2} - \partial_x p|_{j-1/2}\Big), \qquad j = 1, \ldots, N$$

where

$$\partial_x p|_{1/2} = \partial_x p|_{N+1/2} = 0, \qquad \text{(no-flux condition)}$$

and

$$\partial_x p|_{j+1/2} = \frac{p_{j+1}^n - p_j^n}{\Delta x}, \qquad j = 2, \ldots, N-1.$$

Resulting scheme:

$$\begin{aligned} \frac{p_1^{n+1} - p_1^n}{\Delta t} &= \frac{\kappa}{\Delta x}\Big(\partial_x p|_{1+1/2} - 0\Big), \\ \frac{p_j^{n+1} - p_j^n}{\Delta t} &= \frac{\kappa}{\Delta x}\Big(\partial_x p|_{j+1/2} - \partial_x p|_{j-1/2}\Big), \qquad j = 2, \ldots, N-1 \\ \frac{p_N^{n+1} - p_N^n}{\Delta t} &= \frac{\kappa}{\Delta x}\Big(0 - \partial_x p|_{N-1/2}\Big). \end{aligned}$$

Stability condition is: $\kappa \frac{\Delta t}{\Delta x^2} \leq 1/2$.

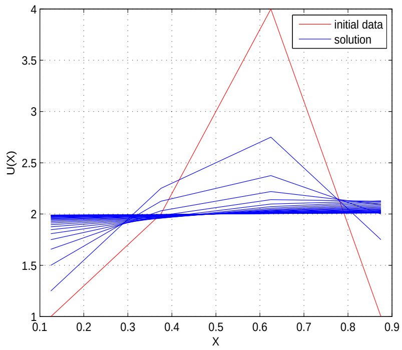

FIGURE 1. Solution of discrete scheme for timestep t^1, t^2, t^3, ...

c)

p^0	1	2	4	1
p^1	$\frac{5}{4}$	$\frac{9}{4}$	$\frac{11}{4}$	$\frac{7}{4}$
p^2	$\frac{12}{8}$	$\frac{17}{8}$	$\frac{19}{8}$	2

d)

$$\nabla \cdot (\overline{\rho}_g \overrightarrow{v}_g + \overline{\rho}_{dg} \overrightarrow{v}_o) = -\frac{\partial}{\partial t}(\phi \overline{\rho}_g s_g + \phi \overline{\rho}_{dg} s_o) + q_g$$

Let $\rho_l^s = \frac{m_l}{[V_l]_{ST}}$ be densities at standard conditions, $l = w, o, g$ and $\rho_g^s = \frac{m_{dg}}{[V_{dg}]_{ST}}$.

$$\overline{\rho}_l = \frac{m_l}{[V_l]_{RC}} = \frac{\rho_l^s}{B_l(p)}, \qquad l = w, o, g$$

$$\overline{\rho}_{dg} = \frac{m_{dg}}{[V_o]_{RC}} = \rho_g^s \frac{[V_{dg}]_{ST}}{[V_o]_{RC}} \cdot \frac{[V_o]_{ST}}{[V_o]_{ST}} = \rho_g^s \frac{R_s(p)}{B_o(p)}$$

where we have used that $\rho_g^s = \frac{m_{dg}}{[V_{dg}]_{ST}}$.
Inserting these relations in the component equations gives the reformulated equations we are searching for.

Gas Condensate Model:
oil component equation:

$$\nabla \cdot (\overline{\rho}_o \overrightarrow{v}_o + \overline{\rho}_{do} \overrightarrow{v}_g) = -\frac{\partial}{\partial t}(\phi \overline{\rho}_o s_g + \phi \overline{\rho}_{do} s_o) + q_o$$

and

$$\overline{\rho}_{do} = \frac{m_{do}}{[V_g]_{RC}} = \rho_o^s \frac{[V_{do}]_{ST}}{[V_g]_{RC}} \cdot \frac{[V_g]_{ST}}{[V_g]_{ST}} = \rho_o^s \frac{R_{sog}(p)}{B_g(p)}, \qquad R_{sog} = \frac{[V_{do}]_{ST}}{[V_g]_{RC}}$$

This gives

$$\nabla \cdot (\frac{1}{B_o} \overrightarrow{v}_o + \frac{R_{sog}(p)}{B_g(p)} \overrightarrow{v}_g) = -\frac{\partial}{\partial t}(\phi \frac{1}{B_o} s_o + \phi \frac{R_{sog}(p)}{B_g(p)} s_g) + q_o/\rho_o^s$$