**FACULTY OF SCIENCE AND TECHNOLOGY**

**DATE: November 20, 2013**

**SUBJECT: PET 510 – Computational Reservoir and Well Modeling**

**TIME: 4 hours**

**AID: No printed or written means allowed. Definite basic calculator allowed.**

**THE EXAM CONSISTS OF 6 PROBLEMS ON 5 PAGES AND 7 ADDITIVES**

**REMARKS: You may answer in English or Norwegian. Exercises 1 and 2 (part A) and exercises 3-6 (part B) are given equal weight.**

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**Problem 1.**

(a) Consider the linear transport equation

$$(*) \qquad u_t + a(x)u_x = b(x,u), \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x, t=0) = u_0(x) = \phi(x).$$

- Describe by words the transport effect represented by the term $a(x)u_x$. Intuitively, what is the impact from the source term $b(x,u)$ on the solution $u(x,t)$?

(b) Let $a(x) = x$ and $b(x,u) = 0$ in (*). Compute the solution $u(x,t)$ by using the method of characteristics. Verify that your solution satisfies (*) and (**).

(c) Let $a(x) = x$ and $b(x,u) = u$ in (*). Compute the solution $u(x,t)$ and verify that your solution satisfies (*) and (**).

(d) Now we choose $\phi(x) = \exp(-x^2)$.
- Make a rough sketch of the solution in (b) at time $t = 1$ and $t = 2$.
- Explain what is the main difference between this solution and the solution computed in (c)?

(e) In the following we set $a(x) = 1$ and $b = 0$ in $(*)$ and consider a discretization of the domain $[0,1] \times [0,T]$ with discretization parameters $\Delta x$ and $\Delta t$.
More precisely, we divide the domain $[0,1]$ into cells $0, 1, \ldots, M, M+1$ and timesteps $t^0 = 0$, $t^1 = \Delta t$, $\ldots$, $t^n = n\Delta t$. For the first and last cell we set $u_0^{n+1} = 0$ and $u_{M+1}^{n+1} = 0$. In the interior part of the domain ($j = 1, \ldots, M$) we consider a discrete scheme for the model $(*)$ of the form

$$(***) \qquad \frac{u_j^{n+1} - u_j^n}{\Delta t} + \frac{1}{\Delta x}\Big(U_{j+1/2}^n - U_{j-1/2}^n\Big) = 0, \qquad j = 1, \ldots, M.$$

- Describe how to define the flux terms $U_{j+1/2}^n$ and $U_{j-1/2}^n$ for the cells $j = 1, \ldots, M$ in order to obtain a stable scheme (up-wind scheme) and find an expression for $u_j^{n+1}$.

(f) It is known that the true solution of $(*)$ with $a(x) = 1$ and $b(x,u) = 0$ satisfies the estimate

$$\int_0^1 |u(x,t)|\, dx \leq \int_0^1 |u_0(x)|\, dx.$$

- Demonstrate how to obtain a corresponding estimate for the discrete scheme obtained from $(***)$.
(**Hint**: use the triangle inequality: $|a+b| \leq |a| + |b|$ and consider summation in space over the cells $j = 1, \ldots, M$).
- What is the condition on the discretization parameters $\Delta t$ and $\Delta x$ we must impose in order to obtain this estimate?

**Problem 2.**

(a) In the following we consider a horizontal 1D reservoir.
- State the single-phase porous media mass balance equation in 1D (without source term) and identify the various variables (rock and fluid).
- Introduce Darcy's law and derive an equation for the pressure where it is assumed that $\phi = \phi(p)$, $\rho = \rho(p)$, and permeability and viscosity are constant.
- Assuming a weakly compressible rock (compressibility $c_r$ is small) we get a linear relation for $\phi(p)$.

$$\phi(p) = \phi_0[1 + c_r(p - p_0)],$$

where $p_0$ and $\phi_0$ are reference pressure and porosity. Use this together with the assumption that the fluid is incompressible and show that we then can obtain a pressure equation of the form

$$(*) \qquad p_t = \kappa p_{xx}, \qquad x \in \mathbb{R} = (-\infty, +\infty),$$

and identify the constant parameter $\kappa > 0$.

(b) We now set $\kappa = 1$ in (*). Verify that

$$(**) \qquad p(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{x}{2\sqrt{t}}} e^{-\theta^2} d\theta$$

satisfies (*).
What must be the initial condition corresponding to the solution (**) ?
(**Hint**: use that $\int_{-\infty}^{\infty} e^{-\theta^2} d\theta = \sqrt{\pi}$ )

(c) Next, consider (*) with $\kappa = 1$ on the spatial domain $[0, 1]$. Formulate a discrete version of (*) based on an explicit time discretization when we assume the following boundary condition:

$$p(x = 0, t) = p(x = 1, t) = 0.$$

Divide the domain into $M$ cells with points $x_1$, $x_2$,..., $x_M$ located at the center of each cell. The cell interface $x_{1/2}$ corresponds to $x = 0$ and $x_{M+1/2}$ to $x = 1$.
What is the stability condition for this scheme?

(d) Explain why we have the estimate

$$\int_0^1 p(x,t)^2 \, dx \le \int_0^1 p_0(x)^2 \, dx$$

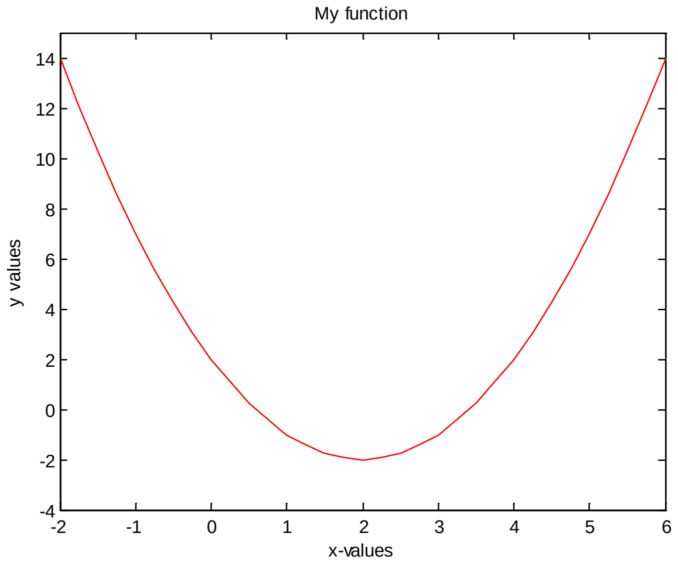
when we consider the domain $[0, 1]$ and boundary conditions $p(x = 0, t) = p(x = 1, t) = 0$ for the model $(*)$ with initial data $p(x, t = 0) = p_0(x)$.

## Exam Part B – Solving Nonlinear Equations & Modelling of Well Flow

There are 13 questions in total. Some formulas, equations and Matlab codes are found in Appendixes. This part constitutes 50 % of exam.

**Exercise 3 – Bisection Method**

a) In Appendix B, we have the function func which calculates the y value of $f(x) = x^2 - 4x + 2 = 0$. for a given argument x. Write down the matlab code lines necessary to produce a figure similar to this. (the x values shall increase in increments of one)




b) The function above has two roots. In Appendix B, you have a copy of three files (main, bisection, func) that can be used for solving this type of problems. Explain how you would change the code to find the "left" root. What is the requirement here?

c) What is the meaning of the variable *ftol* in the bisection function and what will happen if we for instance increase the value for this variable ?

d) Show how the bisection method works by filling out a similar table as below (We assume ftol = 0.1):

| Iteration | x1 | x2 | x3 | f(x1) | f(x2) | f(x3) |
|---|---|---|---|---|---|---|
| 1 | 2 | 4 | 3 | -2 | 2 | -1 |
| 2 | 3 | 4 | 3.5 | -1 | 2 | 0,25 |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |

**Exercise 4 – Cuttings Transport**

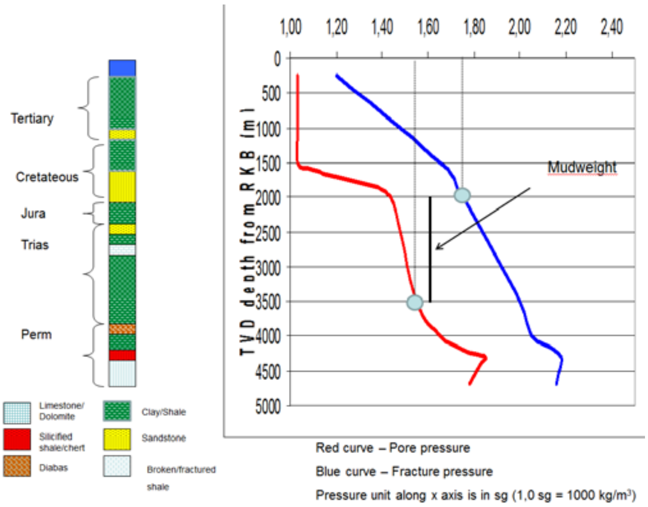
a) An empirical rule of thumb is that the cuttings transport is ensured if the annular flow velocity is larger than 200 ft/min in the horizontal part of the well. We are considering an 8 ½" hole. The drillstring OD is 5". What should the flowrate be in liters per minute ?

The Larsen model is an empirical model derived from a series of experiments on cuttings transport in horizontal/inclined flowloops. It gives a prediction of the minimum required annular velocity required for cuttings transport: $v_{crit} = v_{cut} + v_{slip}(v_{crit})C_{ang}C_{size}C_{mw}$ where the latter was correction factors included to account for effects related to inclination, cuttings size and mudweight.

b) How did cuttings size, mud weight and ROP (rate of penetration) affect the minimum required velocity for cuttings transport when you tested the model?

c) When working with the Larsen model, you were asked to include the inclination angle correction factor and make a plot of the required flowrate for cuttings transport vs inclination angle. In Appendix C, the finished code/solution is given: Write down the statements or explain what had to be added/changed in the program based on the solution proposal given in Appendix C.

**Exercise 5  - Well pressures**




a) We are at 3500 meters. The static mudweight is 1.6 sg. We are circulating the well with 2000 lpm (liters per minute). The annular friction is 25 bars.  What will the Equivalent circulating density ECD (in sg ) be ?

b) Assume that we take a Kick of 4 $m^3$. The kick has a pressure of 550 bar at bottom. When this kick is circulated to surface (open well), what kind of gas volumes can be expected at surface ?

c) Why can there be a need for introducing a discretization of the well in combination with application of conservation laws to calculate well pressures more accurately?

**Exercise 6 – Conservation Laws and Flow Models**

Flowmodels can be used for describing pressures and flow in a well. They are based on a set of conservation laws for mass, momentum and possible energy in combination with closure laws. These are solved for the different boxes in a discretized well by an appropriate numerical method. The steady state conservation laws are given in Appendix D.

a) What is the main difference between a steady state and a transient model with respect to what they describe?

b) In an UBO operation we will inject gas at a rate of 40 $m^3$/min. The pressure at surface is 1 bar. The pressure at bottom of the well is 200 bars. What will the gas volume rate ($m^3$/s) be at bottom. The flow is steady state  (check formulas in Appendix A &D)

c) Describe briefly the solution technique used for solving the steady state two phase  flow conservation equations for a discretized well.

## Appendix A – Some Units & Formulas

1 inch =2.54 cm = 0.0254 m

1 feet = 0.3048 m

1 bar = 100000 Pa

1 sg = 1 kg/l (sg - specific gravity)

$M = Q \cdot \rho$ M massrate (kg/s), Q Volumerate ($m^3$/s), $\rho$ density (kg/$m^3$)

$Q = v \cdot A$ Q Volumerate ($m^3$/s), v velocity m/s. A area $m^2$

$p = \rho \cdot h \cdot 0.0981$ p (bar), $\rho$ density ( sg), h – vertical depth (m)

$\frac{P \cdot V}{T} = C$ , from Ideal gas law

$P \cdot V = C$ , Boyles law (temperature is assumed constant)

# Appendix B

## Main.m

```
% Main program that calls up a routine that uses the bisection
% method to find a solution to the problem f(x) = 0.
% The search intervall [a,b] is specified in the main program.
% The main program calls upon the function bisection which again calls upon
% the function func.

% if error = 1, the search intervall has to be adjusted to ensure
% f(a) x f(b)<0

% Specify search intervall, a and b will be sent into the function
% bisection
 a = 4.0;
 b = 5.0;

% Call upon function bisection which returns the results in the variables
% solution and error.
  [solution,error] = bisection(a,b);

 solution % Write to screen.
 error  % Write to screen.
```

## Bisection.m

```
function [solution,error] = bisection(a,b)

% The numerical solver implemented here for solving the equation f(x)= 0
% is called Method of Halving the Interval (Bisection Method)

% You will not find exact match for f(x)= 0. Maybe f(x) = 0.0001 in the
end.
% By using ftol we say that if abs(f(x))<ftol, we are satisfied. We can
% also end the iteration if the search interval [a,b] is satisfactory
small.
% These tolerance values will have to be changed depending on the problem
% to be solved.


 ftol = 0.01;

% Set number of iterations to zero. This number will tell how many
 % iterations are required to find a solution with the specified accuracy.

  noit = 0;

  x1 = a;
  x2 = b;
```

```
  f1 = func(x1);
  f2 = func(x2);


%   First include a check on whether f1xf2<0. If not you must adjust your
%   initial search intervall. If error is 1 and solution is set to zero,
%   then you must adjust the search intervall [a,b].


 if (f1*f2)>=0
     error = 1;
     solution = 0;
 else
 % start iterating, we are now on the track.
     x3 = (x1+x2)/2.0;
     f3 = func(x3);

     while (f3>ftol | f3 < -ftol)
        noit = noit +1 ;

        if (f3*f1) < 0
           x2 = x3;
        else
           x1 = x3;
        end

        x3 = (x1+x2)/2.0;
        f3 = func(x3);
        f1 = func(x1);

     end
     error = 0;
     solution = x3;
     noit % This statement without ; writes out the number of iterations to
the screen.
 end
```

### func.m

```
function f = func(x)


  f = x^2-4*x+2;
```

## Appendix C

```
% Program where the Larsen Cuttings Transport Model is implemented

%  First specify all input parameters:

do = 8.5; % Outerdiameter (in) ( 1 in = 0.0254 m)
di = 5; % Innerdiameter   (in)
rop = 33 % Rate of Penetration - ROP ft/hr (1 ft = 0.3048m)
pv = 15 % Plastic viscosity (cP)
yp = 16 % Yield point (lbf/100ft2)
dcutt = 0.1 % Cuttings diameter (in) (1 inch = 0.0254 m)
mw = 10.833 % Mudweight (ppg - pounds per gallon) 1 ppg = 119.83 kg/m3.
rpm = 80 % rounds per minute
cdens = 19 % cuttings density (ppg - pounds per gallon)
angstart = 50 % Angle with the vertical

% vcut - Cuttings Transport Velocity (CTF in Larsens paper)
% vcrit - Critical Transport fluid velocity (CTFV) in Larsens paper. This
% is the minimum fluid velocity required to maintain a continously upward
% movement of the cuttings.
% vslip - Equivalent slip velocity (ESV) defined as the velocity difference
% between the cuttings and the drilling fluid
% vcrit = vcut+vslip
% All velocities are in ft/s.
% ua - apparent viscosity

% It should be noted that the problem is nested. Vcrit depends on vslip
% which again depends on an updated/correct value for vcrit. An iterative
% approch on the form  x(n+1) = g(x(n)) will be used.

for i = 1:8

ang(i)=angstart+i*5
vcut = 1/((1-(di/do)^2)*(0.64+18.16/rop));

vslipguess = 3;
vcrit = vcut + vslipguess;

% Find the apparent viscosity (which depends on the "guess" for vcrit)
ua = pv+ (5*yp*(do-di))/vcrit

% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
if (ua <= 53)
 vslip = 0.0051*ua+3.006;
else
 vslip = 0.02554*(ua-53)+3.28;
end

%Now we have two estimates for vslip that can be compared and updated in a
% while loop. The loop will end when the vslip(n+1) and vslip (n) do not
% change much anymore. I.e the iterative solution is found.
n=1;
while (abs(vslip-vslipguess))>0.01
 vslipguess = vslip;
 vcrit = vcut + vslipguess;
% Find the apparent viscosity (which depends on the "guess" for vcrit)
  ua = pv+ (5*yp*(do-di))/vcrit;
```

```
% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
  if (ua <= 53)
   vslip = 0.0051*ua+3.006;
  else
   vslip = 0.02554*(ua-53)+3.28;
  end
  n=n+1;
  vslip  % Take away ; and you will se how vslip converges to a solution
end % End while loop

%
% Cuttings size correction factor: CZ = -1.05D50cut+1.286
 CZ = -1.05*dcutt+1.286
% Mud Weight Correction factor (Buoancy effect)
  if (mw>8.7)
  CMW = 1-0.0333*(mw-8.7)
  else
  CMW = 1.0
  end

% Angle correction factor

CANG = 0.0342*ang(i)-0.000233*ang(i)^2-0.213

vslip = vslip*CZ*CMW*CANG;   % Include correction factors.

% Find final minimum velocity required for cuttings transport (ft/s).

vcrit = vcut + vslip

vcritms = vcrit*0.3048 % Velocity in m/s

Q = 3.14/4*((8.5*0.0254)^2-(5*0.0254)^2)*vcritms % (m3/s)
Q = Q*60*1000  % (lpm)

yrate(i)=Q
end

plot(ang,yrate)
```

# Appendix D – Steady State Model for Two Phase Flow

**Conservation of liquid mass**

$$\frac{\partial}{\partial z}(A\rho_l \alpha_l v_l) = 0$$

**Conservation of gas mass**

$$\frac{\partial}{\partial z}(A\rho_g \alpha_g v_g) = 0$$

**Conservation of momentum.**

$$\frac{\partial}{\partial z} p = -(\rho_{mix} g + \frac{\Delta p_{fric}}{\Delta z})$$

**Gas slippage model (simple):**

$v_g = K v_{mix} + S$ (K=1.2, S = 0.55)

**Liquid density model (simple)**

$\rho_l(p) = \rho_{lo} + \frac{(p - p_O)}{a_L^2}$, assume water: $\rho_{lo} = 1000$ kg/m$^3$, $p_O = 100000 Pa$, $a_L = 1500$ m/s

**Gas density model (simple)**

$\rho_g(p) = \frac{p}{a_g^2}$, ideal gas: $a_g = 316$ m/s.

**Friction model**

The friction model presented here is for a Newtonian fluids like water. The general expression for the frictional pressure loss gradient term is given by:

$$\frac{\Delta p_{fric}}{\Delta z} = \frac{2 f \rho_{mix} v_{mix} abs(v_{mix})}{(d_{out} - d_{in})} \quad \text{(Pa/m)}$$

$A$ - (m$^2$)

$\rho_i$ - phase densities (kg/m$^3$), liquid – > i=l, gas ->i =g

$v_i$ - phase velocities (m/s)

$p$ - pressure (Pa)

$g$ – gravity constant 9.81 m/s$^2$

$\alpha_i$ - phase volume fractions taking values between 0 and 1. $\alpha_l + \alpha_g = 1$.

$\rho_{mix} = \alpha_l \rho_l + \alpha_g \rho_g$ - mixture density

$v_{mix} = \alpha_l v_l + \alpha_g v_g$ - mixture velocity