University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: MPE 690 - PVT analysis and Reservoir modeling DATE: May 7, 2013

TIME: 4 hours

AID: No printed or written means allowed. Definite basic calculator allowed.

THE EXAM CONSISTS OF 5 PROBLEMS ON 6 PAGES AND 1 ADDITIVE (last page).

REMARKS: You may answer in English or Norwegian. Problem 1 and 2 (PVT part) are given equal weight as Problem 3-5 (modeling part).

Problem 1.

a.
Scratch a block diagram to illustrate how the PVT-simulator works.
Describe shortly the content of each block.

- Specify the input data for each component, especially the pseudo components (carbon numbers).
- Specify the parameters needed for each component, and how they are calculated from the input data for the pseudo components (carbon numbers).

b.
The SKR-EOS is given by the formula:

$$P = \frac{RT}{V - b} - \frac{a(T)}{V(V + b)}$$

The following parameters must be specified for each component *i*:

$a_i(T) = a_{ci}\alpha_i(T)$

$$a_{ci} = 0.42747 R^2 \frac{T_{ci}^{\ 2}}{P_{ci}}$$

$$b_i = 0.08664 R \frac{T_{ci}}{P_{ci}}$$

1. Explain what the two terms in the SKR-EOS represent.
2. Explain what the two parameters: a(T) and b, represent.
3. Explain by figure and formula, without doing any calculations, how the parameters a_{ci} and b_i can be determined for the pure component *i*.

c.

By putting $V = \frac{ZRT}{P}$, the SRK-EOS can be written as a 3. order equation in Z:

$Z^3 - Z^2 + (A-B-B^2)Z - AB = 0$

where:

$$A = \frac{a(T)P}{R^2T^2} \quad \text{and} \quad B = \frac{bP}{RT}$$

For a given value of P and T, a reservoir fluid with composition (mole fraction) z_i is in the 2-phase area, i. e. vapor and liquid in equilibrium. The cubic equation in Z is solved to give 2 real roots.
In order to do compositional calculations in the 2-phase region, K_i-values must be known.

Questions:
1. What is the physical meaning of the 2 real roots?
2. How is the K_i-value defined? What is the K_i -value depending on?
3. Explain how the K_i –value is determined. Use formula.

Problem 2.

The following reservoir data are given for an oil reservoir:

$P_i = 450$ bar
$P_b = 253.3$ bar

T_{res} = 80 ^oC
Φ = 0.25
S_{wr} = 0.2
Bulk reservoir volume: V_{bulk}= 10^6 m^3

A Constant Mass Expansion (CME) analysis of the reservoir fluid is performed at T_{res}, and the following data are given:

Pressure Bar	**Rel. Vol. V/V_b**	**Compressibility 1/Bar**	**Y -Factor**
500.0	0.9634	1.134E-04	
450.0	0.9693	1.240E-04	
400.0	0.9759	1.364E-04	
350.0	0.9832	1.510E-04	
300.0	0.9914	1.686E-04	
253.3	1.0000	1.884E-04	
250.0	1.0033		4.02
200.0	1.0725		3.68
150.0	1.2087		3.30
100.0	1.5271		2.91
50.0	2.6158		2.52

A 3 step separator test was simulated, and the following data were obtained:

Pressure Bar	**Temp ^oC**	**GOR Sm^3/Sm^3**	**Gravity air=1**	**Oil Dens g/cm^3**	**FVF m^3/Sm^3**
253.3	80.0			0.732	1.340
100.0	60.0	73.7	0.682	0.796	1.155
30.0	40.0	31.1	0.710	0.834	1.070
1.0	15.0	21.3	1.048	0.865	1.000

The fluid is supposed to be produced through the given separator system, and the HCPV is assumed to be constant during the pressure depletion.

a. Calculate:
1. Initial oil formation volume factor, B_{oi}, at P_i=450 bar.
2. Initial oil in place, IOIP, and initial gas in place, IGIP, as Sm^3.
3. Calculate volume of STO and gas produced by pressure depletion from P_i to P_b as Sm^3.
4. Illustrate the variation in B_o in the pressure interval 1 – 450 bar.

Problem 3.

(a) Consider the linear transport equation

$$(*) \qquad u_t + cu_x = 0, \qquad c > 0 \text{ is constant}, \qquad x \in (0,1),$$

with initial data

$$u(x, t=0) = u_0(x) = \begin{cases} 1, & 0.4 \le x \le 0.6; \\ 0, & \text{otherwise.} \end{cases}$$

and boundary data

$$u(x=0,t) = u(x=1,t) = 0.$$

- Check that a general solution of $(*)$ is $u(x,t) = u_0(x - ct)$.
- Describe by words the behavior reflected by this solution.
- Explain why the right boundary condition is satisfied only for times $t \le T^b = \frac{0.4}{c}$.

(b) In the following we set $c = 1$ in $(*)$ and assume that $T \in (0, T^b)$ and consider a discretization of $[0,1] \times [0,T]$ with discretization parameters Δx and Δt.
More precisely, we divide the domain $[0,1]$ into cells $0, 1, \ldots, M, M+1$. For the first and last cell we set $u_0^{n+1} = 0$ and $u_{M+1}^{n+1} = 0$. In the interior part of the domain $(j = 1, \ldots, M)$ we consider a discrete scheme for the model $(*)$ of the form

$$(**) \qquad \frac{u_j^{n+1} - u_j^n}{\Delta t} + \frac{1}{\Delta x}\Big(U_{j+1/2}^n - U_{j-1/2}^n\Big) = 0, \qquad j = 1, \ldots, M.$$

- Describe how to define the flux terms $U_{j+1/2}^n$ and $U_{j-1/2}^n$ for the cells $j = 1, \ldots, M$ in order to obtain a stable scheme (up-wind scheme) and find an expression for u_j^{n+1}.

It is known that the true solution of $(*)$ satisfies the estimate

$$\int_0^1 |u(x,t)|\, dx \le \int_0^1 |u_0(x)|\, dx.$$

- Demonstrate how to obtain a corresponding estimate for the discrete scheme obtained from $(**)$.
(hint: use the triangle inequality: $|a + b| \le |a| + |b|$ and consider summation in space over the cells $j = 1, \ldots, M$).
- What is the condition on the discretization parameters Δt and Δx that guarantee stability?

(c) In the following we consider a horizontal 1D reservoir.
- State the single-phase porous media mass balance equation in 1D (without source term) and identify the various variables (rock and fluid).
- Introduce Darcy's law and derive an equation for the pressure where it is assumed that $\phi = \phi(p)$ and $\rho = \rho(p)$.
- Assume a weakly compressible rock (compressibility c_r is small) and find a linear relation for $\phi(p)$. Assume that the fluid is incompressible and show that we then can obtain a pressure equation of the form

$$(***) \qquad p_t = \kappa p_{xx},$$

and identify the constant parameter $\kappa > 0$.

(d) Consider the pressure equation $(***)$ on the spatial domain $[0,1]$ and assume that we have the initial data

$$p(x, t=0) = p_0(x) = \begin{cases} 4x + a, & 0 \le x \le \frac{1}{2}; \\ 2(1-x) + b, & \frac{1}{2} < x \le 1. \end{cases}$$

with $a, b > 0$ and boundary data

$$p(x=0,t) = a, \qquad p(x=1,t) = b.$$

- What is the condition on a and b that ensures that $p_0(x)$ is continuous? What is the physical interpretation of this model problem?
- Formulate the model that describes the stationary solution of $(***)$ and solve this equation. What is the corresponding stationary fluid velocity?

(e) Explain why we have the estimate

$$\int_0^1 p(x,t)^2\,dx \le \int_0^1 p_0(x)^2\,dx$$

when we consider the domain $[0,1]$ and boundary conditions $p(x=0,t) = p(x=1,t) = 0$ for the model $(***)$.

Problem 4. This exercise deals with the Black-Oil Model (BOM).

(a) What are the basic assumptions for a Black Oil model?
- Formulate the mass balance equations by using component densities at reservoir condition (RC) denoted as ρ_w, ρ_o, ρ_g, and ρ_{dg}.

(b) Introduce volume factors B_l $(l = w, o, g)$ and gas-oil solution ratio R_s (define them) and demonstrate how to rewrite the model formulated in (a) where these parameters are used.

Problem 5. This exercise deals with the Buckley-Leverett model. The BL model for water flooding is given by

$$(*) \qquad s_t + f(s)_x = 0.$$

(a) A solution of $(*)$ can be constructed by using the equation

$$x_s = f'(s)t, \qquad s \in [0,1].$$

- Explain why we obtain an unphysical solution by using this approach.
- What is the physical principle that is used to determine the front height s^* of the physical correct solution?
- Use this principle and show how to obtain the mathematical equation satisfied by s^*.

(b) Given the fractional flow function $f(s)$ shown in Fig. 1 (left figure). The corresponding derivative is also shown in Fig. 1 (right figure). Sketch the solution at time $T = 0.5$ (dimensionless).

(c) The parameter $M = \frac{\mu_w}{\mu_o}$ (viscosity ratio) used in b) is 4. Now it is set to be $M = 0.5$. The relative permeability functions, which are based on Corey functions, are unchanged.
- Make a rough sketch of the new fractional flow function where you compare with the one used in b).
- Make a rough sketch of the corresponding new solution at time $T = 0.5$. Compare with the solution obtained in b). What is the main difference between the two solutions (front height and position)?

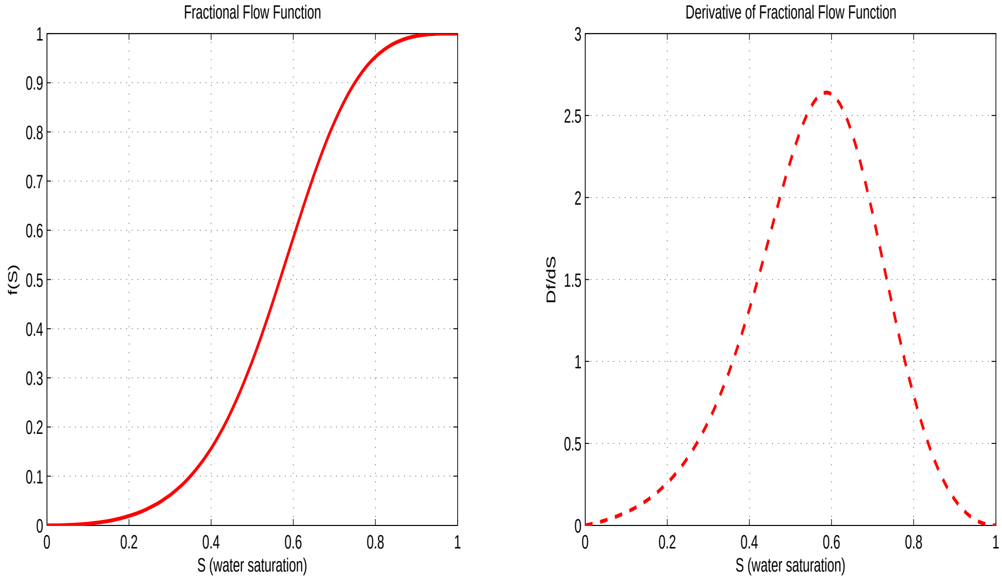

FIGURE 1. **Left:** Plot of $f(s)$. **Right:** Plot of $f'(s)$.

Important formula/correlations in PVT-Analysis.

Temperature:

$^oK = 273.15 + {}^oC$
$^oF = 1.8 \times {}^oC + 32$
$^oR = {}^oF + 459.69$

Pressure:

1atm = 1013.250 mBar = 1.013250 bar = 101.3250 kPa = 0.1013250 MPa = 14.69595 psia
psia = 14.69595 + psig
1 atm = 760.002 mmHg at 0 ^oC

Density:

1 g/cm^3 = 62.43 lb/ft^3 = 350.54 lb/bbl
1 lb/ft^3 = 16.0185 kg/m^3
ρ_w = 0.999015 g/cm^3 (60 ^oF, 1 atm)
ρ_w = 0.9991 g/cm^3 (15 ^oC, 1 atm)

Specific density:

For liquids: Determined relative to water at sc.
For gases: Determined relative to air at sc.

$$\gamma_o = \frac{\rho_o}{\rho_w} = \frac{141.5}{131.5 + {}^oAPI}$$

$$^oAPI = \frac{141.5}{\gamma_o} - 131.5$$

Cragoe`s formula (empirical formula giving molecular weight of hydrocarbons):

$$M_o = \frac{6084}{^oAPI - 5.9}$$

$$\gamma_g = \frac{M_g}{M_{air}} = \frac{M_g}{28.96}$$

Volume:

1 bbl = 5.615 ft^3 = 0.15898 m^3
1 ft^3 = 0.0283 m^3
1 US Gallon = 3.785 litre
1 Imp. Gallon = 4.546 litre
Molar volume of gas at standard conditions:
V_m = 379.51 SCF/lb mole (60 ^oF and 14.69595 psia)
V_m = 23644.7 cm^3/g mole = 23.6447 m^3/kg mole (15 ^oC and 101.3250 kPa)

Air:

Z_{air} = 0.9959 (60 ^oF, 14.69595 psia)
M_{air} = 28.96

Gas constant:

R = 10.732 (psia, ft^3, ^oR, lb mole)
R = 0.082054 (atm, litre, ^oK, g mole)
R = 8.3145 (kPa, m^3, ^oK, kg mole)