University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

DATE: March 5, 2014

SUBJECT: PET 510 – Computational Reservoir and Well Modeling

TIME: 4 hours

AID: No printed or written means allowed. Calculator is allowed.

THE EXAM CONSISTS OF 6 PROBLEMS ON 5 PAGES AND APPENDIX A - D

REMARKS: You may answer in English or Norwegian. Exercises 1 and 2 (part A) and exercises 3-6 (part B) are given equal weight.

HALLIBURTON

Problem 1.

(a) Consider the linear transport equation

$$(*) \qquad u_t + xu_x = q(x,t), \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x, t = 0) = \phi(x).$$

Set $q(x,t) = 0$. Compute the solution $u(x,t)$ by using the method of characteristics. Verify that your solution satisfies (*) and (**).

(b) Consider (*) with $q(x,t) = x$.
- Compute the solution $u(x,t)$ by using the method of characteristics. Verify that your solution satisfies (*) and (**)
- What is the dominating part of the solution when $t \to \infty$ (the long time behavior).

(c) Consider (*) with $q(x,t) = xe^{-t}$.
- Compute the solution ~~$u(x,t)$~~ by using the method of characteristics. Verify that your solution satisfies (*) and (**)
- What is the dominating part of the solution when $t \to \infty$ (the long time behavior).

(d) Now, consider (*) with $q(x,t) = 0$ on the interval $x \in [0,5]$ with initial data

$$(***) \qquad \phi(x) = \begin{cases} 2x, & 0 \le x < 0.5; \\ 2(1-x), & 0.5 \le x \le 1. \end{cases}$$

- State the solution of this this problem in view of the solution found in (a).
- Make a sketch of the initial data and the solution $u(x,t)$ at time $t = 1$ in one and the same figure.
- Show that we can obtain the following characterization of the solution u

$$\int_0^5 u(x,t)^2 \, dx = e^t \int_0^5 \phi(x)^2 \, dx, \qquad 0 \le t \le 1.$$

Hint: Multiply (*) by u and integrate over $[0,5]$.

(e) Consider a discretization of the domain $[0,5] \times [0,T]$ with discretization parameters Δx and Δt. Divide the domain $[0,5]$ into cells $1, \ldots, M$ and timesteps $t^0 = 0$, $t^1 = \Delta t$, $\ldots$, $t^n = n\Delta t$.
- Define a stable discrete scheme for computing the solution in part (d) based on explicit time discretization.

Problem 2.

(a) In the following we consider the following model for single-phase flow in a vertical wellbore:

$$(*) \begin{cases} \rho_t + (\rho u)_x = q_w, \\ (\rho u)_t + (\rho u^2)_x + P(\rho)_x = -\kappa u - g\rho, \qquad x \in [0, L], \end{cases}$$

where κ and g are constants related to friction and gravity, $x = 0$ represents bottom and $x = L$ represents top.

- Assume that we ignore the acceleration effect represented by $(\rho u)_t + (\rho u^2)_x$ in the momentum equation. Show that we obtain an equation for the density ρ of the form

$$(**) \qquad \rho_t + f(\rho)_x = (d(\rho)P(\rho)_x)_x + q_w.$$

In particular, identify $f(\rho)$ and $d(\rho)$.

(b) Consider a grid composed of $1, \dots, M$ cells where x_j refers to the cell center with cell interfaces $x_{j-1/2}$ and $x_{j+1/2}$. Write down a discrete version of (**) for the interior domain, i.e., cells $2, \dots, M-1$ where boundary conditions are not involved.

(c) Consider Fig. 1 where we have specified a liquid fluid rate at left end ($x = 0$). The right end (at $x = L$) is open with atmospheric pressure $p^* = 1$ bar. Focus on pressure P and fluid velocity u along the wellbore.

- Explain the relation between pressure profiles and fluid velocity profiles for the 3 different cases. In particular, make use of the equation that relates u, $P(\rho)_x$, and $g\rho$.

(d) We are interested in the stationary solution of (**). We assume a weakly compressible liquid with a linear pressure law

$$P(\rho) = a_l^2(\rho - \rho^0) + p^0,$$

where a_l represents sound velocity, and ρ^0 is the density corresponding to pressure p^0.

- How will the sound velocity a_l affect the convergence towards the stationary solution

(e) The stationary solution of (**) when $q_w = 0$ satisfies the following ODE

$$(***) \qquad f(\rho) - d(\rho)P(\rho)_x = q_L(t), \qquad \rho|_{x=L} = \rho^*,$$

where $q_L(t)$ is the rate at $x = 0$ and $P(\rho^*) = p^*$ is the pressure at $x = L$ and

$$f(\rho) = -\frac{g}{\kappa}\rho^2, \qquad d(\rho) = \frac{1}{\kappa}\rho.$$

Show that that

$$\rho(x) = \sqrt{\frac{[a + b(\rho^*)^2]e^{2b(L-x)} - a}{b}}, \qquad a = \kappa q_L/a_l^2, \qquad b = g/a_l^2,$$

(a, b, ρ^* are constants relative space variable x) is a solution of $(***)$.

Hint: Show first that

$$\rho_x = -\frac{1}{\rho}[a + b(\rho^*)^2)]e^{2b(L-x)}.$$

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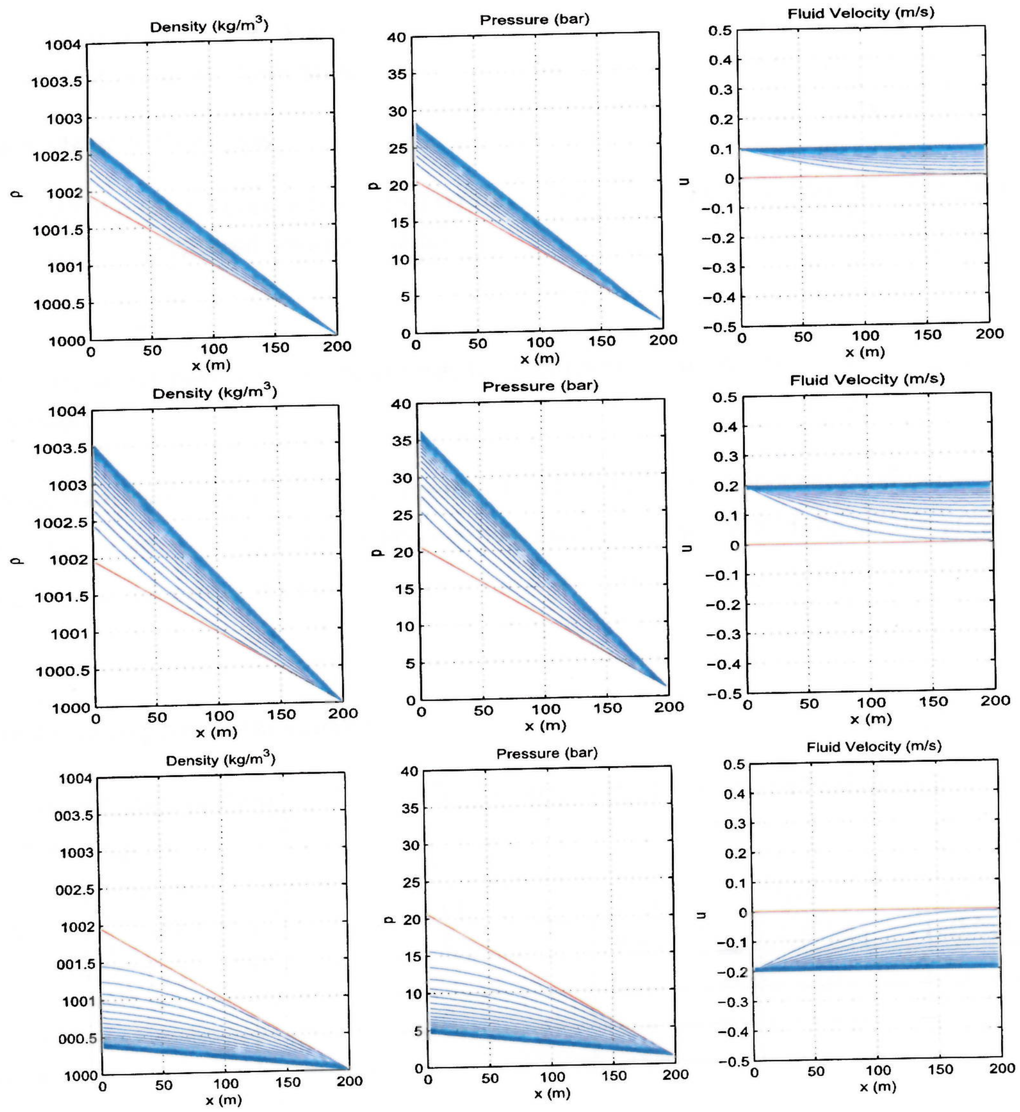


FIGURE 1. Curves show change in profiles from initial state towards stationary state.

Exam Part B – Solving Nonlinear Equations & Modelling of Well Flow

There are 11 questions in total. Some formulas, equations and Matlab codes are found in Appendixes. This part constitutes 50 % of exam.

Exercise 3 – Matlab Questions

a) Explain what is the difference between a script file and a function file in Matlab and explain how information is transferred between the files.

b) In the course, we have learned about three types of control statements in Matlab. Explain how these works.

c) The program given in Appendix B has been written for solving the nonlinear equation $f(x) = x^2 - 4x + 2$ using the bisection method. Explain how you would change this program in order to find the root of the function $f(x) = x^3 + x^2 - 3x - 3$ in the interval [1,2]

Exercise 4 – Solving Nonlinear Equations

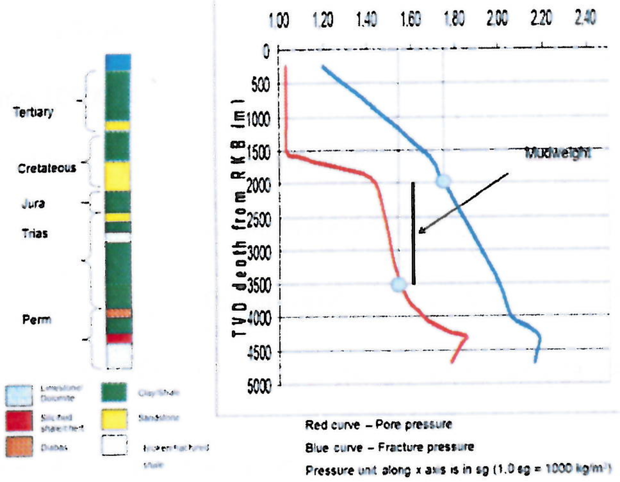
a) We are given the function $f(x) = e^x - 3x$. Show how the bisection method works by filling out the following table.

Iteration	x1	x2	x3	f(x1)	f(x2)	f(x3)
1	1	2	1,5	-0,28172	1,38906	-0,01831
2						
3						
4						

b) In the following, we will consider a horizontal closed pipe. The pipe has an inner diameter of 0.2 meter. The length of the pipe is 5 meters. The pipe contains 141.92 kg with water and 1.57 kg of gas. We want to find the pressure inside this pipe. How can we proceed to solve this problem ? (Hint: Formulas for gas and liquid densities can be found in Appendix D)

c) Consider the following equation: $f(x) = x^2 - 2x - 3$. This has roots x_1=-1 and x_2=3. Use the iterative method to show how we can find one of these roots. Use x_0=4 as a starting point

Exercise 5 – Well pressures



a) We are at 3500 meters. We are circulating the well with 2000 lpm (liters per minute). The Equivalent circulating density ECD is 1.67 sg. During a connection, we saw that the pressure drop in annulus was 25 bars. Find out what the static mudweight is!

b) What is the pump pressure reflecting and what will happen with the pump pressure when the rig pump is turned off ?

c) We take a kick of 4 m^3. The kick has a pressure of 515 bar at bottom. If we let this kick migrate in a closed well of depth 3500 meters, what will the final bottomhole pressure theoretically become? (assume mudweight of 1.5 sg and that temperature effects can be neglected)

Exercise 6 – Conservation Laws

a) What are the three fundamental conservation laws and why is there a need for closure laws

b) What kind of mathematical model is the drift flux model and what role does the eigenvalues of the system have (what do they express) ?

Appendix A – Some Units & Formulas

1 inch =2.54 cm = 0.0254 m

1 feet = 0.3048 m

1 bar = 100000 Pa

1 sg = 1 kg/l (sg - specific gravity)

$M = Q \cdot \rho$ M massrate (kg/s), Q Volumerate (m^3/s), ρ density (kg/m^3)

$Q = v \cdot A$ Q Volumerate (m^3/s), v velocity m/s. A area m^2

$p = \rho \cdot h \cdot 0.0981$ p (bar), ρ density (sg), h – vertical depth (m)

$\frac{P \cdot V}{T} = C$, from Ideal gas law

$P \cdot V = C$, Boyles law (temperature is assumed constant)

Appendix B

Main.m

```
% Main program that calls up a routine that uses the bisection
% method to find a solution to the problem f(x) = 0.
% The search intervall [a,b] is specified in the main program.
% The main program calls upon the function bisection which again calls upon
% the function func.

% if error = 1, the search intervall has to be adjusted to ensure
% f(a) x f(b)<0

% Specify search intervall, a and b will be sent into the function
% bisection
 a = 4.0;
 b = 5.0;

% Call upon function bisection which returns the results in the variables
% solution and error.
  [solution,error] = bisection(a,b);

 solution % Write to screen.
 error  % Write to screen.
```

Bisection.m

```
function [solution,error] = bisection(a,b)

% The numerical solver implemented here for solving the equation f(x)= 0
% is called Method of Halving the Interval (Bisection Method)

% You will not find exact match for f(x)= 0. Maybe f(x) = 0.0001 in the
end.
% By using ftol we say that if abs(f(x))<ftol, we are satisfied. We can
% also end the iteration if the search interval [a,b] is satisfactory
small.
% These tolerance values will have to be changed depending on the problem
% to be solved.

  ftol = 0.01;

% Set number of iterations to zero. This number will tell how many
 % iterations are required to find a solution with the specified accuracy.

   noit = 0;

   x1 = a;
   x2 = b;
```

```
  f1 = func(x1);
  f2 = func(x2);


%   First include a check on whether f1xf2<0. If not you must adjust your
%   initial search intervall. If error is 1 and solution is set to zero,
%   then you must adjust the search intervall [a,b].


 if (f1*f2)>=0
     error = 1;
     solution = 0;
 else
 % start iterating, we are now on the track.
     x3 = (x1+x2)/2.0;
     f3 = func(x3);

     while (f3>ftol | f3 < -ftol)
        noit = noit +1 ;

        if (f3*f1) < 0
           x2 = x3;
        else
           x1 = x3;
        end

        x3 = (x1+x2)/2.0;
        f3 = func(x3);
        f1 = func(x1);

     end
     error = 0;
     solution = x3;
     noit % This statement without ; writes out the number of iterations to
the screen.
 end
```

func.m

```
function f = func(x)


  f = x^2-4*x+2;
```

Appendix C

```
% Program where the Larsen Cuttings Transport Model is implemented

%  First specify all input parameters:

do = 8.5; % Outerdiameter (in)  ( 1 in = 0.0254 m)
di = 5; % Innerdiameter   (in)
rop = 33 % Rate of Penetration - ROP ft/hr (1 ft = 0.3048m)
pv = 15 % Plastic viscosity (cP)
yp = 16 % Yield point (lbf/100ft2)
dcutt = 0.1 % Cuttings diameter (in)  (1 inch = 0.0254 m)
mw = 10.833 % Mudweight (ppg - pounds per gallon) 1 ppg = 119.83 kg/m3.
rpm = 80 % rounds per minute
cdens = 19 % cuttings density (ppg - pounds per gallon)
angstart = 50 % Angle with the vertical

% vcut - Cuttings Transport Velocity (CTF in Larsens paper)
% vcrit - Critical Transport fluid velocity (CTFV) in Larsens paper. This
% is the minimum fluid velocity required to maintain a continously upward
% movement of the cuttings.
% vslip - Equivalent slip velocity (ESV) defined as the velocity difference
% between the cuttings and the drilling fluid
% vcrit = vcut+vslip
% All velocities are in ft/s.
% ua - apparent viscosity

% It should be noted that the problem is nested. Vcrit depends on vslip
% which again depends on an updated/correct value for vcrit. An iterative
% approch on the form  x(n+1) = g(x(n)) will be used.

for i = 1:8

ang(i)=angstart+i*5
vcut = 1/((1-(di/do)^2)*(0.64+18.16/rop));

vslipguess = 3;
vcrit = vcut + vslipguess;

% Find the apparent viscosity (which depends on the "guess" for vcrit)
ua = pv+ (5*yp*(do-di))/vcrit

% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
if (ua <= 53)
 vslip = 0.0051*ua+3.006;
else
 vslip = 0.02554*(ua-53)+3.28;
end

%Now we have two estimates for vslip that can be compared and updated in a
% while loop. The loop will end when the vslip(n+1) and vslip (n) do not
% change much anymore. I.e the iterative solution is found.
n=1;
while (abs(vslip-vslipguess))>0.01
 vslipguess = vslip;
 vcrit = vcut + vslipguess;
% Find the apparent viscosity (which depends on the "guess" for vcrit)
  ua = pv+ (5*yp*(do-di))/vcrit;
```

```
% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
  if (ua <= 53)
   vslip = 0.0051*ua+3.006;
  else
   vslip = 0.02554*(ua-53)+3.28;
  end
  n=n+1;
  vslip  % Take away ; and you will se how vslip converges to a solution
end % End while loop

%
% Cuttings size correction factor: CZ = -1.05D50cut+1.286
 CZ = -1.05*dcutt+1.286
% Mud Weight Correction factor (Buoancy effect)
  if (mw>8.7)
  CMW = 1-0.0333*(mw-8.7)
  else
  CMW = 1.0
  end

% Angle correction factor

CANG = 0.0342*ang(i)-0.000233*ang(i)^2-0.213

vslip = vslip*CZ*CMW*CANG;   % Include correction factors.

% Find final minimum velocity required for cuttings transport (ft/s).

vcrit = vcut + vslip

vcritms = vcrit*0.3048 % Velocity in m/s

Q = 3.14/4*((8.5*0.0254)^2-(5*0.0254)^2)*vcritms % (m3/s)
Q = Q*60*1000  % (lpm)

yrate(i)=Q
end

plot(ang,yrate)
```

Appendix D – Steady State Model for Two Phase Flow

Conservation of liquid mass

$$\frac{\partial}{\partial z}(A\rho_l\alpha_l v_l) = 0$$

Conservation of gas mass

$$\frac{\partial}{\partial z}(A\rho_g\alpha_g v_g) = 0$$

Conservation of momentum.

$$\frac{\partial}{\partial z}p = -(\rho_{mix}g + \frac{\Delta p_{fric}}{\Delta z})$$

Gas slippage model (simple):

$v_g = Kv_{mix} + S$ (K=1.2, S = 0.55)

Liquid density model (simple)

$\rho_l(p) = \rho_{lo} + \frac{(p - p_O)}{a_L^2}$, assume water: $\rho_{lo} = 1000$ kg/m^3, $p_O = 100000 Pa$, $a_L = 1500$ m/s

Gas density model (simple)

$\rho_g(p) = \frac{p}{a_g^2}$, ideal gas: $a_g = 316$ m/s.

Friction model

The friction model presented here is for a Newtonian fluids like water. The general expression for the frictional pressure loss gradient term is given by:

$$\frac{\Delta p_{fric}}{\Delta z} = \frac{2f\rho_{mix}v_{mix}abs(v_{mix})}{(d_{out} - d_{in})} \text{ (Pa/m)}$$

A - (m^2)

ρ_i - phase densities (kg/m^3), liquid –> i=l, gas ->i =g

v_i - phase velocities (m/s)

p - pressure (Pa)

g – gravity constant 9.81 m/s^2

α_i - phase volume fractions taking values between 0 and 1. $\alpha_l + \alpha_g = 1$.

$\rho_{mix} = \alpha_l \rho_l + \alpha_g \rho_g$ - mixture density

$v_{mix} = \alpha_l v_l + \alpha_g v_g$ - mixture velocity