University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

DATE: August 29, 2014

SUBJECT: PET 565 – Core scale modeling and interpretation

TIME: 4 hours

AID: No printed or written means allowed. Definite basic calculator allowed.

THE EXAM CONSISTS OF 4 PROBLEMS ON 5 PAGES

REMARKS: You may answer in English or Norwegian. Exercises 1 and 2 (part A) and exercises 3 and 4 (part B) are given equal weight.

Problem 1.

(a) Consider the conservation law

$$(*) \qquad u_t + f(u)_x = 0, \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x, t = 0) = u_0(x).$$

- We choose $f(u) = \frac{1}{2}u^2$. Compute an expression for the solution by using the method of characteristics.
- Assume that u_0 is given by

$$u_0(x) = \begin{cases} 2x, & 0 \le x \le \frac{1}{2}; \\ 2(1-x), & \frac{1}{2} < x \le 1. \end{cases}$$

Make a sketch of $u_0(x)$. Then, find an expression for the characteristics that starts at different points $x_0 = 0$, $x_0 = \frac{1}{4}$, $x_0 = \frac{1}{2}$, $x_0 = \frac{3}{4}$, and $x_0 = 1$ at time $t = 0$, and make a sketch of them in $x - t$ coordinate system.

(b) Based on the expression for the characteristics found in (a), demonstrate that we get the following expression for x_0:

$$(***) \qquad x_0 = \begin{cases} \frac{x}{1+2t}, & 0 \le \frac{x}{1+2t} \le \frac{1}{2}; \\ \\ \frac{x-2t}{1-2t}, & \frac{1}{2} < \frac{x-2t}{1-2t} \le 1 \end{cases}$$

- Based on (***) find an expression for the solution $u(x,t)$. Give a sketch of the the solution at time $t = \frac{1}{4}$ and identify which parts of the solution that represents, respectively, a rarefaction wave and a compression wave.

(c) After some time $t = T_b$, a discontinuity is formed in the solution $u(x,t)$. Find this time T_b, determine the solution $u(x, T_b)$, and make a sketch of it.

(d) Using the equation for the characteristic, one can construct a solution for times $t > T_b$ by tracking the distance travelled for different values of u. Consider $x_0 = 0$, $x_0 = \frac{1}{4}$, $x_0 = \frac{1}{2}$, $x_0 = \frac{3}{4}$, and $x_0 = 1$ and compute the travelled distance of the corresponding values of u and make a sketch of the resulting solution $u(x,t)$ at time $t = 1$.

(e) The solution computed in (d) is not a physical correct solution. Why not?
Now we want to find an expression for $u(x,t)$ by removing the unphysical part.
- Introduce a height u_s at position x_s in the unphysical solution and use the principle of conservation of mass to compute an expression for $x_s(t)$ and $u_s(t)$.
- Sketch the resulting (physical correct) solution at time $t = 1$.
- Check that this solution is consistent with the Rankine-Hugoniot condition.

Problem 2. The Buckley-Leverett (B-L) equation for water-oil transport (horizontal flow) is given by

$$(A) \qquad \frac{\partial S}{\partial t} + \frac{\partial f(S)}{\partial x} = 0, \qquad x \in [0, 1]$$

where S is the water saturation and x and t are dimensionless variables.

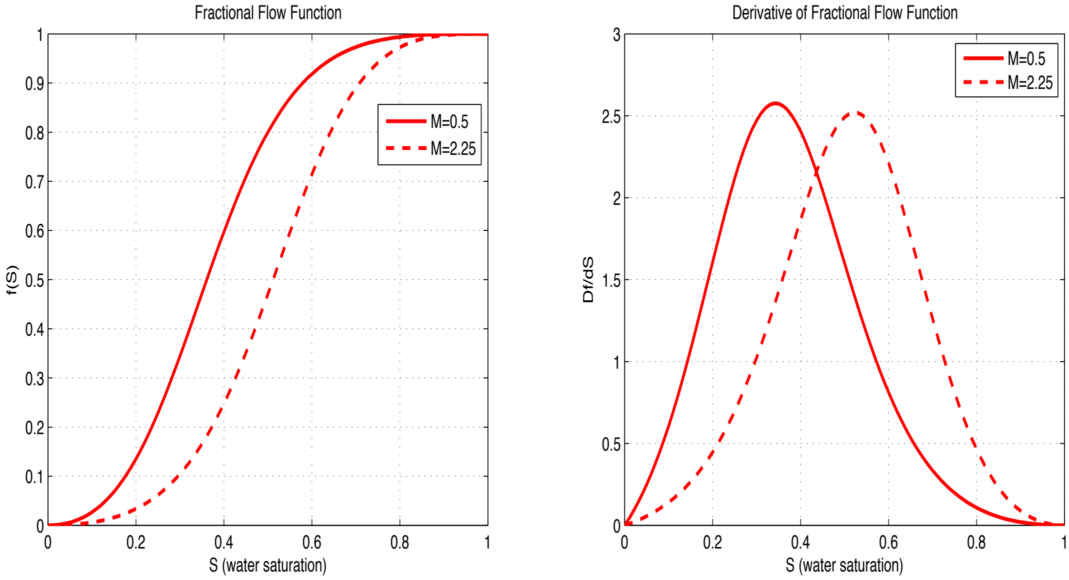

FIGURE 1. **Left:** Plot of $f(S)$. **Right:** Plot of $f'(S)$.

(a) Formulate the two mass balance equations (where Darcys law has been used), respectively, for water and oil that are used to derive (A). State the main assumptions used to derive (A). Demonstrate why the total velocity (sum of water and oil velocity) becomes constant and express the pressure gradient term in terms of total velocity and mobility functions.

(b) Identify the expression of the function $f(S)$ in terms of mobility functions and give a sketch of a typical fractional flow function $f(S)$. Explain how to calculate the water-front solution after a time $T > 0$ when water is injected at $x = 0$. Draw figures and describe the different steps.

(c) A mathematical (but unphysical) solution of the water-flooding problem in point (b) can be constructed by using the equation $x_s = f'(S)$T where $S \in [0, 1]$ and x_s represents the travelled distance of saturation S. Illustrate this solution and explain mathematically why we obtain the equation $f(S^*) = f'(S^*)S^*$ as a characterization of the front height S of the physical solution.

(d) Given the two fractional flow functions shown in Fig. 1 (left figure) corresponding to two different values of the viscosity ratio $M = \frac{\mu_w}{\mu_o}$. The corresponding derivatives $f'(S)$ are also shown in Fig. 1 (right figure). Compute oil recovery for $f(S)$ with $M = 2.25$ at time T_b corresponding to the time when water breakthrough takes place.

(e) Consider now the fractional flow function with $M = 0.5$. What is oil recovery at time T_b for this case, where T_b is the time found in point d)?

<u>Problem 3</u>.

(a) Assume a brine contains 0.01 M Mg^{2+}, 0.02 M Ca^{2+} and 0.03 M SO_4^{2-}.
- Calculate the ionic strength of the solution.
- Calculate the activity coefficients.
- What is the activity of Ca^{2+}?

Relevant equations are:

$$I = 1/2 \sum_i m_i Z_i^2, \qquad \log \gamma_i = -A z_i^2 \left(\frac{\sqrt{I}}{1+\sqrt{I}} - 0.3I\right), \quad A = 0.51$$

(b) For the same brine, assume formation of the complex $CaSO_4^0$.
- Define the relevant dissociation reaction for this complex and corresponding equilibrium equation.
- Write mass balance equations for the species Ca^{2+} and SO_4^{2-}.

(c) For the same brine, assuming the dissociation constant is $K_{CaSO_4^0} = 25$ and the $\gamma_i = 1$, determine the concentrations of free species and complexes.

(d) The reactions for ion exchange of Na^+, Fe^{3+} and Mg^{2+} are given as follows

$$Na^+ + 1/3FeX_3 \rightleftharpoons NaX + 1/3Fe^{3+}$$
$$Na^+ + 1/2MgX_2 \rightleftharpoons NaX + 1/2Mg^{2+}$$

- Use the law of mass action to write equilibrium equations for this system.
- Assuming the equilibrium constants K_{nafe}, K_{namg} are known for the above reactions, derive the equilibrium constant $K_{femg}(K_{nafe}, K_{namg})$ for the exchange reaction of Fe^{3+} and Mg^{2+}:

$$2Fe^{3+} + 3MgX_2 \rightleftharpoons 2FeX_3 + 3Mg^{2+}$$

(e) A solution is exposed to the atmosphere with partial pressure of carbon dioxide given by $P_{co2} = 10^{-3.5}$. Carbon then enters the solution in aqueous forms of H_2CO_3, HCO_3^- and CO_3^{2-}. Relevant reactions are given below:

$$\begin{aligned} CO_2(g) + H_2O(g) &\rightarrow H_2CO_3(aq) \\ H_2CO_3(aq) &\rightarrow H^+(aq) + HCO_3^-(aq) \\ HCO_3^-(aq) &\rightarrow H^+(aq) + CO_3^{2-}(aq) \end{aligned}$$

where the equilibrium constants are respectively $K_H = 10^{-1.5}, K_1 = 10^{-6.3}, K_2 = 10^{-10.3}$.
- Assuming $\gamma_i = 1$, write the equilibrium equations for the system.
- Assuming the pH is 6 at equilibrium, determine the respective concentrations of carbonic species.
- Determine the total concentration of carbon in the solution.

<u>Problem 4</u>.

(a) Brine is injected into a system with velocity v_w, and it carries a species with concentration c which adsorbs at amount $q(c)$ at a given concentration c. This system can be described by the partial differential equation:

$$\partial_t c = -v_w \partial_x c - \partial_t q(c)$$

z	erfc(z)	z	erfc(z)	z	erfc(z)
0	0.0000	0.5	0.5205	1	0.8427
0.05	0.0564	0.55	0.5633	1.05	0.8624
0.1	0.1125	0.6	0.6039	1.1	0.8802
0.15	0.1680	0.65	0.6420	1.15	0.8961
0.2	0.2227	0.7	0.6778	1.2	0.9103
0.25	0.2763	0.75	0.7112	1.25	0.9229
0.3	0.3286	0.8	0.7421	1.3	0.9340
0.35	0.3794	0.85	0.7707	1.35	0.9438
0.4	0.4284	0.9	0.7969	1.4	0.9523
0.45	0.4755	0.95	0.8209	1.45	0.9597

FIGURE 2. Complementary error function table.

- Show that for continuous solutions the speed v_c of a given concentration c is given as

$$v_c = v_w \frac{1}{1 + dq/dc}$$

Assume $v_w = 3$ m/d and the length of the system is 3 m.
- The adsorption term is given as $q = c^2$. The initial concentration is $c = 0$ and the injected concentration is $c = 1$, Plot the concentration profile $c(x)$ after 1 d.
- Assume that the initial concentration is $c = 1$ and the injected concentration is $c = 0$. Plot the concentration profile $c(x)$ after 1 d.

(b) A brine is injected into a rock that can adsorb one of the components in the brine, while the other component acts as a tracer. The system is described by advection, dispersion and adsorption.
- What is a tracer? Write the transport equations for the two components.
- Show mathematically how we can use measurements of the time-concentration profiles $c(t)$ at the outlet to measure how many moles are adsorbed by the rock. Which parameters must be known / measured?

(c) - Explain how an adsorption isotherm can be produced from experimental measurements.
- How can such a tracer-adsorption system be used to interpret wettability in chalk?
- Define a wettability index based on the adsorption test.

(d) The equation for a system with advection and dispersion can be described by

$$\partial_t c = -v_w \partial_x c + D \partial_{xx} c$$

where D is dispersion coefficient. Assuming a fixed concentration c_0 at the inlet boundary and a uniform initial concentration c_i: The concentration $c(x,t)$ is given by

$$c(x,t) = c_i + \frac{c_0 - c_i}{2}\left[erfc(\frac{x - vt}{\sqrt{4Dt}}) + exp(\frac{vx}{D})erfc(\frac{x + vt}{\sqrt{4Dt}})\right]$$

- Assume $c_0 = 10$, $c_i = 0$, $v_w = 10^{-7}$m/s and $D = 10^{-8}$m^2/s. Calculate the concentrations at $x = 0.25, 0.5, 0.75$m after $5 \cdot 10^6$s and sketch the concentration profile. Make use of the $erfc$ table.
- Assume $D = 0$. Sketch the new concentration profile.

(e) Assume a solution of Na_2SO_4 is injected into a chalk core (made of calcite $CaCO_3$). This task will be used to represent that system mathematically.
- Assume $CaCO_3$ can dissolve and $CaSO_4$ can precipitate. Write dissolution reactions for both minerals.
- Define the saturation state for both minerals. Suggest dissolution rate expressions for the minerals, using the saturation states.
- Write transport equations for the aqueous species Ca^{2+}, Na^+, SO_4^{2-}, and the minerals $CaCO_3$ and $CaSO_4$ (but not the carbon species). Describe the mechanisms and variables you include.