University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

DATE: May 6, 2014

SUBJECT: PET 565 – Core scale modeling and interpretation

TIME: 4 hours

AID: No printed or written means allowed. Definite basic calculator allowed.

THE EXAM CONSISTS OF 4 PROBLEMS ON 5 PAGES

REMARKS: You may answer in English or Norwegian. Exercises 1 and 2 (part A) and exercises 3 and 4 (part B) are given equal weight.

Problem 1.

(a) Consider the conservation law

$$(*) \qquad u_t + f(u)_x = 0, \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x, t = 0) = \phi(x).$$

- Explain by using the *method of characteristics* why a general solution of (*) and (**) takes the form

$$(***) \qquad u(x,t) = \phi\Big(x - f'(u(x,t))t\Big)$$

(b) Based on (***) compute an expression for u_x.
- Explain under what circumstances u_x might blow up (i.e., becomes infinitely large) when we assume that f is convex ($f'' > 0$)

- Verify by direct calculation that (***) satisfies (*) and (**) subject to the condition

$$1 + \phi'(x_0)f''(\phi(x_0))t \neq 0.$$

(c) Generally, we must consider *weak solutions* of (*) and (**).
- Give the mathematical description of a weak solution in terms of an integral equality.
- What is the motivation for working with this class of solutions?

(d) Consider (*) with $f(u) = \frac{1}{4}u^2$ and

$$\phi(x) = \begin{cases} 4, & 0 \leq x < 1; \\ 0, & \text{otherwise} \end{cases}$$

- Consider the two Riemann problems, one at $x = 0$ and the other at $x = 1$. Compute solutions in terms of shock wave and/or rarefaction wave.
- In particular, compute the time T_c when the two waves will start interacting.

(e) For the problem discussed in (d), compute the solution for $t > T_c$
- either by using mathematical relations that characterize the behaviour of the two interacting waves (Rankine-Hugoniot condition, etc)
- or by using the "Equal-Area Rule"

Problem 2.

(a) Mass conservation of water and oil in a 1D reservoir is represented by the following equations:

$$\frac{\partial}{\partial x}\Big[\frac{kk_{rl}}{\mu_l}(p_x+\gamma_l)\Big]=\phi\frac{\partial S_l}{\partial t}, \qquad l=w,o$$

where $\gamma_l=\rho_l g\sin(\alpha)$ accounts for the gravity force.

- list some of the main assumptions for deriving the Buckley-Leverett (BL) model and define the different quantities
- introduce mobility functions λ_l and demonstrate how we can find the following expression for the pressure gradient

$$p_x=-\frac{u_T+\lambda_w\gamma_w+\lambda_o\gamma_o}{\lambda_T}, \qquad \lambda_T=\lambda_w+\lambda_o.$$

- what does u_T represent and can you express this in terms of some other available variables?

(b) Explain how to obtain the BL formulation

$$\phi\frac{\partial}{\partial t}S+u_T\frac{\partial F(S)}{\partial x}=0, \qquad S=S_w$$

In particular, find the expression for the fractional flow function $F(S)$ when you let $f(S)=\frac{\lambda_w(S)}{\lambda_T(S)}$.

(c) Now, we consider a horizontal reservoir with fractional flow function $f(S)$ as shown in Fig. 1. The BL model takes the following form in dimensionless variables x_D and t_D

$$\frac{\partial S}{\partial t_D}+\frac{\partial f(S)}{\partial x_D}=0.$$

Based on Fig. 1, compute the solution (saturation distribution) after a time $T=0.5$.

(d) Use the plot of $f'(S)$ and give a sketch of the unphysical solution after a time $T=0.5$ based on the method of characteristics.
- Use the principle of mass conservation and derive the general mathematical expression for the front height S^* satisfied by the physical correct solution.

(e) For the flux function $f(S)$ shown in Fig. 1, we now want to include gravity. It is assumed that $\rho_w>\rho_o$. In particular, consider the following two cases:
(i) upwards dip ($\sin(\alpha)>0$);
(ii) downward dip ($\sin(\alpha)<0$).
- Explain (by sketching an approximate solution) how the solution will change compared to the one computed in (c). Back up your explanation by referring to a sketch of the corresponding fractional flow function for (i) and (ii).

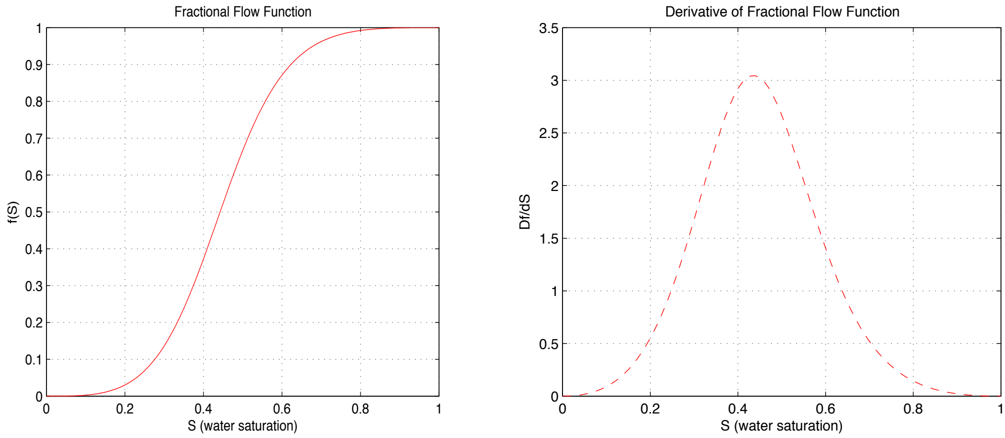

FIGURE 1. Left: $f(S)$. Right $f'(S)$

<u>Problem 3</u>.

(a) Assume a brine contains 0.05 M Na^+, 0.02 M Ca^{2+} and 0.03 M SO_4^{2-}.
- Calculate the ionic strength of the solution.
- Calculate the activity coefficients.
- What is the ion activity product of anhydrite $CaSO_4$ (s)?

Relevant equations are:

$$I = 1/2 \sum_i m_i Z_i^2, \qquad \log \gamma_i = -A z_i^2 \left(\frac{\sqrt{I}}{1+\sqrt{I}} - 0.3I\right), \quad A = 0.51$$

(b) For the same brine, assume formation of the complexes $NaSO_4^-$ $CaSO_4^0$.
- Describe the relevant dissociation reaction for these complexes and corresponding equilibrium equations.
- Write mass balance equations for the species Na^+, Ca^{2+} and SO_4^{2-}.

(c) The reactions for ion exchange of Na^+, Ca^{2+} and Mg^{2+} are given as follows

$$Na^+ + 1/2CaX_2 \rightleftharpoons NaX + 1/2Ca^{2+}$$
$$Na^+ + 1/2MgX_2 \rightleftharpoons NaX + 1/2Mg^{2+}$$

- Use the law of mass action to write equilibrium conditions for this system.
- Define, in words and mathematically, the mass balance condition for the surface activities.

Assume $\gamma_i = 1$ and the solution contains 0.1 M Na^+, 0.02M Ca^{2+} and 0.03 M Mg^{2+}. Also assume $K_{naca} = 0.4$ and $K_{namg} = 0.5$.
- Use the 3 equations to determine the surface composition.

Problem 4.

(a) Brine is injected into a system with velocity v_w, and it carries a species with concentration c which adsorbs at amount $q(c)$ at a given concentration c. This system can be described by the partial differential equation:

$$\partial_t c = -v\partial_x c - \partial_t q(c)$$

- Show that for continuous solutions the speed v_c of a given concentration c is given as

$$v_c = v_w \frac{1}{1 + dq/dc}$$

Assume $v_w = 1$ m/d and the length of the system is 1 m.
- The adsorption term is given as $q = c^{0.5}$. What type of adsorption isotherm is this? What other types do you know?
- The initial concentration is $c = 1$ and the injected concentration is $c = 4$, Plot the concentration profile $c(x)$ after 1 d.
- Assume that the initial concentration is $c = 4$ and the injected concentration is $c = 1$. Plot the concentration profile $c(x)$ after 1 d.

(b) Assume we inject a brine into a rock that can adsorb one of the components in the brine.
- Which mathematical model can be used to describe the system?
- Show mathematically how we can use measurements of the concentration at the outlet of the sample vs time to measure how many moles are adsorbed by the rock.
- Explain how an adsorption isotherm can be produced from experimental measurements. There are several ways to do this.
- How can a tracer-adsorption system be used to interpret wettability in chalk?

(c) The equation for a system with advection, dispersion and adsorption can be described as

$$\partial_t(c + q(c)) = -v_w\partial_x c + D\partial_{xx} c$$

where D is dispersion coefficient. Assuming a fixed concentration c_0 at the inlet boundary and a uniform initial concentration c_i: If there is no adsorption $q(c) = 0$ the concentration $c(x,t)$ is given by

$$c(x,t) = c_i + \frac{c_0 - c_i}{2}\left[erfc(\frac{x - vt}{\sqrt{4Dt}}) + exp(\frac{vx}{D})erfc(\frac{x + vt}{\sqrt{4Dt}})\right]$$

- Using this result, derive an analytical expression for the solution of (1) in the case where $q(c) = k \cdot c$ (a linear adsorption isotherm).
- Find the analytical solution if there is no advection or adsorption.
- Assume $c_0 = 10$, $c_i = 0$ and $D = 10^{-9}$. Calculate the position of the concentration $c = 5$ after 5 hrs (still assuming no advection).

(d) Assume a solution of $MgCl_2$ is injected into a chalk core (made of calcite $CaCO_3$). This task will be used to represent that system mathematically.

z	erfc(z)	z	erfc(z)	z	erfc(z)
0	0.0000	0.5	0.5205	1	0.8427
0.05	0.0564	0.55	0.5633	1.05	0.8624
0.1	0.1125	0.6	0.6039	1.1	0.8802
0.15	0.1680	0.65	0.6420	1.15	0.8961
0.2	0.2227	0.7	0.6778	1.2	0.9103
0.25	0.2763	0.75	0.7112	1.25	0.9229
0.3	0.3286	0.8	0.7421	1.3	0.9340
0.35	0.3794	0.85	0.7707	1.35	0.9438
0.4	0.4284	0.9	0.7969	1.4	0.9523
0.45	0.4755	0.95	0.8209	1.45	0.9597

FIGURE 2. Complementary error function table.

- Assuming $CaCO_3$ can dissolve and $MgCO_3$ can precipitate. Write dissolution reactions for both minerals.
- Also, simplify into 1 reaction by assuming dissolution of one mineral results in precipitation of the other mineral. Suggest a reaction rate expression for dissolution of calcite in this reaction. What are the units?
- Assume ions Ca^{2+} and Mg^{2+} can adsorb on the surface via ion exchange. Write the corresponding reaction and equilibrium condition.
- Assume Cl^- behaves like a tracer. What does this mean?
- Write transport equations for the aqueous species Ca^{2+}, Mg^{2+}, Cl^-, the adsorbing cations Ca^{2+}, Mg^{2+} and the minerals $CaCO_3$ and $MgCO_3$. Describe the mechanisms and variables you include and how the transport and distribution of the different species are related.