University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

DATE: May 5, 2015

SUBJECT: PET 565 – Core scale modeling and interpretation

TIME: 4 hours

AID: No printed or written means allowed. Definite basic calculator allowed.

THE EXAM CONSISTS OF 7 PROBLEMS ON 3 PAGES

REMARKS: You may answer in English or Norwegian. Exercises 1 - 3 (part A) and exercises 4 - 7 (part B) are given equal weight.

Problem 1.

(a) We focus on the model for spontaneous imbibition

$$\phi \partial_t S_w + \partial_x \Big(K \frac{\lambda_w \lambda_o}{\lambda_t} \partial_x P_c(S_w) \Big) = 0, \qquad \lambda_i = \frac{k_{ri}}{\mu_i}, \qquad i = w, o \tag{1}$$

where $\lambda_t = \lambda_w + \lambda_o$ is the total mobility. K and ϕ are, respectively, absolute permeability and porosity.

(i) Formulate the mass balance equations as well as Darcy's law for water and oil.

(ii) Assume countercurrent flow (i.e., $u_w = -u_o$) and use this to find an expression for p_{wx} expressed in terms of P_{cx}, where p_w is water pressure and $P_c = p_o - p_w$ is capillary pressure.

(iii) Finally, explain how to obtain (1).

(b) We introduce the normalized saturation $S = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}}$, and the dimensionless space variable $x_D = x/L$. Use the notation $P_c(S_w) = \tilde{P}_c(S)$.

(i) Rewrite (1) such that it is expressed in terms of x_D and S.

(ii) In the following we skip the "D" index such that $x = x_D$. Let $\overline{S} = \overline{S}(t) = \int_0^1 S(x,t)\,dx$ and assume that $S|_{x=0} = 1$ and $\partial_x S|_{x=1} = 0$. Derive an equation for $\overline{S}(t)$.

(c) Following the approach discussed in the paper by Tavassoli et al, the following equation can be obtained for $\overline{S} = \int_0^1 S(x,t)\,dx$ (after having specified Corey functions, capillary pressure function, dimensionless time, etc.):

$$\frac{d}{dt}\overline{S} = -k_{ro}^{max} J' \varepsilon^b (S_x|_{x=0}), \qquad (\varepsilon = 1 - S \quad \text{as } x \to 0^+), \tag{2}$$

where k_{ro}^{max} and J' are constants related to relative permeability and capillary pressure, respectively, and b is a Corey exponent. In order to derive an approximate solution it is assumed that $S(x,t)$ is of the form

$$S(x,t) = \begin{cases} 1 - A(t)x^f, & 0 \le x \le x^0; \\ 0, & x^0 \le x < 1. \end{cases} \tag{3}$$

where $x^0(t) < 1$ represents the foot of the front such that $S(x^0(t), t)) = 0$. Here $A(t)$ and f are unknown and must be determined.

(i) Use (3) and derive an expression for $\overline{S}(t)$. Find also an expression for $x^0(t)$ and use that in combination with the newly found $\overline{S}(t)$.

(ii) Explain **briefly**, expressed in words, how to proceed in order to find expressions for $A(t)$ and f.

(d) After a series of calculations, it is found that

$$A(t) = \frac{1}{(\beta t)^{f/2}}, \qquad \beta = \frac{2(2+b)}{1+b} k_{ro}^{max} J', \qquad f = \frac{1}{1+b}$$

(i) Use this information to find an expression for the oil recovery $\overline{S}(t)$ that reveals how it depends on time t.

(ii) What is the corresponding expression for the position of the foot of the front $x^0(t) < 1$?

Problem 2. The Buckley-Leverett model for injection of water in an oil reservoir takes the form in dimensionless variables

$$\begin{cases} S_t + f(S)_x = 0, & x \in [0,1] \\ S(x=0,t) = 1, \end{cases} \tag{4}$$

where S represents the water saturation.

(a) Give a description (without mathematical derivation) of how to construct the solution after a time $t > 0$ before the front has reached the producer at $x = 1$ for a typical choice of the fractional flow function $f(S)$.

(b) Derive (by mathematical calculations) an expression for the oil recovery $R(t)$ for the solution discussed in (a).
Compare this expression for $R(t)$ with oil recovery for spontaneous imbibition found in (1d) and represented by $\overline{S}(t)$. What does these two expressions for oil recovery tell you?

Problem 3. Conservation of mass and Darcy's law can be used to derive a conservation law model

$$S_t + g(S)_x = 0, \qquad g(S) = -S(1-S), \qquad x \in [0,1], \tag{5}$$

for flow of water (S is water saturation) and oil ($1-S$ is oil saturation) in a vertical core plug that is closed at both ends.

(a) Consider the Riemann problem where the following initial data is assumed

$$S_0(x) = \begin{cases} 1, & x \le 0.5; \\ 0, & x > 0.5. \end{cases}$$

Compute the solution at time $T = 0.5$.

(b) Consider the Riemann problem where the following initial data is assumed

$$S_0(x) = \begin{cases} 0, & x \le 0.5; \\ 1, & x > 0.5. \end{cases}$$

Compute the solution at time $T = 0.5$.

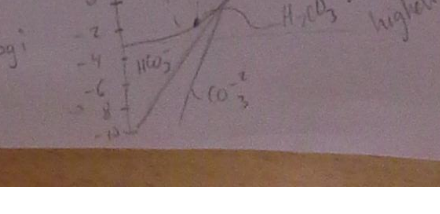
(c) Give a physical interpretation of the solutions obtained in (a) and (b) where it is assumed that $\rho_w > \rho_o$ (density of water versus oil).

OK **Problem 4.** Consider the analysis of a brine sample. The carbon concentration, pH and ionic strength were measured to 0.002mol/L, 5 and 0.5. Carbon species dissociate according to the following reactions.

$$H_2CO_3^0 \rightleftharpoons HCO_3^- + H^+, \qquad K_1 = 10^{-6.3} \tag{6}$$
$$HCO_3^- \rightleftharpoons CO_3^{2-} + H^+, \qquad K_2 = 10^{-10.3} \tag{7}$$

- Write down the equilibrium equations for the two reactions and the mass balance relation between the carbon species concentrations for CO_3^{2-}, HCO_3^-, H_2CO_3 and the total carbon concentration.
- Determine activity coefficients for all species involved in the reactions.
- Which carbon species has the highest concentration? PS: you dont need to calculate the individual concentrations.

at high PH the cons. of CO_3^{-2} and H_2CO_3 becomes large. while HCO_3 remaining const.



Problem 5. The ions Na^+, Ca^{2+} and K^+ adsorb on a surface which is in equilibrium with a brine of composition $C_{na} = 0.05, C_{ca} = 0.02, C_k = 0.03$. Exchange reactions are given as follows:

$$Na^+ + 1/2CaX_2 \rightleftharpoons NaX + 1/2Ca^{2+}, \qquad K_{na/ca} = 0.4 \tag{8}$$

$$Na^+ + KX \rightleftharpoons NaX + K^+, \qquad K_{na/k} = 0.2 \tag{9}$$

- Write down equilibrium and mass balance equations for the system.
- Calculate the composition on the rock surface in terms of equivalent fractions.
- Define and calculate the equilibrium constant for the exchange reaction

$$2K^+ + CaX_2 \rightleftharpoons 2KX + Ca^{2+} \tag{10}$$

Problem 6. A species adsorbs according to $q = 0.5c^2$ where c is concentration and q(mmol/L pore) is the adsorption isotherm (amount adsorbed species as function of brine concentration). A 0.1m sample contains 2mmol/L formation water and we flood with a brine containing 3mmol/L. The crossectional area is 0.0005 m^2 and the porosity of the sample is 0.2.

- At the time the water front has reached the core end, 0.1m, sketch the concentration profile $c(x)$.
- How many moles are retained from the injected fluid by adsorption after flooding (at a time when the injected concentration is obtained at the outlet)?

Problem 7. A rock sample contains amounts of the salt anhydrite $CaSO_4$. The core is flooded with distilled water. The salt dissolves according to the reaction

$$CaSO_4 \rightleftharpoons Ca^{2+} + SO_4^{2-} \tag{11}$$

- Suggest a rate expression for the dissolution reaction based on the saturation state of $CaSO_4$.
- Write a transport model for the ions Ca^{2+}, SO_4^{2-} and the salt $CaSO_4$. Include dispersion, advection and dissolution.
- Sketch the expected time profile of the effluent of the ions Ca^{2+}, SO_4^{2-}. List your assumptions.
- Would it make a difference for the process to flood the core with NaCl instead of distilled water? Discuss.

Appendix A. Formulas

Davies formula (at 25°C):

$$\log_{10}\gamma_i = -0.5085 Z_i^2\left(\frac{\sqrt{I}}{1+\sqrt{I}} - 0.3I\right) \tag{12}$$

Retardation formula

$$R_c = 1 + \frac{dq}{dc}, \qquad \text{(broadening front)} \tag{13}$$

$$R_f = 1 + \frac{\Delta q}{\Delta c}, \qquad \text{(sharpening front)} \tag{14}$$

pH

$$pH = -log_{10}([H^+]) \tag{15}$$