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University of Stavanger

English text.

FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: MPE 750 Reservoir simulation **DATE: November 24, 2011**

TIME: 4 hours

AID: No printed or written means allowed. Definite basic calculator allowed.

THE EXAM CONSISTS OF 6 PROBLEMS ON 3 PAGES + APPENDIX

REMARK: You may answer in English or Norwegian. All problem parts are given equal weight.

Problem 1

a) Consider the differential equation

$$u_{xx} = u_t, \quad 0 \le x \le 1.$$

with standard discretization

$$\frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta x^2} = \frac{u_i^{n+1} - u_i^n}{\Delta t}$$

Write implicit and explicit discretized equations.
Show how to determine the order of the approximations.

b) State stability properties for the equations in a).
Derive exact stability criteria.

c) Given the following boundary conditions:

- $u(0,t)=1,\ u(1,t)=0, \quad t \geq 0$.
- $u(x,0)=1, \quad 0<x<1$

Use 5 equally spaced computational points 1,2,3,4,5 in x-direction with 1 corresponding to end point 0 and 5 corresponding to endpoint 1.
Use explicit method to compute the solution after 2 steps using constant time step length 0.025.

Problem 2

a) Given $f(x)=\frac{1}{3}x^3-x^2+2x-6$.

Use Newton's method with initial estimate 4 to determine an estimate for the solution of $f(x)=0$ such that $|f| \leq 1$.

b) Consider a part of the oil flow term in the oil equation

$$F=(T_o)_{i+1/2}(p_{i+1}-p_i)$$

Use upstream evaluation of inter block terms.

Let p_s denote saturation pressure.
Consider two cases

i) $p_i > p_{i+1}, \quad p_i > p_{s,i}$

ii) $p_{s,i+1} > p_{i+1} > p_i$

Linearize F for both cases.

Problem 3

a) Why is history matching a useful tool in a reservoir study?

b) Make a list of parameters that are frequently matched in a history procedure.
Make a list of parameters frequently modified to improve a match.

Problem 4

a) What will be the structure of the linear equations for a 3D problem when implicit solution of the compositional equations is used?

b) Present the main steps in an AIM (Adaptive Implicit Method) solution procedure?
It is not necessary to depict matrix structures at each step. Give a sketch of the structure of the final set of equations only.
For b) you can stick to the Black Oil case.

c) Explain why AIM (Adaptive Implicit Method) is usually preferred for solving the compositional equations.

Problem 5

a) What are the main steps in an IMPES solution procedure?
You can stick to a two phase water/oil case for the above exposition.

b) IMPES solution will in many cases produce a more accurate solution than implicit solution (confer simulation project 1).
Explain why this is the case.

Problem 6

a) Conjugate gradients are frequently used for solving linear reservoir equations.
What is meant by restart of ORTHOMIN?
Why is restart needed?

b) Explain shortly the construction of Nested Factorization preconditioning.

APPENDIX

Let J denote the complex unit, $J=\sqrt{-1}$, a, b and ϕ real numbers.

$$|a+Jb|^2=a^2+b^2$$
$$e^{J\phi}=\cos\phi+J\sin\phi$$
$$e^{J\phi}+e^{-J\phi}=2\cos\phi$$
$$e^{J\phi}-e^{-J\phi}=2J\sin\phi$$
$$\cos^2\phi+\sin^2\phi=1$$
$$\cos(-\phi)=\cos\phi$$
$$\sin(-\phi)=-\sin\phi$$
$$1-\cos\phi=2\sin^2(\phi/2)$$

Let A_i denote values of a parameter distributed in a grid with block lengths Δx_i .

Arithmetic mean:

$$\frac{\Delta x_i A_i+\Delta x_{i+1}A_{i+1}}{\Delta x_i+\Delta x_{i+1}}.$$

Harmonic mean:

$$\frac{A_i A_{i+1}(\Delta x_i+\Delta x_{i+1})}{A_{i+1}\Delta x_i+A_i\Delta x_{i+1}}.$$

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FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: MPE 750 Reservoir simulation **DATE December 3, 2013**

TIME: 4 hours

AID: Calculator

THE EXAM CONSISTS OF 5 PROBLEMS ON 3 PAGES + APPENDIX

REMARK: You may answer in English or Norwegian. All problem parts are given equal weight.

Problem 1

Consider the equation $u_x = -u_t$.

a) Determine the order of the approximation

$$u_x \approx \frac{u_{i-2} - 4u_{i-1} + 3u_i}{2\Delta x}$$

b) Standard discretization of $u_x = -u_t$ is given by

$$\frac{u_{i-1} - u_i}{\Delta x} = \frac{u_i^{n+1} - u_i^n}{\Delta t}.$$

Show that explicit difference method is stable if $\frac{\Delta t}{\Delta x} \le 1$.

c) Suppose $x \in [0,1]$, $t \geq 0$.
Use explicit discretization of $u_x = -u_t$ with 5 computational points along x where the first point 1 is the left boundary $x = 0$ and the last point 5 is the right boundary $x = 1$. The following boundary conditions are specified:

$$u(0,t) = 0, \quad t \geq 0$$
$$u(x,0) = 1, \quad x > 0$$

Use explicit discretization of $u_x = -u_t$ with maximal stable time step length. Compute solution after 4 steps.

Problem 2

All questions in this problem refer to a 3D Black Oil model.

a) Which input data are needed for initialization?

b) Outline the main steps in pressure initialization.

c) What are the initial saturations if capillary pressures are assumed to be zero?

d) Outline the modifications needed if capillary pressures are non-zero.

Problem 3

Linearization of the flow term of oil between blocks i and i+1 in x-direction

$$(T_o)_{i+1/2}(p_{i+1} - p_i).$$

Upstream evaluation is used for all terms.
Write linearized term for computations at step k+1 for two cases:

a) $p_i > p_{i+1}$, $p_i < p_{s,i}$

b) $p_i < p_{i+1}$, $p_{i+1} > p_{s,i+1}$

where p_s denotes saturation pressure.

Problem 4

a) Write the discretized mass balance equations for a compositional model.

b) Which unknowns are determined when compositional equations are solved?

c) Give reasons why the AIM solution method is usually preferred for solving compositional equations.

Problem 5

a) What are the iterative methods used for numerical solution of mass balance equations? What is used for initial estimates?

b) Why is the parameter N_{cycle} (called *NSTACK* in ECLIPSE) introduced in ORTHOMIN? Algorithm for ORTHOMIN with preconditioning is given in Appendix.

c) Explain why ORTHOMIN with preconditioning matrix $B = A$ converges in one iteration regardless of initial estimate.
The equations to be solved are $A\vec{x} = \vec{r}$.

APPENDIX

Let J denote the complex unit, $J = \sqrt{-1}$, a, b and ϕ real numbers.

$$|a + Jb|^2 = a^2 + b^2$$
$$e^{J\phi} = \cos\phi + J\sin\phi$$
$$e^{J\phi} + e^{-J\phi} = 2\cos\phi$$
$$e^{J\phi} - e^{-J\phi} = 2J\sin\phi$$
$$\cos^2\phi + \sin^2\phi = 1$$
$$\cos(-\phi) = \cos\phi$$
$$\sin(-\phi) = -\sin\phi$$
$$1 - \cos\phi = 2\sin^2(\phi/2)$$

Let A_i denote values of a parameter distributed in a grid with block lengths Δx_i .

Arithmetic mean:

$$\frac{\Delta x_i A_i + \Delta x_{i+1} A_{i+1}}{\Delta x_i + \Delta x_{i+1}} .$$

Harmonic mean:

$$\frac{A_i A_{i+1} (\Delta x_i + \Delta x_{i+1})}{A_{i+1}\Delta x_i + A_i \Delta x_{i+1}} .$$

Algorithm for ORTHOMIN with restart and preconditioning matrix B for solving $A\vec{x} = \vec{r}$:

– *given* $\vec{x}^0$ *and set* $\vec{r}^0 = \vec{b} - A\vec{x}^0$, $\vec{q}^0 = B^{-1}\vec{r}^0$ (*)

for $k = 0$ *to convergence do*:

– *compute*

$$\omega^k = \frac{\vec{r}^k \circ A\vec{q}^k}{A\vec{q}^k \circ A\vec{q}^k}$$

– *set* $\vec{x}^{k+1} = \vec{x}^k + \omega^k \vec{q}^k$, $\vec{r}^{k+1} = \vec{r}^k - \omega^k A\vec{q}^k$

– *compute* $\vec{y}^{k+1} = B^{-1}\vec{r}^{k+1}$

– *generate numbers*

$$\alpha_i^k = -\frac{A\vec{y}^{k+1} \circ A\vec{q}^i}{A\vec{q}^i \circ A\vec{q}^i}, \quad i = 0\ldots, k$$

– *set* $\vec{q}^{k+1} = \vec{y}^{k+1} + \sum_{i=0}^{k} \alpha_i^k \vec{q}^i$

– *if* $k+1 = N_{cycle}$, *set* $\vec{x}^0 = \vec{x}^{N_{cycle}}$, $\vec{r}^0 = \vec{r}^{N_{cycle}}$ *and go to* (*)

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FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: MPE 750 Reservoir simulation **DATE: December 3, 2012**

TIME: 4 hours

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THE EXAM CONSISTS OF 6 PROBLEMS ON 3 PAGES + APPENDIX

REMARK: You may answer in English or Norwegian. All problem parts are given equal weight.

Problem 1

Consider difference formulas

i) $\frac{u_{i-1}-u_i}{\Delta x} = \frac{u_i^{n+1}-u_i^n}{\Delta t}$ of $u_x = -u_t$

ii) $\frac{u_{i-1}-2u_i+u_{i+1}}{\Delta x^2} = \frac{u_i^{n+1}-u_i^n}{\Delta t}$ of $u_{xx} = u_t$.

For this Problem no derivations or computations are required.

a) a1. Write explicit and implicit formulas.

a2. What are the orders of the approximations used?

b) State stability properties for the formulas.
Give exact stability criteria.

Problem 2

Suppose $u(x,t)$ is defined for $0 \le x \le 1,\ t \ge 0$, with boundary conditions

$$u(0,t) = 1,\ \ t \ge 0$$
$$u(x,0) = 0,\ \ 0 < x \le 1.$$

Solve $u_x = -u_t$ numerically using four equally spaced computational points where point 1 corresponds to left boundary point x = 0 and point 4 corresponds to right boundary point x = 1. Use explicit formula and time step length 1/3 . Compute solution after 3 steps.

Problem 3

For this problem consider a 1D horizontal reservoir with two flowing fluids water and oil. Assume zero capillary pressure.

a) What is the structure of linear equations if implicit solution method is used?

b) Derive the pressure equation to be used for IMPES solution.

Problem 4

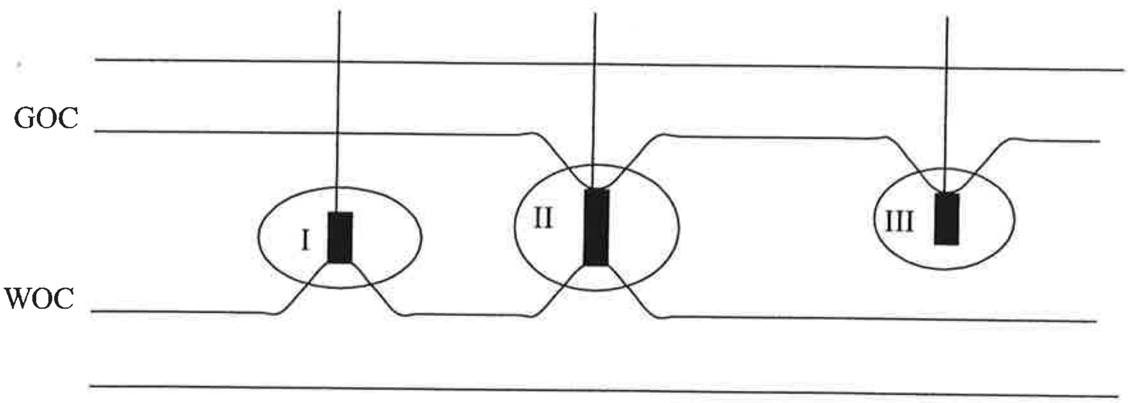
In this problem the general Black Oil equations are considered.

a) Outline the differences in computer work and stability for implicit, IMPES and AIM methods.

b) What is the degree of implicitness for AIM?

In the figure below three production wells are located between WOC and GOC.

What will be a likely degree of implicitness for regions I, II and III?



c) Consider the matrix structure for a 3 x 2 grid

x x x x x x x x x	x x x x x x x x x		x x x		
x x x x x x	x x x x x x x x x	x x x		x x x x x x	
	x x x x x x x x x	x x x x x x x x x			x x x
x x x x x x			x x x x x x x x x	x x x x x x	
	x x x x x x x x x		x x x	x x x x x x x x x	x x x
		x x x		x x x x x x	x x x x x x x x x

AIM solution is used and flow terms corresponding to explicit unknowns are set to 0 in the figure above.

Show the structure of the final set of linear equations.

Problem 5

a) What are the unknowns when compositional equations are solved?

b) Write the discretized compositional mass balance equations.
Use standard difference operators without expanding the various terms.

c) When are compositional models needed?

Problem 6

a) Write the well inflow equations for a well perforated in one block.

b) Which modifications are needed for vertical wells perforated in several blocks?

c) Which modifications are needed for horizontal wells?

APPENDIX

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