Suggested solution to the final exam in Welltest Analysis, Dec. 11, 2009

Problem 1

a) Line 1 clearly represents the period dominated by wellbore storage, while Line 2 is likely to represent the period of radial flow. Line 3 is likely to represent hemi-radial flow, i.e., the period with a doubling in semilog slope caused by a nearby boundary. With fully developed hemi-radial flow the derivatives on Line 3 should have twice the value of those on Line 2.

b) If we use the shut-in pressure (the first entry) and the first buildup point from Table 2 we get the storage constant

$$C = \frac{qB}{24}\frac{\Delta t}{\Delta p} = \frac{(2800)(1.26)(0.0022)}{(24)(3601.91-3570.68)} = 0.01036 \text{ RB/psi.}$$

The corresponding dimensionless wellbore storage then takes the value

$$C_D = \frac{5.615C}{2\pi\phi h c_t r_w^2} = \frac{(5.615)(0.01036)}{(6.2832)(0.22)(63)(3.2\times10^{-5})(0.354)^2} = 166.6$$

To determine *kh* and *k* by a similar approach we can assume that Line 2 represents radial flow, and hence that data points from the range 3 – 5 hrs can be used to determine the semilog slope. If we use the points at 3.081 and 4.3521 with equivalent flow times (to be on the safe side)

$$\frac{t\cdot\Delta t}{t+\Delta t} = \frac{(72)(3.081)}{72+3.081} = 2.955 \quad \text{and} \quad \frac{t\cdot\Delta t}{t+\Delta t} = \frac{(72)(4.3521)}{72+4.3521} = 4.104$$

then we get the slope

$$m = \frac{4848.16-4835.68}{\log 4.104-\log 2.955} = \frac{12.48}{0.14265} = 87.49 \text{ psi/log-cycle.}$$

From the slope *m* we get the flow capacity

$$kh = \frac{162.6qB\mu}{m} = \frac{(162.6)(2800)(1.26)(1.03)}{87.49} = 6753.5 \text{ md}\cdot\text{ft}$$

and the permeability

$$k = \frac{kh}{h} = \frac{6753.5}{63} = 107.2 \text{ md.}$$

c) Line 1 on the Horner plot in Fig. 2 should represent the radial-flow period. We can therefore use the points chosen above or slightly different values to determine the semilog slope

$$m = \frac{4860.47 - 4835.68}{\log\left[6.1475/(72+6.1475)\right] - \log\left[3.081/(72+3.081)\right]} = \frac{24.79}{0.2826} = 87.72 \text{ psi/log-cycle.}$$

The pressure p_{1hr} at $\Delta t = 1$ hr can be determined by extrapolation from the pressure $p_{ws} = 4835.68$ psi at $\Delta t = 3.081$ hrs as follows:

$$p_{1hr} = 4835.68 + m\left\{\left[\log 1/(72+1)\right] - \log\left[3.081/(72+3.081)\right]\right\}$$

$$= 4835.68 + 87.72(-1.8633 + 1.3868) = 4835.68 - 41.8 = 4793.88 \text{ psi.}$$

From the slope above we also get

$$kh = \frac{162.6 qB\mu}{m} = \frac{(162.6)(2800)(1.26)(1.03)}{87.72} = 6735.8 \text{ md}\cdot\text{ft}$$

and

$$k = \frac{kh}{h} = \frac{6735.8}{63} = 106.9 \text{ md.}$$

The skin value can be determined from the formula

$$S = 1.151\left(\frac{p_{1hr} - p_{wf,s}}{m} - \log\frac{t}{t+1} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.23\right)$$

$$= 1.151\left[\frac{4793.88 - 3570.68}{87.72} - \log\frac{72}{72+1} - \log\frac{106.9}{(0.22)(1.03)(3.2\times10^{-5})(0.354)^2} + 3.23\right]$$

$$= 1.151(13.944 + 0.006 - 8.0706 + 3.23) = 10.48\,.$$

The added pressure drop at the wellbore can now also be computed as

$$\Delta p_S = \frac{m}{1.151} S = \frac{87.72}{1.151}(10.48) = 798.7 \text{ psi.}$$

d) From Fig. 3 the intersection of the two straight lines is seen to occur at about the value

$$\log\left[\Delta t/(t+\Delta t)\right] = \log\left[\Delta t/(72+\Delta t)\right] = -0.395$$

which corresponds to

$$(72+\Delta t)/\Delta t = 10^{0.395} = 2.48$$

and hence the equivalent time

$$\Delta t_e = \frac{t \cdot \Delta t}{t + \Delta t} = \frac{72}{2.48} = 29.03 \text{ hrs}$$

to be used in the "distance-to-fault" analysis. The distance to the boundary can now be estimated to be

$$d = 0.01217\sqrt{\frac{k\Delta t_e}{\phi\mu c_t}} = 0.01217\sqrt{\frac{(106.9)(29.03)}{(0.22)(1.03)(3.2\times 10^{-5})}} = 251.8 \text{ ft.}$$

If no additional effects of boundaries are observed, then the radius of investigation at 240 hrs can be used to indicate how close additional boundaries can be placed without being observed, i.e., at the distance

$$r_{inv} = 0.0246\sqrt{\frac{k\Delta t}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(106.9)(240)}{(0.22)(1.03)(3.2\times 10^{-5})}} = 1463 \text{ ft.}$$

e) The best option is to extrapolate the last line to an infinite shut-in time, corresponding to a Horner time ratio of 1, with an assumed semilog slope of 2*m* from the last point. We can then estimate the formation pressure p_{res} to be

$$p_{res} = 4981.1 + 2m\left\{\log 1 - \log\left[240/(72+240)\right]\right\}$$

$$= 4981.1 + (2)(87.72)(0 + 0.1139) = 4981.1 + 19.98 = 5001.08 \text{ psi.}$$

If the buildup ended after 20 hrs, corresponding the Horner time ratio 20/(72 + 20) = 0.2174 with log value log(0.2174) = -0.66, then it would be possible to place a line with double slope consistent with gradual increase in slope observed towards the end of the data. Distance-to-fault analysis could then be used as in the analysis above. It would also be possible to use the onset of boundary effects and the radius of investigation at that time to estimate the distance t the boundary.

f) The storage constant can be determined from the slope of a line through the early data. Consistency of the data can be determined by extrapolating this line to $\Delta t = 0$ hrs and compare with the shut-in pressure $p_{wf,s} = 3570.68$ psi. If we now use the first two points from Table 2 we get the slope

$$m' = \frac{qB}{24C} = \frac{3632.16 - 3601.91}{0.0043 - 0.0022} = \frac{30.25}{0.0021} = 14405 \text{ psi/hr,}$$

and hence the storage constant

$$C = \frac{qB}{24m'} = \frac{(2800)(1.26)}{(24)(14405)} = 0.0102 \text{ RB/psi.}$$

The corresponding dimensionless wellbore storage then takes the value

$$C_D = \frac{5.615C}{2\pi\phi h c_t r_w^2} = \frac{(5.615)(0.0102)}{(6.2832)(0.22)(63)(3.2\times 10^{-5})(0.354)^2} = 164\,.$$

The extrapolated pressure at shut-in, p_0, based on the analysis above must satisfy the identity

$$m' = 14405 = \frac{3601.91 - p_0}{0.0022} \text{ psi/hr},$$

and hence will take the value

$$p_0 = 3601.91 - (14405)(0.0022) = 3570.22 \text{ psi},$$

implying that the shut-in pressure is consistent with the analysis above.

Problem 2

a) The key to *well productivity* is the term

$$\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2} + S\,,$$

while the *field productivity* depends on this term and the number of wells, with the well productivity increased if this term is reduced. For fractured wells the skin values will depend on fracture half-length through the identity

$$S = \ln\frac{a r_w}{x_f}$$

with a = 2 or e. If the number of wells is doubled with the pattern kept, then we get the new area A/2 and the new productivity term

$$\frac{1}{2}\ln\frac{4A/2}{e^{\gamma}C_A r_w^2} + S = \frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2} - \frac{1}{2}\ln 2 + S\,.$$

If the fracture half-length is instead doubled, then we get the new terms

$$\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2} + \ln\frac{a r_w}{2x_f} = \frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2} + \ln\frac{a r_w}{x_f} - \ln 2 = \frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2} + S - \ln 2\,.$$

With twice as many wells we can double the key term and still have the same field productivity, in other words, if

$$\frac{1}{2}\left(\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2}-\frac{1}{2}\ln 2+S\right)=\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2}+S-\ln 2\,,$$

then the two options have the same result, i.e., if

$$\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2}+S=\frac{3}{2}\ln 2\,.$$

The result therefore depends on the initial productivity

Problem 3

a) The pressures are high and therefore justify the use of a simple pressure formulation. Since we only have stabilized data, we can use the first and last points in the LIT analysis, i.e., the values

$$\frac{\Delta p}{q}=\frac{520-448.72}{330000}=0.000216 \quad \text{and} \quad \frac{\Delta p}{q}=\frac{520-299.48}{740000}=0.000298\,.$$

From these we get the slope

$$b=\frac{0.000298-0.000216}{740000-330000}=2E-10\,.$$

From the first we can also determine the value

$$a=\frac{520-448.72}{330000}-(2E-10)(330000)=0.00015$$

The deliverability of the well will therefore be given by the identity

$$\bar{p}-p_{wf}=aq+bq^2=0.00015q+(2E-10)q^2\,.$$

Furthermore we get

$$\text{AOF}=\frac{1}{(2)(2E-10)}\left[-0.00015+\sqrt{0.00015^2+(4)(2E-10)(520)}\right]=1280000 \ \text{Sm}^3\text{/d}$$

b) The same points give us

$$\Delta p_1=520-448.72=71.28 \ \text{bar at} \ q_1=330000 \ \text{Sm}^3\text{/d}$$

and

$\Delta p_2 = 520 - 299.48 = 220.52$ bar at $q_2 = 740000$ Sm^3/d.

From these points we get the slope

$$\frac{1}{n} = \frac{\log \Delta p_2 - \log \Delta p_1}{\log q_2 - \log q_1} = \frac{\log(220.52) - \log(71.28)}{\log(74000) - \log(330000)} = 1.4 \,,$$

and hence the exponent $n = 0.714$ for the back-pressure equation $q = C(\overline{p} - p_{wf})^n$. Using the first point we next get

$330000 = C(520 - 448.72)^{0.714} = C(21.04)$, and therefore $C = 15684$ with

$AOF = C(\overline{p})^n = (15684)(520)^{0.714} = 1364000$ Sm^3/d.

Note: We could also have used the identity $q = C(\overline{p}^2 - p_{wf}^2)^n$ in this analysis.