Suggested solution to the final exam in Well-Test Analysis, Dec. 9, 2013

Problem 1

a) Two flow regimes are directly evident in the data: 1) Early linear flow, from the beginning to about 0.06 hrs, and (2) hemi-radial flow from about 3 hrs to the end of the data. The last part must be hemi radial because a boundary effect is starting to show during the period with linear flow.

b) With hemi-radial flow after 3 hrs we can for instance use the data points $p_{wf} = 6213.4$ psia at $t = 10.169$ hrs and $p_{wf} = 6192.5$ psia at $t = 24$ hrs to determine the double semi-log slope

$$2m = \frac{|6192.5 - 6213.4|}{\log 24 - \log 10.169} = \frac{20.9}{0.3729} = 56.05 \text{ psi/log-cycle},$$

and hence the actual semi-log slope

$$m = \frac{56.05}{2} = 28.025 \text{ md.}$$

From this slope we get

$$kh = \frac{162.6 qB\mu}{m} = \frac{(162.6)(3300)(1.23)(1.15)}{28.025} = 27082.7 \text{ md·ft}$$

and

$$k = \frac{kh}{h} = \frac{27082.7}{59} = 459.03 \text{ md.}$$

c) With linear flow ending around 0.06 hrs we can for instance use the points $p_{wf} = 6298.5$ psia at $t = 0.003$ hrs and $p_{wf} = 6294.7$ psia at $t = 0.034$ hrs, to determine the slope

$$m' = \frac{|6294.7 - 6298.5|}{\sqrt{0.034} - \sqrt{0.003}} = \frac{3.8}{0.12962} = 29.32 \text{ psi} / \sqrt{\text{hr}}$$

of the linear-flow data. Since we must also have

$$m' = \frac{|6298.5 - p_i|}{\sqrt{0.003} - \sqrt{0}} = \frac{p_i - 6298.5}{0.054772} = 29.32 \text{ psi} / \sqrt{\text{hr}},$$

it follows that we must have

$p_i = 6298.5 + (29.32)(0.054772) = 6300.1$ psia.

Moreover, from the slope and permeability obtained above we get the fracture half-length

$$x_f = \frac{4.064qB}{hm'}\sqrt{\frac{\mu}{k\phi c_t}} = \frac{(4.064)(3300)(1.23)}{(59)(29.32)}\sqrt{\frac{1.15}{(459.03)(0.09)(2.6\times10^{-5})}} = 312 \text{ ft.}$$

d) Deviation from simple linear flow is indicated at a time of 0.06 hrs. Based on the radius of investigation at this time,

$$r_{inv} = 0.0246\sqrt{\frac{kt}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(459.03)(0.06)}{(0.09)(1.15)(2.6\times10^{-5})}} = 78.7 \text{ ft,}$$

we can estimate the distance to the nearest boundary to be 79 ft. The radius of investigation at the end of the data will be

$$r_{inv} = 0.0246\sqrt{\frac{(459.03)(24)}{(0.09)(1.15)(2.6\times10^{-5})}} = 1574 \text{ ft.}$$

e) We can determine t_{Dxf} at the end of simple linear-flow data at $t = 0.06$ hrs and compare that we the bounds for linear-flow behavior. This corresponds to

$$t_{Dxf} = \frac{0.000264kt}{\varphi\mu c_t x_f^2} = \frac{(0.000264)(459.03)(0.06)}{(0.09)(1.15)(2.6\times10^{-5})(296.5)^2} = 0.03073,$$

which is beyond the end of linear-flow data for a fracture with infinite conductivity. The fracture must therefore have uniform flux.

<u>Problem 2</u>

a) The required equation can be derived from the basic skin equation

$$S = 1.151\left(\frac{p_i - p_{1hr}}{m} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.098\right),$$

which can be rewritten in the form

$$p_{1hr} = p_i - m\left(\log\frac{k}{\phi\mu c_t r_w^2} - 3.098 + \frac{S}{1.151}\right),$$

and also in the general form

$$p_{wf}(t) = p_i - m\left(\log t + \log\frac{k}{\phi\mu c_t r_w^2} - 3.098 + \frac{S}{1.151}\right).$$

From the listed properties we get the semi-log slope

$$m = \frac{21.49 qB\mu}{kh} = \frac{(21.49)(500)(1.27)(0.74)}{(9.5)(36)} = 29.53 \text{ bar/log-cycle.}$$

We therefore get the pressure

$$p_{wf}(300) = 495 - 29.53\left(\log 300 + \log\frac{9.5}{(0.21)(0.74)(1.74\times10^{-4})(0.108)^2} - 3.098 + \frac{6}{1.151}\right)$$

$$= 495 - 29.53(2.477 + 7.479 - 3.098 + 5.213) = 138.54 \text{ bar.}$$

b) The skin value for a fracture with infinite conductivity will be given by

$$S = \ln\frac{2r_w}{x_f} = \ln\frac{(2)(0.108)}{130} = -6.4.$$

The computation above can be repeated with the new skin value, or final value modified with the difference in skin effect, which will be

$$29.53\left(\frac{6+6.4}{1.151}\right) = 29.53\frac{12.4}{1.151} = 318.13 \text{ bar.}$$

The bottom-hole pressure after 300 hours with the new skin value will therefore be

$$p_{wf}(300) = 138.54 + 318.13 = 456.67 \text{ bar.}$$

Problem 3

a) Since the pressures are high we should use a direct pressure formulation. Therefore, if we use the first and the last points we get

$$\frac{\Delta p}{q} = \frac{6940 - 6662}{17325} = 0.016046$$

and

$$\frac{\Delta p}{q} = \frac{6940 - 6141}{31480} = 0.025381.$$

From these we get the slope

$$b = \frac{0.025381 - 0.016046}{31480 - 17325} = 6.5948\text{E-7}\,.$$

From the first point we can next determine

$$a = 0.016046 - (6.5948\text{E-7})(17325) = 0.0046205\,.$$

The deliverability of the well will therefore be given by the identity

$$\bar{p} - p_{wf} = aq + bq^2 = 0.0046205q + (6.5948\text{E-7})q^2\,.$$

We furthermore get the open flow potential

$$\text{AOF} = \frac{1}{(2)(6.5948\text{E-7})}\left[-0.0046205 + \sqrt{0.0046205^2 + (4)(6.5948\text{E-7})(6940)}\right]$$

$$= 99140 \text{ Mscf/d.}$$

b) The points above can be re-used here, with (can also use p^2 formulation)

$$\Delta p = 6940 - 6662 = 278 \text{ psi}$$

and

$$\Delta p = 6940 - 6141 = 799 \text{ psi}$$

to determine the slope

$$\frac{1}{n} = \frac{\log(799) - \log(278)}{\log(31480) - \log(17325)} = 1.7678\,,$$

and hence the exponent $n = 0.5657$ for the back-pressure equation $q = C(\bar{p} - p_{wf})^n$. Using the first point we next get

$17325 = C(6940 - 6662)^{0.5657} = C(24.1322)$, and therefore $C = 717.92$ with

$$AOF = C(\bar{p})^n = (717.92)(6940)^{0.5657} = 106938 \text{ Mscf/d.}$$