Suggested solution to the final exam in Well-Test Analysis, Dec. 9, 2014

Problem 1

a) Three flow regimes are evident in the data: 1) Wellbore storage dominated early data, from the beginning to about 0.003 hrs, (2) radial-flow data in the middle, from about 0.5-4 hrs, and (3) hemi-radial data caused by a single no-flow boundary from about 40-300 hrs. Data from the first can be used to determine the wellbore storage constant (*C*). Data from the second can be used to determine the flow capacity (*kh*) and hence the permeability (*k*).

The storage constant can be determined through the steps

$$C = \frac{qB}{24}\frac{\Delta t}{\Delta p} = \frac{(11200)(1.53)}{24}\frac{0.0012}{4363.49 - 4273.03} = \frac{(11200)(1.53)(0.0012)}{(24)(90.46)} = 0.00947 \ \text{RB/psi}$$

by using the first entry in Table 2 and the pressure at shut-in.

Since we have radial-flow buildup data between 0.5 and 4 hrs, we can use the data points $p_{ws} = 6033.96$ psia at $\Delta t = 1.2118$ hrs and $p_{ws} = 6147.56$ psia at $\Delta t = 3.832$ hrs to determine the semi-log slope

$$m = \frac{6147.56 - 6033.96}{\log(3.832/243.832) - \log(1.2118/241.2118)} = \frac{113.6}{0.4953} = 229.355 \ \text{psi/log-cycle,}$$

or alternatively just

$$m = \frac{6147.56 - 6033.96}{\log(3.832) - \log(1.2118)} = \frac{113.6}{0.5} = 226.2 \ \text{psi/log-cycle}$$

due to the long producing time. From the slope we next get

$$kh = \frac{162.6 qB\mu}{m} = \frac{(162.6)(11200)(1.53)(0.78)}{229.355} = 9475.8 \ \text{md·ft}$$

and therefore

$$k = \frac{kh}{h} = \frac{9475.8}{102} = 92.9 \ \text{md.}$$

b) We can use the points and steps above to determine *m*, *kh* and *k*. Moreover, with the point $p_{ws} = 6033.96$ psia at $\Delta t = 1.2118$ hrs an on the semi-log straight line by construction, we can extrapolate to p_{1hr} at $\Delta t = 1$ hr as follows:

$$p_{1hr} = 6033.96 + m\left\{\log\left[1/(240+1)\right] - \log\left[1.2118/(240+1.2118)\right]\right\}$$

$$= 6033.96 + 229.355(-2.38202 + 2.298968) = 6033.96 - 19.05 = 6014.91 \ \text{psi.}$$

The skin value can next be determined from the formula

$$S = 1.151\left(\frac{p_{1hr} - p_{wf,s}}{m} - \log\frac{t}{t+1} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.23\right)$$

$$= 1.151\left[\frac{6014.91 - 4273.03}{229.355} - \log\frac{240}{240+1} - \log\frac{92.9}{(0.13)(0.78)(2.9\times10^{-5})(0.354)^2} + 3.23\right]$$

$$= 1.151(7.59469 + 0.00181 - 8.401573 + 3.23) = 2.79\,.$$

The added pressure drop at the wellbore can now also be computed as

$$\Delta p_S = \frac{m}{1.151}S = \frac{229.355}{1.151}(2.79) = 555.95\ \text{psi.}$$

c) The easiest is to use the radius of investigation at the end of simple radial flow, roughly at $\Delta t = 4$ hrs in view of the plot in Fig. 1. The distance to the nearest boundary should then be obtained from the result

$$r_{inv} = 0.0246\sqrt{\frac{k\Delta t}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(92.9)(4)}{(0.13)(0.78)(2.9\times10^{-5})}} = 276.5\ \text{ft}$$

if we just use time since shut-in. With effective flow time

$$\Delta t_e = \frac{t\Delta t}{t+\Delta t} = \frac{(240)(4)}{240+4} = 3.934\ \text{hrs}$$

we get only a 2% reduction in distance, which is small in comparison with the uncertainty in the onset of boundary effects based on Fig. 1.

If we instead use the Horner plot and estimate that the two straight lines intersect at

$$\log\frac{\Delta t}{t+\Delta t} = -1.2\,,$$

then we get

$$\frac{\Delta t}{t+\Delta t} = 10^{-1.2} = 0.0631,$$

and hence the equivalent time

$$\Delta t_e = \frac{t\Delta t}{t+\Delta t} = (240)(0.0631) = 15.14\ \text{hrs.}$$

From this value we can estimate the distance from the identity

$$d = 0.01217\sqrt{\frac{kt}{\phi\mu c_t}} = 0.01217\sqrt{\frac{(92.9)(15.14)}{(0.13)(0.78)(2.9\times10^{-5})}} = 266.2 \text{ ft.}$$

d) In terms of a simple analysis the best one can do is to use the radius of investigation when the derivatives have started to decline significantly, for instance at 400 hrs. There are two possible choices of time to use in the expression for the radius of investigation: (1) the equivalent time (240)(400)/(240 + 400) = 150 hrs and (2) the time since shut-in 400 hrs. Form the first we get

$$r_{inv} = 0.0246\sqrt{\frac{k\Delta t}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(92.9)(150)}{(0.13)(0.78)(2.9\times10^{-5})}} = 1693 \text{ ft,}$$

and from the second

$$r_{inv} = 0.0246\sqrt{\frac{k\Delta t}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(92.9)(400)}{(0.13)(0.78)(2.9\times10^{-5})}} = 2765 \text{ ft.}$$

The last one is most realistic since we are looking boundary effects from all sides, but this type of analysis is normally only used as a rough starting point for a direct analysis based on matching a model to the data.

e) Based on the severe drop in derivatives at the end of buildup in Fig. 1, we can assume that we can assume that the pressure has become essentially constant at the end of the, with the last pressure value from Table 2 being a good estimate of the static pressure or reservoir pressure after shut-in at 6756.73 psia, or perhaps 6757 psia. Direct extrapolation can be used based on the last data points, but the result will be similar. Extrapolating the steepest line towards the end of the data is not a good idea here.

f) An estimate of the initial pressure before the producing period can be back-calculated from the pressure estimated above and material-balance considerations based on the distances to boundaries estimated above. With the nearest boundary at d = 276.5 ft and distant boundaries at r_e = 2765 ft, the drainage area can be estimated to be

$$A = 2dr_e + \frac{1}{2}\pi r_e^2 = (2)(276.5)(2765) + \pi(0.5)2765^2 = 1.35\text{E}7 .$$

The dimensionless producing time based on this area

$$t_{DA} = \frac{0.000264kt}{\varphi\mu c_t A} = \frac{(0.000264)(92.9)(240)}{(0.13)(0.78)(2.9\times10^{-5})(1.35\,\text{E}\,7)} = 0.1483 .$$

We therefore get

$$p_i - \bar{p} = \frac{141.2qB\mu}{kh} 2\pi t_{DA} = \frac{m}{1.151} 2\pi t_{DA},$$

and hence

$$p_i = \bar{p} + \frac{m}{1.151} 2\pi t_{DA} = 6756.73 + \frac{229.355}{1.151} 2\pi 0.1483 = 6942.41 \text{ psia.}$$

g) We clearly have storage dominated data in the beginning of the buildup, and hence can use the first two points from Table 2 to determine the slope

$$m' = \frac{4446.75 - 4363.49}{0.0024 - 0.0012} = \frac{83.26}{0.0012} = 69383.3 \text{ psi/hr,}$$

which in turn yields the storage constant

$$C = \frac{qB}{24m'} = \frac{(11200)(1.53)}{(24)(69383.3)} = 0.01029 \text{ RB/psi,}$$

corresponding to the dimensionless value

$$C_D = \frac{5.615C}{2\pi\varphi h c_t r_w^2} = \frac{(5.615)(0.01029)}{(2\pi)(0.13)(102)(2.9\text{E-}5)(0.354)^2} = 190.8$$

<u>Problem 2</u>

a) The semi-log slope for Well 1

$$m = \frac{21.49qB\mu}{kh} = \frac{(21.49)(150)(1.47)(1.14)}{(7.3)(12)} = 61.67 \text{ bar/log-cycle.}$$

b) The critical distance corresponds to the midpoint between the wells, i.e., at a distance of 230/2 = 115 m. The onset of interference between the two wells can therefore be estimated from the identity for the radius of investigation by setting

$$r_{inv} = 0.0286\sqrt{\frac{kt}{\phi\mu c_t}} = 115 \text{ m,}$$

and hence

$$0.0286^2 \frac{(7.3)t}{(0.08)(1.14)(1.98\times10^{-4})} = 115^2$$

with the result

$$t = \frac{(115^2)(0.08)(1.14)(1.98\times10^{-4})}{(0.0286^2)(7.3)} = 40 \text{ hrs.}$$

c) The late-time semi-log slope combining the response from both wells is similar to the result from the first point, only with total rate 150 + 65 = 215 Sm^3/d. We therefore get the late-time semi-log slope

$$m_{late-time} = \frac{21.49qB\mu}{kh} = \frac{(21.49)(215)(1.47)(1.14)}{(7.3)(12)} = 88.39 \text{ bar/log-cycle.}$$

Problem 3

a) Since the pressures are low we should use a pressure-squared formulation. Therefore, if we use the first and the last of the transient points we get

$$\frac{\Delta p^2}{q} = \frac{1998.5^2 - 1943.9^2}{4800} = 44.845$$

and

$$\frac{\Delta p^2}{q} = \frac{1998.5^2 - 1839.3^2}{8600} = 71.044 .$$

From these we get the slope

$$b = \frac{71.044 - 44.845}{8600 - 4800} = 0.006895 .$$

From the last point (the stable point) we get

$$\frac{\Delta p^2}{q} = \frac{1998.5^2 - 1765.8^2}{9540} = 91.819 ,$$

and hence

$$a = 91.819 - (0.006895)(9540) = 26.045 .$$

The deliverability of the well will therefore be given by the identity

$$\bar{p}^2 - p_{wf}^2 = aq + bq^2 = 26.045q + 0.006895q^2 .$$

We furthermore get the open flow potential

$$\text{AOF} = \frac{1}{(2)(0.006895)}\left[-26.045 + \sqrt{26.045^2 + (4)(0.006895)(1998.5^2)}\right]$$

$$= 22253 \text{ Mscf/d.}$$

b) The transient points used above can be re-used here, with

$$\Delta p = 1998.5^2 - 1943.9^2 = 215255 \text{ psi}^2$$

and

$$\Delta p = 1998.5^2 - 1839.3^2 = 610978 \text{ psi}^2$$

to determine the slope

$$\frac{1}{n} = \frac{\log(610978) - \log(215255)}{\log(8600) - \log(4800)} = 1.789 ,$$

and hence the exponent $n = 0.559$ for the back-pressure equation $q = C(\bar{p}^2 - p_{wf}^2)^n$. Using the last point (the stable point) we next get

$9540 = C(1998.5^2 - 1765.8^2)^{0.559} = C(2098.2)$, and therefore $C = 4.547$ with

$$AOF = C(\bar{p}^2)^n = (4.547)(1998.5^2)^{0.559} = 22279 \text{ Mscf/d.}$$