Suggested solution to the final exam in Well-Test Analysis, Dec. 9, 2015

Problem 1

a) Three flow regimes are evident in the data: (1) Early linear flow to the fracture from the beginning to about 0.02 hrs, (2) radial-flow data approached in the range 0.3-0.8 hrs, (3) hemi-radial data caused by a single no-flow boundary in the range 10-30 hrs, and additional boundary effects at the end, apparently from linear flow, e.g., between parallel boundaries, after 60 hours.

b) Since we have radial-flow data between 0.3 and 0.8 hrs, we can use $p_{ws} = 3103.557$ psia at $\Delta t = 0.3635$ hrs and $p_{ws} = 3145.613$ psia at $\Delta t = 0.5762$ hrs to determine the semi-log slope

$$m = \frac{3145.613 - 3103.557}{\log(0.5762) - \log(0.3635)} = \frac{42.056}{0.20007} = 210.21 \text{ psi/log-cycle.}$$

Here we have used the buildup time directly since 0.576 is much smaller than 720. From the slope we next get

$$kh = \frac{162.6 qB\mu}{m} = \frac{(162.6)(12500)(1.21)(1.8)}{210.21} = 21058.9 \text{ md}\cdot\text{ft}$$

and therefore

$$k = \frac{kh}{h} = \frac{21058.9}{110} = 191.4 \text{ md.}$$

The pressure at 1 hour can be determined by extrapolation from either of the two points used to determine the slope *m*, for instance in the form

$$p_{1hr} = 3103.557 + m[\log 1 - \log(0.3635)] = 3103.557 + 210.21[-\log(0.3635)]$$

$$= 3103.557 + 210.21(0.4395) = 3103.557 + 92.386 = 3195.943 \text{ psia.}$$

The skin value can next be determined from the formula

$$S = 1.151\left(\frac{p_{1hr} - p_{wf,s}}{m} - \log\frac{t}{t+1} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.23\right)$$

$$= 1.151\left[\frac{3195.943 - 2811.889}{210.21} - \log\frac{720}{721} - \log\frac{191.4}{(0.08)(1.8)(1.3\times10^{-5})(0.5)^2} + 3.23\right]$$

$$= 1.151(1.827 + 0.0006 - 8.6117 + 3.23) = -4.09\,.$$

The “added” pressure drop at the wellbore can now also be computed as

$$\Delta p_S = \frac{m}{1.151} S = \frac{210.21}{1.151}(-4.09) = -746.97 \text{ psi.}$$

If we repeat the semi-analysis with a full superposition time axis, then we get the slope

$$m = \frac{3145.613 - 3103.557}{\log(0.5762/(720+0.5762)) - \log(0.3635/(720+0.3635))}$$

$$= \frac{42.056}{0.19994} = 210.34 \text{ psi/log-cycle.}$$

From the slope we next get

$$kh = \frac{162.6 qB\mu}{m} = \frac{(162.6)(12500)(1.21)(1.8)}{210.34} = 21045.85 \text{ md·ft}$$

and therefore the permeability

$$k = \frac{kh}{h} = \frac{21045.85}{110} = 191.33 \text{ md.}$$

The pressure at 1 hour will now take the value

$$p_{1hr} = 3103.557 + m\left[\log(1/721) - \log(0.3635/720.3635)\right]$$

$$= 3103.557 + 210.34(0.4391) = 3103.557 + 92.36 = 3195.917 \text{ psia.}$$

We can now compute the skin value from the formula

$$S = 1.151\left[\frac{3195.917 - 2811.889}{210.34} - \log\frac{720}{721} - \log\frac{191.33}{(0.08)(1.8)(1.3\times10^{-5})(0.5)^2} + 3.23\right]$$

$$= 1.151(1.826 + 0.0006 - 8.6115 + 3.23) = -4.09\,.$$

The "added" pressure drop at the wellbore can now also be computed as

$$\Delta p_S = \frac{m}{1.151} S = \frac{210.34}{1.151}(-4.09) = -747.43 \text{ psi.}$$

c) There appear to be two boundaries affecting the data. The easiest way to estimate the distance to the first boundary is to use the radius of investigation at the end of simple radial flow, roughly at $\Delta t = 1$ hrs in view of the plot in Fig. 1. To this end we can use the result

$$r_{inv} = 0.0246\sqrt{\frac{k\Delta t}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(191.33)(1)}{(0.08)(1.8)(1.3\times10^{-5})}} = 248.7 \text{ ft}$$

as estimated distance to this boundary.

With the same approach we can assume the onset of effects from the next boundary to happen at 35 hours, and use this to estimate the second distance by the value

$$r_{inv} = 0.0246\sqrt{\frac{k\Delta t}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(191.33)(35)}{(0.08)(1.8)(1.3\times10^{-5})}} = 1471 \text{ ft.}$$

d) Two periods with constant or near constant derivatives are indicated in Fig. 1. We should therefore expect to observe two straight-line segments in Fig. 2. The first corresponding to simple radial flow between $\Delta t = 0.3$ and $\Delta t = 0.8$ hrs, and the second to hemi-radial flow between $\Delta t = 10$ and $\Delta t = 30$ hrs, according to Point a). With respect to the superposition time axis, the start and end of the first straight-line segment will be

$$\log(\Delta t/(t+\Delta t)) = \log(0.3/720.3) = -3.38$$

and

$$\log(\Delta t/(t+\Delta t)) = \log(0.8/720.8) = -2.95$$

and for the second line

$$\log(\Delta t/(t+\Delta t)) = \log(10/730) = -1.86$$

and

$$\log(\Delta t/(t+\Delta t)) = \log(30/750) = -1.4$$

In order to estimate the reservoir pressure from the buildup data based on the semi-log plot there is only one option, namely to extrapolate s straight line from the end of the data to infinite shut-in time, corresponding to $\log(\Delta t/(t+\Delta t)) = 0$. We therefore first of all need to determine the slope at the end of the data in Fig. 2. We can use the two last points from Table 2 to this end. From these two points we get

$$m = \frac{4005.403 - 4001.311}{\log(192/(720+192)) - \log(188.5787/(720+188.5787))}$$

$$= \frac{4.092}{0.006176} = 662.56 \text{ psi/log-cycle.}$$

We can now extrapolate to the reservoir pressure by setting

$$p_{res} = 4005.403 + 662.56\left[0 - \log(192/(720+192))\right]$$

$$= 4005.403 + 662.56(0.6767) = 4005.403 + 448.354 = 4453.76 \text{ psia.}$$

e) With linear flow from the beginning we can for instance use $p_{ws} = 2834.946$ psia at $\Delta t = 0.0014$ hrs and $p_{ws} = 2844.515$ psia at $\Delta t = 0.0029$ hrs to determine linear-flow slope

$$m' = \frac{2844.515 - 2834.946}{\left|\left(\sqrt{720+0.0029} - \sqrt{0.0029}\right) - \left(\sqrt{720+0.0014} - \sqrt{0.0014}\right)\right|}$$

$$= \frac{9.569}{0.0164} = 583.22 \text{ psi}/\sqrt{\text{hr}} \text{ .}$$

Note that the slope will be 583.95 with t = 240 hours (mistakenly used to set up Fig. 3). From the slope we get the fracture half-length

$$x_f = \frac{4.064qB}{hm'}\sqrt{\frac{\mu}{k\varphi c_t}} = \frac{(4.064)(12500)(1.21)}{(110)(583.22)}\sqrt{\frac{1.8}{(191.4)(0.08)(1.3\times10^{-5})}} = 91.5 \text{ ft.}$$

For a fracture with infinite conductivity we should get

$$S = \ln\frac{2r_w}{x_f} = \ln\frac{2(0.5)}{91.1} = -4.51$$

For a fracture with uniform flux we should get

$$S = \ln\frac{er_w}{x_f} = \ln\frac{(2.7183)(0.5)}{91.1} = -4.21$$

The result $S = -4.09$ from b) is closest to that of a uniform-flux fracture.

With distances 248.7 and 1471 ft to the boundaries, the width of the model will be 1719.7 ft. The half-width will be 1719.7 /2 = 859.8 ft. The half-width should be used to determine the slope

$$m' = \frac{(4.064)(12500)(1.21)}{(110)(859.8)}\sqrt{\frac{1.8}{(191.4)(0.08)(1.3\times10^{-5})}} = 61.803 \text{ psi}/\sqrt{\text{hr}}$$

With

$$\sqrt{t+\Delta t} - \sqrt{\Delta t} = \sqrt{720+192} - \sqrt{192} = 16.34\sqrt{\text{hr}}$$

at the last buildup point, and $\sqrt{t+\Delta t}-\sqrt{\Delta t}=0$ at infinite shut-in, the reservoir pressure can be estimated by the extrapolated value

$$p_{res}=4005.403+(61.803)(16.34)=5015.264 \text{ psia.}$$

This is likely to be a better estimate of the reservoir pressure with simple linear flow compared to the semi-log extrapolation if the boundaries are parallel.

Problem 2

a) We must have

$$p_{ws}(\Delta t)=p_i-\left\{\left[p_i-p_{wf}(\mathrm{t}+\Delta t)\right]-\left[p_i-p_{wf}(\Delta t)\right]\right\}=p_i-\left[p_{wf}(\Delta t)-p_{wf}(\mathrm{t}+\Delta t)\right]$$

$$=p_i-\left[p_i-\left(\frac{a}{\sqrt{\Delta t}}+b\right)-p_i+\left(\frac{a}{\sqrt{t+\Delta t}}+b\right)\right]=p_i-\left(\frac{a}{\sqrt{t+\Delta t}}-\frac{a}{\sqrt{\Delta t}}\right)$$

$$=p_i+a\left(\frac{1}{\sqrt{\Delta t}}-\frac{1}{\sqrt{t+\Delta t}}\right).$$

In particular, note that we must have $a<0$ since the time expression will be positive.

Problem 3

a) Since the pressures are high we should use a direct pressure formulation. Therefore, if we use the first and the last of the transient points we get

$$\frac{\Delta p}{q}=\frac{8765-8370}{10300}=0.03835$$

and

$$\frac{\Delta p}{q}=\frac{8765-7650}{21000}=0.0531.$$

From these we get the slope

$$b=\frac{0.0531-0.03835}{21000-10300}=1.3785\mathrm{E}-6.$$

From the last point (the stable point) we get

$$\frac{\Delta p}{q}=\frac{8765-7100}{19500}=0.08539,$$

and hence

$$a = \frac{\Delta p}{q} - bq = 0.08539 - (1.3785\text{E}-6)(19500) = 0.05851.$$

The deliverability of the well will therefore be given by the identity

$$\bar{p} - p_{wf} = aq + bq^2 = 0.05851q + (1.3785\text{E}-6)q^2.$$

We furthermore get the open flow potential

$$\text{AOF} = \frac{1}{(2)(1.3785\text{E}-6)}\left[-0.05851 + \sqrt{0.05851^2 + (4)(1.3765\text{E}-6)(8765)}\right]$$

$$= 61326 \text{ Mscf/d.}$$

b) The transient points used above can be re-used here, with

$$\Delta p = 8765 - 8370 = 395 \text{ psi}$$

and

$$\Delta p = 8765 - 7650 = 1115 \text{ psi}$$

to determine the slope

$$\frac{1}{n} = \frac{\log(1115) - \log(395)}{\log(21000) - \log(10300)} = 1.4567,$$

and hence the exponent $n = 0.6865$ for the back-pressure equation $q = C(\bar{p} - p_{wf})^n$. Using the last point (the stable point) we next get

$19500 = C(8765 - 7100)^{0.6865} = C(162.74)$, and therefore $C = 119.82$ with

$$AOF = C(\bar{p})^n = (119.82)(8765)^{0.6865} = 60986 \text{ Mscf/d.}$$

Note: For this point we could also use the expression $q = C(\bar{p}^2 - p_{wf}^2)^n$ to carry out the analysis and determine the deliverability of the well.