Suggested solution to the re-sit exam in Well-Test Analysis, Feb. 26, 2016

Problem 1

a) Four flow regimes are evident in the data: (1) Linear flow towards the fracture in early data, from the beginning to about 0.1 hrs, (2) radial flow in the range 1-10 hrs, (3) linear flow between parallel boundaries in the range 30-150 hrs, and (4) pseudosteady-state flow (simple depletion) from about 200 hrs to the end.

b) Since we have radial-flow data between 1 and 10 hrs, we can use $p_{wf} = 4764.798$ psia at $t = 1.2743$ hrs and $p_{wf} = 4713.967$ psia at $t = 8.0403$ hrs to determine the semi-log slope (absolute value)

$$m = \frac{|4713.967 - 4764.798|}{\log(8.0403) - \log(1.2743)} = \frac{50.83}{0.8} = 63.539 \text{ psi/log-cycle.}$$

From the slope we next get

$$kh = \frac{162.6 qB\mu}{m} = \frac{(162.6)(3190)(1.29)(1.8)}{63.539} = 18955.4 \text{ md·ft}$$

and therefore

$$k = \frac{kh}{h} = \frac{18955.4}{47} = 403.3 \text{ md.}$$

The pressure at 1 hour can be determined by extrapolation from either of the two points used to determine the slope *m*, for instance in the form

$$p_{1hr} = 4713.967 - m[\log 1 - \log(8.0403)] = 4713.967 + 63.539[\log(8.0403)]$$

$$= 4713.967 + 63.539(0.90527) = 4713.967 + 57.52 = 4771.487 \text{ psi,}$$

where the fact that the actual slope is negative has been used.

The skin value can next be determined from the formula

$$S = 1.151\left(\frac{p_i - p_{1hr}}{m} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.23\right)$$

$$= 1.151\left[\frac{4850 - 4771.487}{63.539} - \log\frac{403.3}{(0.21)(1.8)(1.08\times10^{-5})(0.354)^2} + 3.23\right]$$

$$= 1.151(1.23567 - 8.89671 + 3.23) = -5.10\,.$$

The “added pressure drop” at the wellbore can now also be computed as

$$\Delta p_S = \frac{m}{1.151} S = \frac{63.539}{1.151}(-5.1) = -281.537 \text{ psi.}$$

c) It appears that two boundaries are starting to show after about 15 hrs. The radius of investigation at time is given by

$$r_{inv} = 0.0246\sqrt{\frac{kt}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(403.3)(15)}{(0.21)(1.8)(1.08\times10^{-5})}} = 947 \text{ ft}$$

This can be used as estimated distance to the first two boundaries (both).

With the same approach we can assume the onset of effects from the last boundaries to happen at 200 hours (after the end of the second period with linear flow), and use this to estimate the distance to the last two boundaries (both) by the value

$$r_{inv} = 0.0246\sqrt{\frac{kt}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(403.3)(200)}{(0.21)(1.8)(1.08\times10^{-5})}} = 3458 \text{ ft.}$$

d) Since we have linear-flow data until about 0.1 hr, we can for instance use the points $p_{wf} = 4845.757$ psia at $t = 0.002$ hrs and $p_{wf} = 4836.581$ psia at $t = 0.0202$ hrs to determine the slope (absolute value)

$$m' = \frac{|4836.581 - 4845.757|}{\sqrt{0.0202} - \sqrt{0.002}} = \frac{9.176}{0.0974} = 94.21 \text{ psi}/\sqrt{\text{hr}} \text{ .}$$

From this slope we can determine the fracture half-length from the identity

$$x_f = \frac{4.064qB}{hm'}\sqrt{\frac{\mu}{k\varphi c_t}} = \frac{(4.064)(3190)(1.29)}{(47)(94.21)}\sqrt{\frac{1.8}{(403.3)(0.21)(1.08\times10^{-5})}} = 130 \text{ ft.}$$

If the fracture has uniform flux, then the corresponding skin should be

$$S = \ln\frac{er_w}{x_f} = \ln\frac{e(0.354)}{130} = -4.906 \text{ ,}$$

while it should be

$$S = \ln\frac{2r_w}{x_f} = \ln\frac{(2)(0.354)}{130} = -5.213$$

for an infinite-conductivity fracture. The last value is closest to the value $S = -5.1$ from (b), so this suggests that the fracture has infinite conductivity.

e) This corresponds to the second period with linear flow in the range 30 – 150 hrs, where the same linear-flow analysis as used above will give us the half-width of the flow unit. For this analysis we can use the points $p_{wf} = 4641.778$ psia at $t = 60.809$ hrs and $p_{wf} = 4602.667$ psia at $t = 114.809$ hrs to determine the slope (absolute value)

$$m' = \frac{|4602.667 - 4641.778|}{\sqrt{114.809} - \sqrt{60.809}} = \frac{39.111}{2.9169} = 13.408 \text{ psi}/\sqrt{\text{hr}} .$$

From this slope we can determine the fracture half-width from the identity

$$\frac{W}{2} = \frac{4.064qB}{hm'}\sqrt{\frac{\mu}{k\varphi c_t}} = \frac{(4.064)(3190)(1.29)}{(47)(13.408)}\sqrt{\frac{1.8}{(403.3)(0.21)(1.08\times10^{-5})}} = 913 \text{ ft},$$

and hence the width W = 1826 ft.

f) We can use data after 200 hrs in a reservoir-limit analysis. For instance, if we use the points $p_{wf} = 4420.249$ psia at $t = 521.609$ hrs and $p_{wf} = 4340.541$ psia at $t = 720$ hrs , then we get the slope (absolute value)

$$m' = \frac{|4340.541 - 4420.249|}{720 - 521.609} = \frac{79.708}{198.391} = 0.402 \text{ psi/hr} .$$

From the slope we can determine the drainage area from the identity

$$A = \frac{0.2339qB}{\varphi c_t hm'} = \frac{(0.2339)(3190)(1.29)}{(0.21)(1.08\times10^{-5})(47)(0.402)} = 2.25E7 \text{ sqft.}$$

In order to determine C_A we need to use the identity

$$p_0 = p_i - \frac{141.2qB\mu}{kh}\left(\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2} + S\right),$$

where p_0 is the extrapolated pressure to t = 0. We can determine p_0 by extrapolating from the last point, for instance by setting

$$4340.541 = p_{wf}(720) = p_{wf}(t) = p_0 - m't = p_0 - (0.402)(720) = p_0 - 289.44 \text{ psia},$$

and hence get $p_0 = 4629.981$ psia. To proceed from here we can rewrite the equation above in the form

$$\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2}+S=\frac{kh}{141.2qB\mu}(p_i-p_0),$$

or in the form

$$\frac{1}{C_A}=\frac{e^{\gamma}r_w^2}{4A}\exp\left[\frac{kh}{70.6qB\mu}(p_i-p_0)-2S\right].$$

If we use the last identity, then we get

$$\frac{1}{C_A}=\frac{(1.781)(0.354)^2}{(4)(2.25E7)}\exp\left[\frac{(403.3)(47)}{(70.6)(3190)(1.29)(1.8)}(4850-4629.981)+10.2\right]=0.194$$

and finally $C_A=5.155$.

Problem 2

a) The skin S must first be determined as

$$S=\ln\frac{2r_w}{x_f}=\ln\frac{(2)(0.108)}{30}=-4.934.$$

The drawdown will therefore be given by

$$\bar{p}-p_{wf}=\frac{18.66qB\mu}{kh}\left(\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2}+S\right)$$

$$=\frac{(18.66)(200)(1.23)(2.3)}{(73)(18)}\left(\frac{1}{2}\ln\frac{4(1000)(2000)}{(1.781)(21.84)(0.108)^2}-4.934\right)=27.388 \text{ bar.}$$

b) The skin value changes to 0, so we then get with reduced area and reduced rate:

$$\bar{p}-p_{wf}=\frac{18.66qB\mu}{kh}\left(\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2}+S\right)$$

$$=\frac{(18.66)(100)(1.23)(2.3)}{(73)(18)}\left(\frac{1}{2}\ln\frac{4(1000)(1000)}{(1.781)(30.88)(0.108)^2}\right)=31.428 \text{ bar.}$$

Problem 3

a) Since the pressures are high we should use a direct pressure formulation. Therefore, if we use the first and last points we get

$$\frac{\Delta p}{q} = \frac{8655-8392}{8300} = 0.03169$$

and

$$\frac{\Delta p}{q} = \frac{8655-7320}{22000} = 0.06068 \, .$$

From these we get the slope

$$b = \frac{0.06068-0.03169}{22000-8300} = 2.1164\text{E}-6 \, .$$

From the last point (for instance) we get

$$a = \frac{\Delta p}{q} - bq = 0.06068 - (2.1164\text{E}-6)(22000) = 0.01412 \, .$$

The deliverability of the well will therefore be given by the identity

$$\bar{p} - p_{wf} = aq + bq^2 = 0.01412q + (2.1164\text{E}-6)q^2 \, .$$

We furthermore get the open flow potential

$$\text{AOF} = \frac{1}{(2)(2.1164\text{E}-6)}\left[-0.01412 + \sqrt{0.01412^2 + (4)(2.1164\text{E}-6)(8655)}\right]$$

$$= 60700 \text{ Mscf/d.}$$

b) If we use the same points as above, then we get

$$\Delta p = 8655 - 8392 = 263 \text{ psi}$$

and

$$\Delta p = 8655 - 7320 = 1335 \text{ psi}$$

to determine the slope

$$\frac{1}{n} = \frac{\log(1335) - \log(263)}{\log(22000) - \log(8300)} = 1.6666,$$

and hence the exponent $n = 0.6$ for the back-pressure equation $q = C(\overline{p} - p_{wf})^n$. Using the last point we next get

$22000 = C(8655 - 7320)^{0.6} = C(75.039)$, and therefore $C = 293.2$ with

$AOF = C(\overline{p})^n = (293.2)(8655)^{0.6} = 67534$ Mscf/d.

Note: If we use the expression $q = C(\overline{p}^2 - p_{wf}^2)^n$ to carry out the same analysis, then we get

$\Delta p^2 = 8655^2 - 8392^2 = 4483361$ psi

and

$\Delta p^2 = 8655^2 - 7320^2 = 21326625$ psi

to determine the slope

$$\frac{1}{n} = \frac{\log(21326625) - \log(4483361)}{\log(22000) - \log(8300)} = 1.5999,$$

and hence the exponent $n = 0.625$. From the last point we next get

$22000 = C(8655^2 - 7320^2)^{0.625} = C(38069)$, and therefore $C = 0.5779$ with

$AOF = C(\overline{p}^2)^n = (0.5779)(8655^2)^{0.625} = 48243$ Mscf/d.

This value is much lower than the others, clearly because the pressures are too high for this approach.