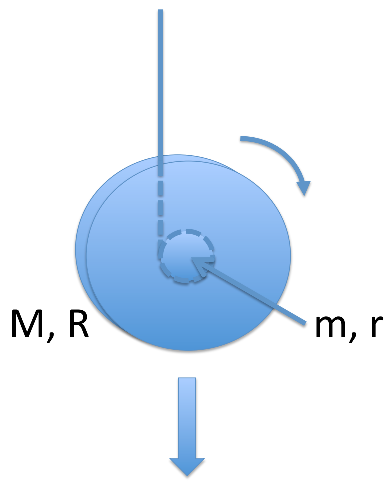
Våren 2016

FYS100 Fysikk
Kontinuasjonseksamen/Re-Exam
Solutions

Problem 1: Yo! Yoda! Yo-Yo! (20 percent)



A yo-yo roughly speaking consists of two round, uniform discs, sandwiched around a third smaller disc. A string is wound around the middle disc, and so the yo-yo may roll up and down as the string winds and unwinds. Consider such a yo-yo, with the two bigger discs having radius $R = 4.00$ *cm and mass* $M = 30.0$ *g each; and the smaller disc in the middle having radius* $r = 0.700$ *cm and mass* $m = 5.00$ *g. The string is taken to be massless, and infinitely thin. The moment of inertia of a disc of mass* m *and radius* r *is* $I = mr^2/2$*. There is gravity* $g = 9.80$ m/s^2*.*

a) What is the total moment of inertia of the yo-yo, around an axis going through the centre of the discs?

Solution: Each big disc has $MR^2/2$ and the small one $mr^2/2$. Add them up to get

$$I = 2 \times MR^2/2 + mr^2/2 = 4.81 \times 10^{-5} \text{ kg m}^2. \quad (1)$$

The end of the string is now fastened to something at a fixed position (like a finger), and the yo-yo is let drop towards the floor.

b) Identify the forces acting on the yo-yo, and for each, indicate whether they provide torque, work, impulse and/or acceleration to the yo-yo.

Solution: The finger acts as a normal force on the string. The string tension acts on the finger. The string doesn't move, so no work, impulse, acceleration is given to it.
The string tension acts on the yo-yo ultimately via the static friction at the string-inner-disc interface, which we may model as a force acting tangentially at the edge of the inner disc. This force provides a contribution to the

acceleration; a torque relative to the CM; and impulse; but no work, since the point of application does not move relative to the string (it is static, not kinetic friction).
Gravity works at the centre of mass of the yo-yo. It contributes to the acceleration, the impulse and does work, but does not give any torque relative to the CM, because the lever arm is zero. (It would give torque around any other axis, but this is not relevant here.)

c) What is the acceleration of the yo-yo downwards; what is its angular acceleration? How large is the string force?

Solution: We can now write down the relevant equations, focusing on linear acceleration of the CM a and angular acceleration α.

$$(2M+m)a = F_g - T, \tag{2}$$
$$I\alpha = Tr, \tag{3}$$
$$a = r\alpha, \tag{4}$$

where the last equation is the rolling without sliding criterion. Using $F_g = (2M+m)g$, we can solve for a, α and T

$$a = \frac{2M+m}{M\left(2+\frac{R^2}{r^2}\right)+\frac{3}{2}m}g = 0.608 \text{ m/s}^2, \tag{5}$$
$$\alpha = \frac{a}{r} = 86.9 \text{ rad/s}^2, \tag{6}$$
$$T = (2M+m)(g-a) = 0.597 \text{ N}. \tag{7}$$

d) How big a fraction of the total kinetic energy goes into the rotating motion?

Solution: When at some point the speed of the CM is v, we have

$$E_{kin,tot.} = E_{kin,CM} + E_{kin,rot.} = \frac{1}{2}(2M+m)v^2 + \frac{1}{2}\frac{I}{r^2}v^2, \tag{8}$$
$$\frac{E_{kin,rot.}}{E_{kin,tot.}} = \frac{1}{1+\frac{(2M+m)r^2}{I}} = 0.938. \tag{9}$$

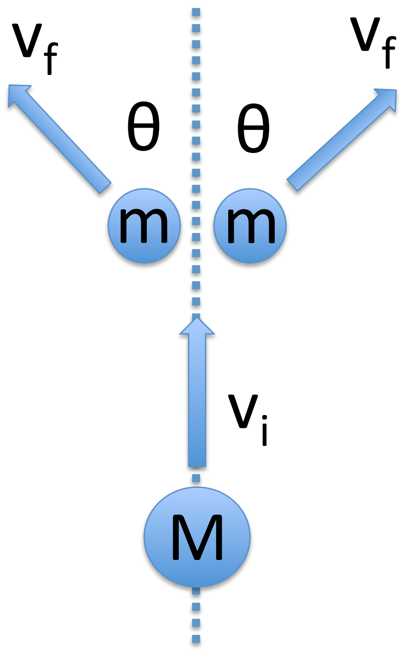
By far most energy goes into the rotating motion.

For each question, provide an algebraic expression, as well as a numerical result with appropriate units.

Typical mistakes: Thinking that T gives work; that F_g gives torque. Errors in computation. Not making a sketch as required. Adding a friction force.

Partial credit given for: Writing down the correct expressions in a) and d) although the result was wrong.

Problem 2: Double score (15 percent)



A hockey puck of mass M hits two other, identical pucks of mass m. The two pucks fly off with the same speed v_f at angles of $\pm\theta$ relative to the direction the original puck was travelling (see figure). The original puck had initial speed v_i, and the two other pucks were initially at rest. We will assume that the pucks are sliding frictionlessly on the ice.

a) If the collision is elastic and the first puck ends up at rest after the collision, what is the final speed v_f of the two other pucks and the angle θ?

Solution: Momentum conservation perpendicular to the original motion gives that the angles are the same ($\pm\theta$), so there is no more information to be gotten from there. Momentum conservation along the original motion gives the relation

$$Mv_i = 2mv_f \cos\theta, \tag{10}$$

and since the collision is elastic, energy conservation gives the relation

$$\frac{1}{2}Mv_i^2 = \frac{1}{2}(2m)v_f^2 \rightarrow v_f = \sqrt{\frac{M}{2m}}v_i. \tag{11}$$

Putting these together, we also get

$$\cos\theta = \frac{M}{2m}\frac{v_i}{v_f} = \sqrt{\frac{M}{2m}}. \tag{12}$$

b) What relation must one require between m and M in order for the scenario of a) to be possible? What happens if this requirement on m and M is not met?

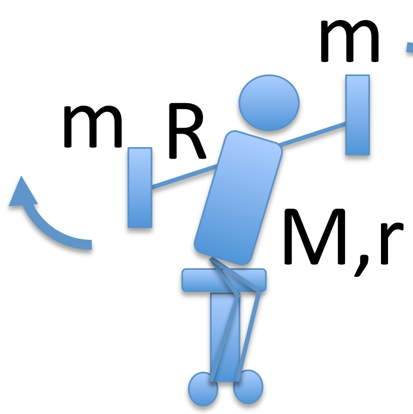
Solution: The relation $\cos\theta = \sqrt{M/2m}$ only makes sense when $M < 2m$ (since $\cos\theta$ must be less than, or equal to, 1). If $M > 2m$ it is not possible for the initial puck to remain at rest after the collision.

For each question, provide an algebraic expression in terms of (some or all of) M, m, v_i.

Typical mistakes: Forgetting that energy is conserved in an elastic collision (very common). Forgetting that momentum is conserved. Thinking that the angle θ is 45°. Calculating errors. Not making a sketch as required.

Partial credit given for: Remembering momentum conservation. Remembering energy conservation. Saying something about b), even if incorrect and/or for the wrong reasons.

Problem 3: Rotating fitness (15 percent)



A man spins around on an office chair. In his hands, he is holding two weights of $m = 5.00$ kg each. He himself weighs $M = 80.0$ kg (including the chair). He starts out rotating with a period of 1.00 second per full revolution, with his arms completely stretched out. We will take the man+chair to have the moment of inertia of a cylinder with radius $r = 30.0$ cm ($I = Mr^2/2$), and his arms to have negligible mass. The arms have length $R = 75.0$ cm. Ignore friction.

He now slowly pulls his arms with the weights in towards his body, so that at the end, they are at a distance $r = 30.0$ cm from the axis.

a) What is the period of rotation at the end?

Solution: Angular momentum is conserved, and so

$$I_i\omega_i = I_f\omega_f \rightarrow T_f = T_i\frac{I_f}{I_i} \tag{13}$$

We find

$$I_i = 2mR^2 + \frac{1}{2}Mr^2, \qquad I_f = 2mr^2 + \frac{1}{2}Mr^2, \tag{14}$$

and find

$$T_f = T_i\frac{2mr^2 + \frac{1}{2}Mr^2}{2mR^2 + \frac{1}{2}Mr^2} = T_i\frac{1 + 4\frac{m}{M}}{1 + 4\frac{m}{M}\frac{R^2}{r^2}} = \frac{1}{2.05} = 0.488 \text{ s.} \tag{15}$$

b) How much work has been done on the system, and where did it come from?

Solution: Energy is not conserved, and we can calculate the change as

$$\Delta E = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}I_i\omega_i^2\left(\frac{I_i}{I_f} - 1\right), \tag{16}$$

where the last relation follows from using conservation of angular momentum again,

$$I_i\omega_i = I_f\omega_f \rightarrow \omega_f = \frac{I_i}{I_f}\omega_i. \tag{17}$$

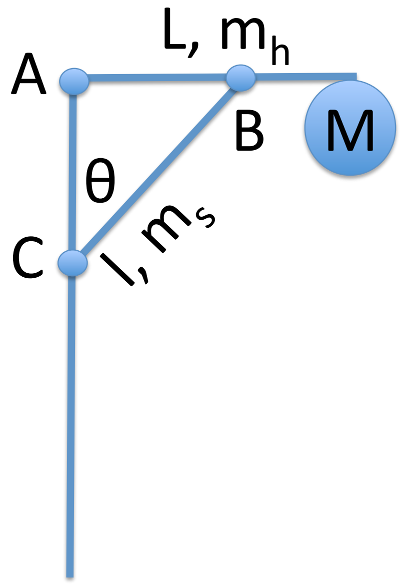
Inserting the numbers, we get $\Delta E = 191$ J.

For each question, provide an algebraic expression, as well as a numerical result with appropriate units.

Typical mistakes: Not using angular momentum conservation. Using energy conservation (even when afterwards calculating a change in energy). Calculational errors. Thinking the work comes from somewhere else (from gravity, chair, masses, angular momentum, kinetic energy, ...). Not making a sketch as required.

Partial credit given for: Remembering angular momentum conservation, even when getting the answer wrong. Trying to compute the work in various (not quite right) ways. Knowing where the work came from.

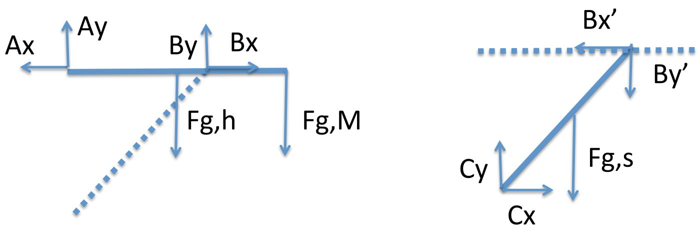
Problem 4: Street lamp (35 percent)



A street lamp of mass $M = 4.00$ kg hangs from a lamp post. The lamp post is composed of a long vertical pole, a horizontal stick of length $L = 50.0$ cm and mass $m_h = 2.00$ kg and a diagonal support stick of length $l = 40.0$ cm and mass $m_s = 1.50$ kg. The lamp is attached to the end of the horizontal stick (see figure). The angle θ between the pole and the support is $45°$. The system is in static equilibrium. There is gravity $g = 9.80$ m/s^2.

a) Make two sketches: 1) The horizontal stick with all the forces acting on it. 2) The support stick with all the forces acting on it. Hint: Decompose the forces at the points A, B and C into $x-$ and $y-$ components $\mathbf{A} = (A_x, A_y)$ and so on.

Solution: For the horizontal stick, include force from the lamp, the force of gravity at the CM of the stick, a general force (A_x, A_y) at the point A, and a general force (B_x, B_y) at the point B, which is $l \sin\theta$ from the point A. Choose directions as you please, but be consistent. I have chosen them so that given the directions on the figure, all the quantities are positive. For the support stick, include the force of gravity at the CM, the force from the horizontal stick at B ((B'_x, B'_y) which is the reaction force of the force of the support stick onto the horizontal stick), and a general force (C_x, C_y) at the point C. Choose directions as you please, but be consistent. Again I have chosen so that all components are positive.



b) Write down the equilibrium equations for both the horizontal stick and the support stick.

Solution: First the force equations. Horizontal stick

$$-A_x + B_x = 0, \qquad A_y + B_y = Mg + m_h g, \tag{18}$$

Support stick

$$C_x - B_x = 0, \qquad C_y - B_y = m_s g. \tag{19}$$

And then we have

$$B_x' = B_x, \qquad B_y' = B_y. \tag{20}$$

Then there are many options, but one simple choice is to compute the torque around A and around C:

$$-m_h g \frac{L}{2} - MgL + B_y l \sin\theta = 0, \qquad -m_s g \frac{l}{2} \cos\theta - B_y l \cos\theta + B_x l \sin\theta = 0. \tag{21}$$

c) Solve these equations for $A_x, A_y, B_x, B_y, C_x, C_y$. Which of the three forces **A**, **B**, **C** *is the largest?*

Solution: The simplest is to first solve for B_y from the torque-around-A equation, to find

$$B_y = \frac{(m_h/2 + M)g}{\sin\theta} \frac{L}{l} = 43.3 \text{ N}. \tag{22}$$

Then insert this to find B_x, C_y and A_y

$$B_x = \frac{m_s g \cos\theta l/2 + l \cos\theta B_y}{l \sin\theta} = 50.7 \text{ N}, \tag{23}$$

$$C_y = m_s g + B_y = 58.0 \text{ N}, \tag{24}$$

$$A_y = (m_h + M)g - B_y = 15.5 \text{ N}. \tag{25}$$

And finally $A_x = B_x = 43.3$ N and $C_x = B_x = 50.7$ N. Computing the length of the three unknown forces, we find

$$|\mathbf{A}| = \sqrt{A_x^2 + A_y^2} = 46.0 \text{ N}, \tag{26}$$

$$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2} = 66.7 \text{ N}, \tag{27}$$

$$|\mathbf{C}| = \sqrt{C_x^2 + C_y^2} = 77.0 \text{ N}, \tag{28}$$

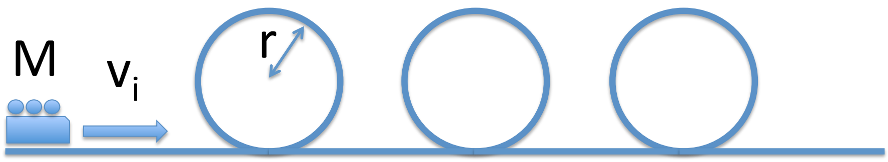
C is the largest.

For each question, provide an algebraic expression, as well as a numerical result with appropriate units.

Typical mistakes: Forgetting gravity on the two sticks. Introducing some additional normal forces". Thinking that the forces at A, B, C only have x-(or y-) components. Not writing down the force equations in x and y. Not writing down the torque equations around A and/or B and/or C. Thinking that gravity of the horizontal beam acts in the same point as force B from the support beam. Calculational mistakes. This was the "challenge-problem this time, and no-one solved it. Not making sketches as required.

Partial credit given for: Getting most of the forces right in the sketch. Writing down some of the equilibrium equations. Having a go at solving them, even though they are wrong. Understanding how to in principle proceed to the end of the problem.

Problem 5: The Loopy Looper (15 percent).



In an amusement park, the new roller-coaster ride The Loopy Looper is composed of a number of full loops, where the car goes round upside down. The car is attached to the track, but there are no seat belts. Consider one such loop, which can be taken as a circle with radius r. The mass of the roller-coaster car is M, including the weight of the people in it, and you may ignore all sources of friction. The car starts out at ground level with initial speed v_i, and after that the engine is turned off. There is gravity g.

a) What is the minimum value of v_i, for the car to get to the top of the loop, assuming that the passengers hold on to something, to not fall out?

Solution: Energy conservation tells us that since mechanical energy is conserved, the minimum initial kinetic energy should just be final potential energy, when at the top of the loop. Hence the minimum speed is

$$\frac{1}{2}Mv_i^2 = (2r)Mg \rightarrow v_i = \sqrt{4rg}. \tag{29}$$

b) What is the minimum value of v_i, for the car to get to the top of the loop, and for the people to not fall out as it goes round, even when not holding on to anything?

Solution: Now we need more speed so that at the top, the centrifugal force keeps the people in place. More precisely, the limiting speed is when the force of gravity is exactly enough to force the people to follow the radius of curvature of the loop. Introducing v_t as the speed at the top, we have

$$\frac{1}{2}Mv_i^2 = \frac{1}{2}Mv_f^2 + (2r)Mg \tag{30}$$

where

$$Mg = M\frac{v_f^2}{r} \tag{31}$$

which when combined gives the minimum speed

$$v_i = \sqrt{5gr} \tag{32}$$

c) The first loop is followed by a second, identical loop. What is the minimum value of v_i, for the car and passengers to go safely round the second loop as well?

Solution: Since energy is still conserved the result is the same as in question b)

For each question, provide an algebraic expression in terms of (some or all of) M, r, g.

Typical mistakes: Not using energy conservation. Using (in various ways) centripetal force in a). Using r instead of $2r$ for the height. Not adding the "$4rg$from a) to the additional "rg"in b). Thinking in c) that it takes more speed (typically 2 times) to get through the second loop. Not making a sketch as required.

Partial credit given for: Writing down the centripetal force in b). Writing down energy conservation in a). Explaining that the same speed works in c) as in b), even without having the right result.