Høsten 2016

FYS100 Fysikk
Solutions week 34

First some problems from the book:

- 1.9, 1.12
- 3.23, 3.36

Solution: 1.9. Badly posed problem, since there is no specification of what a, x, y, v are. Of course, they mean that x, y are lengths, a is an acceleration and v are speed. In this case a) is wrong, because the unit of ax is m^2/s^2 rather than m/s. And b is correct, since y and "2 m"have the same unit, and kx inside the cos has no unit. Worth noting that stuff inside functions (e^x, $\cos(x)$, $\sin(x)$, $\log(x)$) must be unit-free!

Solution: 1.12. One may match units as follows:

$$[F] = kg\frac{m}{s^2} = [G]\frac{kg^2}{m^2} \rightarrow [G] = \frac{m^3}{s^2 kg} \text{ or } \frac{Nm^2}{kg^2}. \tag{1}$$

Solution: 3.23
Given $\overrightarrow{\mathbf{A}} = (3, -2)$ and $\overrightarrow{\mathbf{B}} = (-1, -4)$, we find

$$\overrightarrow{\mathbf{A}} + \overrightarrow{\mathbf{B}} = (2, -6), \qquad \overrightarrow{\mathbf{A}} - \overrightarrow{\mathbf{B}} = (4, 2), \qquad |\overrightarrow{\mathbf{A}} + \overrightarrow{\mathbf{B}}| = \sqrt{40}, \qquad |\overrightarrow{\mathbf{A}} - \overrightarrow{\mathbf{B}}| = \sqrt{20}. \tag{2}$$

and

$$\theta_{\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}} = \tan^{-1}\frac{-6}{2} = -71.6°, \qquad \theta_{\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}} = \tan^{-1}\frac{2}{4} = 26.6°. \tag{3}$$

Solution: 3.36
Given $\overrightarrow{\mathbf{A}} = (3, -4, 4)$ and $\overrightarrow{\mathbf{B}} = (2, 3, -7)$, we find

$$\overrightarrow{\mathbf{A}} + \overrightarrow{\mathbf{B}} = (5, -1, -3), \qquad 2\overrightarrow{\mathbf{A}} - \overrightarrow{\mathbf{B}} = (4, -11, 15), \tag{4}$$

with magnitudes $\sqrt{35}$ and $\sqrt{362}$, respectively.

Additional Problem 1: Find all the solutions for θ to the following equations, expressed in both radians and degrees. Draw the solutions on a unit circle:

a) $\cos\theta = 1$. **Solution:** $\theta = 0$ (radians or degrees) as well as $2\pi \times n$ (or $360 \times n$), for any integer n (positive or negative).

b) $\sin\theta = 0.4$. **Solution:** $\theta = 0.412$ (radians, 23.6 degrees). But also $\pi - 0.412$ (180-23.6) and adding $2\pi \times n$ (or $360 \times n$).

c) $\tan\theta = -2$. **Solution:** $\theta = -1.11$ (-63.4 degrees). But also $\pi - 1.11$ (180-63.4) and adding $2\pi \times n$ (or $360 \times n$).

d) $\cos\theta + \sin\theta = 0$. **Solution:** This is the same as $\tan\theta = -1$, $\theta = -\pi/4$ (-45 degrees). But also $\pi - \pi/4$ (180-45) and adding $2\pi \times n$ (or $360 \times n$).

e) $\cos\theta + 3\sin\theta = 0$. **Solution:** This is the same as $\tan\theta = -1/3$, $\theta = -0.322$ (-18.4 degrees). But also $\pi - 0.322$ (180-18.4) and adding $2\pi \times n$ (or $360 \times n$).

Additional Problem 2: Consider a right triangle with sides a and b, and hypotenuse c. Expressed in a, b, c, what is:

a) The sine, cosine and tangent of the angle opposite a? **Solution:** $\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$, $\tan A = \frac{a}{b}$.

b) The sine, cosine and tangent of the angle opposite b? **Solution:** $\sin B = \frac{b}{c}$, $\cos B = \frac{a}{c}$, $\tan B = \frac{b}{a}$.

c) The sine, cosine and tangent of the angle opposite c? **Solution:** It's the right angle. $\sin C = 1$, $\cos C = 0$, $\tan C = \infty$.

Additional Problem 3: How many significant digits and decimals are there in the following numbers:

a) 1 **solution**: 1 significant, 0 decimals

b) 0.12 **solution**: 2 significant, 2 decimals

c) 0.0034 **solution**: 2 significant, 4 decimals

d) 117.8 **solution**: 4 significant, 1 decimal

e) π **solution**: infinite significant, infinite decimals

Additional Problem 4: Assume that you have measured two quantities $a = 0.4$ and $b = 0.2$ with an error of $\Delta a = 0.1$ and $\Delta b = 0.03$, respectively. Here are some functions $f(a, b)$ of a and b:

a) $f(a,b) =: a+b$; $a-b$; a/b; b/a; ab.

b) $f(a,b) =: a^2$; a^3; a^2b.

c) $f(a,b) =: \log(a)$

Compute their value, and compute an estimate for the error in three ways:

1. Computing "worst case scenarios"(the extreme cases, where the errors on the input are the largest possible). Then use as the estimated error half the difference between the largest and smallest possible values of the output.

2. Computing (you may want to compare this with B.8 of the book)

$$\Delta f(a,b) = \frac{df}{da}\Delta a + \frac{df}{db}\Delta b. \tag{5}$$

3. Computing

$$\Delta f(a,b) = \sqrt{\left(\frac{df}{da}\Delta a\right)^2 + \left(\frac{df}{db}\Delta b\right)^2}. \tag{6}$$

Solution: One gets, in order Value, Error 1, Error 2, Error 3 (note that I am not following the rounding-off conventions here; this is to illustrate the difference between the methods, and so I need to keep significant digits far enough that I see that difference!):

0.6, 0.13, 0.13, 0.104;
0.6, 0.13, 0.13, 0.104;
2, 0.82, 0.8, 0.583;
0.5, 0.213, 0.6, 0.146.
0.08, 0.032, 0.0233.
0.16, 0.08, 0.08;
0.064, 0.049, 0.048, 0.048.
0.032, 0.0211, 0.0208, 0.0167.
-0.916, 0.255, 0.25, 0.25.

Since we are not going to perform full error propagation in this course, we will stick to the rule of thumb: **As many significant digits in the output as the lowest number in the input. This will typically be 3. Unless it is just addition/subtraction: then it is the smallest number of decimals that count.**