Høsten 2016

FYS100 Fysikk
Problems week 36

Have a go at these. And for each, make a little sketch to illustrate the solution.

First some problems from the book:

- 4.12, 4.13, 4.22, 4.25.

Solution 4.12: Use the range equation

$$R = \frac{v^2}{g}\sin(2\theta). \tag{1}$$

Maximal distance is when $\theta = 45°$, and solving for g, one finds $g = v^2/R_{max} = 3^2/15 = 0.6$ m/s^2.

Solution 4.13: This is projectile motion, with initial velocity only in the x-direction, $y_i = 1.22$ m and final distance $d = 1.40$ m. We write a)

$$y_i - \frac{g}{2}t^2 = 0, \tag{2}$$
$$v_i t = d, \tag{3}$$
$$t_{land} = \sqrt{\frac{2y_i}{g}} \to v_i = \frac{d\sqrt{g}}{2y_i} = 2.81 \text{ m/s}. \tag{4}$$

b) The final velocity is then

$$(v_x, v_y) = (v_i, -gt_{land}) = (2.81, -4.89) \tag{5}$$

or $v = 5.64$, and an angle of $\theta = -60.2$ with the horizontal.

Solution 4.22: This is very similar to 4.13, but now with water. Now we have to decide how wide ad high a walkway should be. Let's require that a person of height 180 cm should have 1.00 m wide head space. Then the question is whether the water has dropped below 180 cm, 1 m away from the wall. We calculate at which value of x, the water has dropped to 180 cm, starting from $h = 2.35$ m, that means a drop of $\Delta y = -55$ cm. WIth initial horizontal velocity of $v = 1.70$ m/s:

$$-\frac{g}{2}t^2 = \Delta y, \tag{6}$$

$$t_{land} = \sqrt{\frac{-2\Delta y}{g}}, \tag{7}$$

$$vt = 0.570 \text{ m}. \tag{8}$$

Hence, according to our definition of an appropriate height and width of a walkway, there isn't room.

Solution 4.25: Again projectile motion. a) We write

$$v_i \cos\theta t = d \to v_i = \frac{d}{\cos\theta t} = 18.1 \text{ m/s}. \tag{9}$$

$$\tag{10}$$

b) Now the height at distance d, minus the height of the wall h is

$$\left(v_i \sin\theta t - \frac{g}{2}t^2\right) - h = 1.13 \text{ m}. \tag{11}$$

c) It lands on the roof, when it reaches a height of $h - 1 = 6$ m. We want to know how far beyond the wall (d) it gets to (let's call it l). We calculate

$$\left(v_i \sin\theta t - \frac{g}{2}t^2\right) = 6.00 \text{ m}, \tag{12}$$

$$l = v_i \cos\theta t - d = 2.79 \text{ m}. \tag{13}$$

Additional Problem 1 (Prob. 5, Oblig. 1, 2013):
Wile E. Coyote is keen to catch the Road-Runner (check it on Wikipedia, if you don't know the reference. It won't matter for the following). He hides behind a big rock, and as the Road-Runner zooms past at constant speed $v = 15.0$ m/s, Coyote lights up his ACME rocket pack. After waiting $t_0 = 2.00$ s for the rocket fuse to burn down, he accelerates at a constant rate of $a = 5.00$ m/s^2, in pursuit of the Road-Runner.

a) At what time, with what speed and after what distance does he catch the Road-Runner?

b) What if the rocket would stop working after 4 seconds of acceleration and Coyote would continue at constant speed?

Solution: a) Putting $t = 0$ when the RR passes WEC, and defining $x = 0$ at that position, we have

$$x_{RR} = vt, \qquad x_{WEC} = \frac{a}{2}(t - t_0)^2. \tag{14}$$

Setting these equal at the time t_{catch}, we have

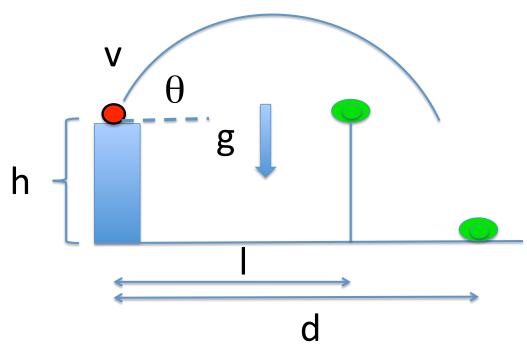
$$vt_{catch} = \frac{a}{2}(t_{catch} - t_0)^2 \rightarrow t_{catch} = \text{ or } t_{catch} = (v + at_0)\left(1 \pm \sqrt{\frac{a^2 t_0^2}{(at_0 + v)^2}}\right) = 9.58 \text{ s.} \quad (15)$$

The other solution with the minus sign (0.417 s) corresponds to the curves passing after RR passes WEC, but when WEC has not yet started, and so it doesn't apply. Distance one may get from putting the time into either of the equations $\rightarrow$ 144 m. The speed then of WEC is

$$a(t_{catch} - t_0) = 37.9 \text{ m/s.} \quad (16)$$

b) If acceleration stops after 4 s (at time 4+2=6 s), then WEC would have final speed $4 \times a = 20$ m/s. He would therefore still catch up, eventually. At $t = 6$, RR is at a position $x = 6 \times 15 = 90$ m, and WEC is at $x = a/2(6 - t_0)^2 = 40$ m. So WEC has to catch up 50 m with a speed advantage of $20 - 15 = 5$ m/s. That takes him $50/5 = 10$ s, for a total time of $10 + 6 = 16$ s. By then, they will both be at a distance $x = 90 + 10 \times 15 = 240 = 40 + 20 \times 10$ m from the origin. And WEC will of course have a speed of 20 m/s when he passes the RR.

Additional Problem 2: (Prob. 1, Hand-in 1, 2015):



You are the chief of the Angry Birds (Google it, if you don't know the reference...it's not important). You can shoot red birds from a catapult, and they will fly as projectiles under the effect of gravity. Gravity points downwards and has magnitude $g = 9.80$ m/s^2. Your goal is to hit some nasty green pigs, who have stolen your Angry Bird Eggs. You are able to adjust the angle θ and speed v of the projectile, as it leaves the catapult. The projectile leaves the catapult at a height h above the ground.

a) First, consider the case when a single green pig is placed on the ground, at a distance d from the foot of the catapult. Find a relation between the angle and speed required to hit it. Remember to draw a sketch of the situation.

Solution: This is simple projectile motion, starting at $y_i = h$ hitting a target at $y = 0$. So

$$d = v\cos\theta t, \tag{17}$$
$$y = h + v\sin\theta t - \frac{g}{2}t^2 = 0. \tag{18}$$

[And now we do it the other way around, as was suggest dot me by one of you...]
Solving the x-equation for t and inserting into the y-equation, one gets

$$h + d\tan\theta - \frac{gd^2}{2v^2}\frac{1}{\cos\theta^2} = 0, \tag{19}$$
$$\to v^2 = \frac{gd^2}{2\cos\theta^2(h + d\tan\theta)} \tag{20}$$

b) Next consider the case where there are two green pigs. One on the ground at a distance d as before, the other at a distance $l < d$, and placed on a pole of height h. What should you choose for θ and v in order to hit both pigs with the same red bird? Why must one require $l < d$? Remember to draw a sketch of the situation.

Solution: Now we have two criteria:

$$d = v\cos\theta t, \tag{21}$$
$$0 = h + v\sin\theta t - \frac{g}{2}t^2, \tag{22}$$
$$v^2 = \frac{gd^2}{2\cos\theta^2(h + d\tan\theta)} \tag{23}$$

and

$$l = v\cos\theta t, \tag{24}$$
$$h = h + v\sin\theta t - \frac{g}{2}t^2 = 0, \tag{25}$$
$$v^2 = \frac{gl^2}{2\cos\theta^2(l\tan\theta)} = \frac{gl}{2\sin\theta\cos\theta}. \tag{26}$$

Inserting one in the other and moving things around a bit, we find

$$\tan\theta = \frac{lh}{d(d-l)}, \qquad v^2 = \frac{gl}{\sin(2\theta)}. \tag{27}$$