Høsten 2016

FYS100 Fysikk
Problems week 37

Have a go at these. And for each, make a little sketch to illustrate the solution.

First some problems from the book:

- 4.33, 4.38, 4.43, 4.51, 4.54

Solution 4.33: This is really non-Uniform circular motion, but we are only asked about the maximal acceleration. The radial acceleration is then just the centripetal acceleration, which is largest at the largest speed.

$$|\mathbf{a_r}| = \frac{v_{max}^2}{r} = 377\text{m/s}^2 \tag{1}$$

Note that the mass does not enter.

Solution 4.38: a) With a length of $r = 0.6$m, and 8 rev/s the speed is $8 \times 2\pi 0.6$m/s= 30.2m/s. With $r = 0.9$m and 6 rev/s, it is $6 \times 2\pi 0.9$m/s= 33.9m/s. So the longer chain gives the fastest motion. b) and c) The centripetal accelerations are $v^2/r = 1279$ and 1520 m/s^2, respectively.

Solution 4.43: a) Yes. The radial acceleration is the centripetal one $v^2/r =$ 4.5 m/s^2. Then if the total acceleration has magnitude 6 m/s^2, there must be a tangential acceleration, so that

$$6^2 = 4.5^2 + |\mathbf{a_t}|^2 \to |\mathbf{a_t}| = 3.97\text{m/s}^2. \tag{2}$$

b) No. The radial acceleration required by the path is already larger than 4 m/s^2.

Solution 4.51: Seen from the bank, moving with the stream, the swimmer has speed $c + v$. Going against the stream, $c - v$. Hence going one way takes $t_1 = d/(c + v)$, going the other way takes $t_2 = d/(c - v)$. Adding them up, one gets a)

$$t_1 + t_2 = \frac{2d}{c}\frac{1}{1 - \frac{v^2}{c^2}}. \tag{3}$$

b) If the stream is still, just take $v = 0$, to get $\frac{2d}{c}$ (as one would expect).
c) It is always slower when the stream moves, since the factor

$$\frac{1}{1 - \frac{v^2}{c^2}} \tag{4}$$

is always larger than 1. Note that if $v \to c$ the swimming time goes to infinity. Swimmer gets swept away by the river.

Solution 4.54: Let's assume that there is no air resistance. Then **a)** the boy throws the can straight upwards (vertical) in the frame of the truck+boy. **b)** This is 1-D motion under gravity, and so

$$y = v_i t - g/2t^2 \to t_{catch} = \frac{2v_i}{g}. \tag{5}$$

On the other hand, since it comes back down $d = 16$ m further down the road, we have $d = v_t t_{catch}$, with v_t the speed of the truck. Eliminating t_{catch}, we get

$$\frac{2v_i}{g} = \frac{d}{v_t} \to v_i = \frac{gd}{2v_t} = 8.25 \text{ m/s}. \tag{6}$$

This is the vertical initial speed of the can relative to the truck. **c)** the shape of the trajectory seen from the boy+truck is a straight line up and down. **d)**ÊSeen from the ground, the boy throws the can forwards, and its path is a parabola, a "2-D projectile motionpath. The initial velocity seen from the ground $\mathbf{v_g}$ follows from a Galilean transformation,

$$\mathbf{v_g} = \mathbf{v_i} + \mathbf{v_t} = (0, v_i) + (v_t, 0) = (v_t, v_i) = (9.50, 8.25) \text{ m/s}. \tag{7}$$

In polar coordinates $(12.5 \text{ m/s}, 41.0°)$.

Additional Problem 1 (OblII, 2013)

A submarine targets a battleship at a distance $d = 20.0$ km, in the direction $\theta_1 = 15.0°$ East of North. The battleship is travelling at $v_1 = 30.0$ km/h in a direction $\theta_2 = 40.0°$ degrees East of North. The torpedoes of the submarine can move at a speed of $v_2 = 100$ km/h.

a) In which direction relative to North should the submarine fire its torpedo to hit the battleship?

Solution: The easiest is to rotate the entire problem, so that the y-axis is 15° East of North. Then the battleship is moving at 25.0° to the right of that. Then one just has to aim the torpedo, so that the movement in x (perpendicular to y) is the same as the battleship's movement in x. So

$$v_1 \sin(\theta_2 - \theta_1) = v_2 \sin(\theta_3 - \theta_1),$$

$$\rightarrow \qquad \theta_3 = \theta_1 + \sin^{-1}\left(\frac{v_1}{v_2}\sin(\theta_2 - \theta_1)\right) = 22.3 \text{ degrees},$$

where θ_3 is the direction of the torpedo relative to North.

b) How long will it take the torpedo to reach the target?

Solution: The relative speed in the y-direction is

$$v_{\text{rel,x}} = v_2 \cos(\theta_3 - \theta_1) - v_1 \cos(\theta_2 - \theta_1),$$

so that to cover the initial distance d, it takes

$$\frac{d}{v_{\text{rel,x}}} = 0.278 \text{ hours} = 1000s.$$