Høsten 2016

FYS100 Fysikk
Problems week 38

Have a go at these. And for each, make a little sketch to illustrate the solution.

Some problems from the book:

- 5.27, 5.28, 5.29, 5.46, 5.52, 5.65, 5.88.

Solution 5.27: Since the boat has constant speed, there is no acceleration, and so the forces must cancel out.
a) x is East, y is North. Then $\overrightarrow{\mathbf{n}} = (-|\overrightarrow{\mathbf{n}}|, 0)$, $\overrightarrow{\mathbf{P}} = (|\overrightarrow{\mathbf{P}}|\cos\theta, |\overrightarrow{\mathbf{P}}|\sin\theta)$ and $F_{\text{drag}} = (0, -220N)$. We then have

$$|\overrightarrow{\mathbf{P}}|\sin\theta = 220N \rightarrow |\overrightarrow{\mathbf{P}}| = \frac{220N}{\sin 40^\circ}, \quad (1)$$

$$|\overrightarrow{\mathbf{P}}|\cos\theta = |\overrightarrow{\mathbf{n}}| = \frac{220N}{\tan 40^\circ}, \quad (2)$$

and so $\overrightarrow{\mathbf{n}} = (-262N, 0)$ and $\overrightarrow{\mathbf{P}} = (262N, 220N)$, length 342 N.

b) Now use axes along $\overrightarrow{\mathbf{P}}$ and orthogonal to it. Then instead one finds

$$|\overrightarrow{\mathbf{P}}| = 220N\sin\theta + |\overrightarrow{\mathbf{n}}\cos\theta, \quad (3)$$

$$|\overrightarrow{\mathbf{n}}|\sin\theta = 220N\cos\theta, \quad (4)$$

which of course gives the same result. Probably a) should be considered the simplest.

Solution 5.28: Think carefully about this one. First b) Clearly, the string/spring is carrying $5 \times 9.8 = 49$ N. Now consider a): it is essentially twice b) and so the reading must be the same, 49 N. (Imagine in b) that you would reach down and grab the rope with both hands and cut it between your hands. Now you are holding both weights, in the same way as the wall in a). It would be quite weird if as you did this, the spring would suddenly be stretched more!). In c), the Newtonmeter is carrying both weights, to 98 N. And in d) it is opposing the component of gravity along the incline, $g\sin 30^\circ$ so a weight of $24.5N$.

Solution 5.29: Draw a picture of the situation. Let's call the masses $m_{1,2,3}$ according to their weight. And let's split the system up in to system 1,2,3 accordingly. Let us put the positive x-axis in the direction of the external force F. The string tension is T, the forces between m_1 and m_2 are $F_{12} = -F_{21}$. All the blocks accelerate at the same rate a.

System 1: $m_3 a = F - T$,
System 2: $m_2 a = F_{12}$,
System 3: $m_1 a = T - F_{12}$.

Add them up and solve for a to get

$$a = \frac{F}{m_1 + m_2 + m_3} = 7.0 \text{ m/s}^2 \tag{5}$$

(not a great surprise!). Reinserting this, one finds

$$T = F - m_3 a = 21N, \tag{6}$$
$$F_{12} = m_2 a = 14N. \tag{7}$$

Solution 5.46: a) The first important thing to notice is that if m_1 moves a distance d, P_2 also moves a distance d, but m_2 moves $2d$. As a result (differentiating twice), $a_2 = 2a_1$. The second is that the tension of the string attached to the wall (T_2) and attached at the other end to m_2 together with the tension in the first string (T_1) provide acceleration for the second pulley, which has mass 0. Splitting up into 3 systems (m_1, m_2 and pulley), we have, putting the positive x-axis downwards:

$$m_1 a_1 = m_1 g - T_1, \tag{8}$$
$$m_2 a_2 = T_2, \tag{9}$$
$$m_{\text{pulley}} a_3 = T_1 - 2T_2 = 0, \tag{10}$$

Inserting $a_2 = 2a_1$, we find

$$a_1 = \frac{a_2}{2} = g\frac{m_1}{m_1 + 4m_2}, \tag{11}$$
$$T_1 = g\frac{4m_1 m_2}{m_1 + 4m_2}, \tag{12}$$
$$T_2 = g\frac{2m_1 m_2}{m_1 + 4m_2}. \tag{13}$$

Solution 5.52:
b) The mass of the truck is irrelevant to the question of slowing down the load.

a) The load is slowed down by the truck slowing down "under"it, and static friction opposing the impending motion of the load "continuing-sliding forward relative to the truck. The force is at most $F_s = \mu_s|n| = \mu_s M g$, with M the mass of the load. That means that is has a constant acceleration of $F_s/M = \mu_k g$ opposite the motion relative to the road. It stops in a (minimum) time $v_i + at = 0 \rightarrow t = v_i/\mu_k g$, and over a (minimum) distance of

$$d = v_i t - \frac{\mu_k g}{2} t^2 = \frac{v_i^2}{2\mu_k g} = 14.7 \text{ m}. \tag{14}$$

Solution 5.65: Splitting up in to 2 systems, and putting the positive x-axis in the direction for he external force, we have:

System 1: x: $T - \mu_k|\overrightarrow{\mathbf{n}}|_1 = m_1 a$, y: $m_1 g = |\overrightarrow{\mathbf{n}}_1|$,
System 2: x: $F - T - \mu_k|\overrightarrow{\mathbf{n}}|_2 = m_2 a$, y: $m_2 g = |\overrightarrow{\mathbf{n}}_2|$.

Adding up, we have

$$F - \mu_k(m_1 + m_2)g = (m_1 + m_2)a \rightarrow a = \frac{F - \mu_k(m_1 + m_2)g}{m_1 + m_2}, \tag{15}$$

$$T = \frac{m_1}{m_1 + m_2} F. \tag{16}$$

Solution 5.88: One may split up in three systems:
1) $Mg = T_2$
2) $2mg \sin\theta = T_1$
3) $mg \sin\theta + T_1 = T_2$
That gives a) + b)

$$M = 3m \sin\theta, \qquad T_2 = 3mg \sin\theta, \qquad T_1 = 2mg \sin\theta. \tag{17}$$

Now $M = 6m \sin\theta$, and we have for the three systems (a positive when M falls down)
1) $Ma = Mg - T_2$
2) $2ma = T_1 - 2mg \sin\theta$
3) $ma = T_2 - T_1 - mg \sin\theta$
Adding these up, and solving for a, we get c)

$$a = \frac{Mg - 3mg \sin\theta}{M + 3m} = \frac{\sin\theta}{1 + 2\sin\theta} g, \tag{18}$$

and for the tensions d)

$$T_1 = 2m(a + g\sin\theta) = 2mg\frac{2\sin\theta(1+\sin\theta)}{1+2\sin\theta}, \tag{19}$$

$$T_2 = M(g-a) = 6mg\sin\theta\frac{1+\sin\theta}{1+2\sin\theta}. \tag{20}$$

Now M is again unknown, and there is static friction m_s between blocks and incline. Then we have
1) $Mg = T_2$
2) $2mg\sin\theta \pm 2mgm_s\cos\theta = T_1$
3) $mg\sin\theta + T_1 \pm mgm_s\cos\theta = T_2$
Adding these up, we get e)+ f)

$$M = 3m(\sin\theta \pm m_s\cos\theta). \tag{21}$$

In those two cases, we find g)

$$T_2 = Mg = 3m(\sin\theta \pm m_s\cos\theta)g. \tag{22}$$