Høsten 2016

FYS100 Fysikk
Problems week 39

Have a go at these. And for each, make a little sketch to illustrate the solution.

Some problems from the book:

- 6.6, 6.7, 6.20, 6.21, 6.31, 6.51, 6.65.

Solution, 6.6:
a) Since we are told that it goes with uniform speed, that speed is $l/t = 235/36 = 6.53$ m/s. The radius of the circle is $r = 235/(\pi/2) = 149.6$ m, since the arc is a quarter circle. Then the acceleration is the centripetal one,

$$a_c = \frac{v^2}{r} = \frac{l^2}{t^2} \times \frac{\pi}{2l} = \frac{\pi l}{2t^2} = 0.285 \text{ m/s}^2. \tag{1}$$

We are asked to provide it at an angle of 35.0°. Lets have the x-axis towards East, y-axis towards North. Since the centripetal acceleration is towards the centre, along the radius vector, it is simply $(-a_c \cos\theta, a_c \sin\theta) = (0.233, 0.163)$ m/s^2.

b) The car's average speed is the one found above, 6.53 m/s.

Solution, 6.7: The fake gravity is the normal force resisting the centrifugal force (in the rotating frame)...or the normal force providing the centripetal force for the rotation (in the inertial frame). It has magnitude $mv^2/r = mr\omega^2$. We want the corresponding acceleration to be 3 m/s^2, and $r = 60$ m. So we have

$$\omega = \sqrt{\frac{a_c}{r}} = 0.224 \text{ s}^{-1}. \tag{2}$$

One revolution of 2π takes $2\pi/\omega = 28.1$ s. It therefore does 2.14 revolutions per minute.

Solution, 6.20: a) From the point of view of the inertial frame on the ground, the spring is providing an acceleration of a, corresponding to a force of ma. If that force is 18 N, then the acceleration is $18/5 = 3.6$ m/s^2.

b) If the car moves with constant velocity, there is no acceleration, and the spring shows 0 N.

c+d) Seen from the inertial frame, there is a spring force accelerating the mass. Seen from the non-inertial car frame, there is a fictitious force pulling the mass backwards, thereby stretching the spring.

Solution, 6.21: This is exactly like Example 6.7 (it's a truck instead of a train...). So we have a)

$$\tan\theta = \frac{a}{g} \to \theta = \tan^{-1}\frac{3}{9.8} = 17.0°. \tag{3}$$

and b)

$$T = \frac{mg}{\cos\theta} = 5.12N. \tag{4}$$

Solution, 6.31: a) The terminal speed is $v_T = mg/b$, and so

$$b = \frac{mg}{v_T} = 1.47. \tag{5}$$

b) The solution to the evolution is $v(t) = v_T(1 - e^{-t/\tau})$, where $\tau = m/b$, which is equal to 0.632τ when

$$1 - e^{-t/\tau} = 0.632 \to t = -Ln(0.368)\tau = \tau = 0.0204s \tag{6}$$

c) The terminal speed is never exactly reached, but the resistive force goes asymptotically to exactly cancel the force of gravity $mg = 0.0294$ N.

Solution, 6.51: This is very similar to 6.21, only now going uphill. We choose the coordinate system to decompose on to be along the incline (x positive up the hill) and orthogonal to that (y positive up from the incline). Then we have

$$x: \quad ma = T\sin\theta - mg\sin\phi, \tag{7}$$
$$y: \quad T\cos\theta - mg\cos\phi = 0. \tag{8}$$

which agrees with 6.21 in the limit $\phi = 0$. Getting the T-alone on the LHS and dividing the two equations, we have

$$\tan\theta = \tan\phi + \frac{a}{g\cos\phi} \to a = (\tan\theta - \tan\phi)g\cos\phi. \tag{9}$$

Solution, 6.65: We know the relation

$$v(t) = v_T(1 - e^{-t/\tau}), \qquad \tau = \frac{m}{b}, \qquad v_T = g\tau. \tag{10}$$

a) If $v = v_T/2$, then $e^{-t/\tau} = 1/2$, and $t/\tau = \log(2)$, so $v_T = g\tau = gt/L\log(2) =$ 78.3 m/s.
b) If $v = 3v_T/4$ then $e^{-t\tau} = 1/4 = (1/2)^2 = (e^{-5.54/\tau})^2 = e^{-2\times 5.54/\tau}$. So that happens after 11.1s (twice as long).
c) Integrating the speed equation, we find

$$x(t) = v_T(t + \tau(e^{-t/\tau} - 1)) = v_T(5.54 - 5.54/\log(2) \times 1/2) = 121m. \quad (11)$$