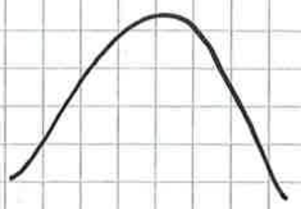
Hand-In I 10.11.16

Problem 4 - Ugly duckling 15%

Average speed



$V_2 \rightarrow$ Up: 15 km/h

$V_1 \rightarrow$ down: 30 km/h

$$V_{avg} = \frac{D}{t} = \frac{2d}{t_1 + t_2}$$

t_1: down, t_2: up

$$t_1 = \frac{d}{V_1}$$

$$t_2 = \frac{d}{V_2}$$

$$= \frac{2d}{\frac{d}{V_1} + \frac{d}{V_2}} = \frac{2}{\frac{1}{V_1} + \frac{1}{V_2}} = \frac{2 V_1 V_2}{V_2 + V_1} = \frac{2 \cdot 15 \cdot 30}{15 + 30} = \underline{\underline{20 \text{ km/h}}}$$

NB! Potential for exam

Problem 5 - Tesla

$X_2 = 60\,m$

$|V_1| = 40\,km/h$

$|V_2| = 80\,km/h$

$|a_2| - |a_1| = 5\,m/s^2$

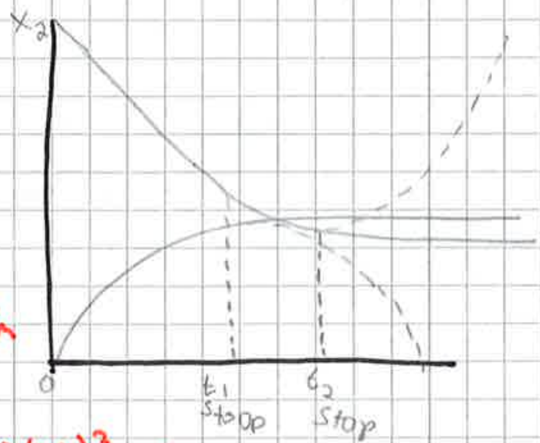
a) 1-D, constant a

$X_1(t) = V_i t - \frac{a}{2} t^2$

$X_2(t) = X_2 - V_2 t + \frac{a}{2} t^2$

$\rightarrow t < t_1^{stop}$; X^1_{final}, $t > t_1^{stop}$

$\rightarrow t < t_2^{stop}$; X^2_{final}, $t > t_2^{stop}$



$V_1(t) = V_1 - at$

$V_2(t) = -V_2 + at$

t^1_{stop} : $V_1(t) = 0$: $t = \frac{V_1}{a} = 2.22\,s$

$X^1_{final} = V_1 t - \frac{a}{2} t^2 = V_1 \frac{V_1}{a} - \frac{a}{2}\left(\frac{V_1}{a}\right)^2 = \frac{V_1^2}{2a} = 12.3\,m$

$t^2_{stop} = V_2(t) = 0$: $t = \frac{V_2}{a} = 4.44$

$X^2_{final} = X_2 - V_2 t + \frac{a}{2} t^2 = X_2 - \frac{V_2^2}{2a} = 60 - \frac{(80/36)^2}{2 \cdot 5} = 10{,}6\,m$

t_{crash} : $X_2(t) = X_1^{final}$

$X_2 - V_2 t + \frac{a}{2} t^2 = \frac{V_1^2}{2a}$: $t = \frac{V_2}{a}\left(1 \pm \sqrt{1 - \frac{2a(V_2 - V_1^2/2a)}{V_2^2}}\right) = \underline{\underline{3.61\,s}}$

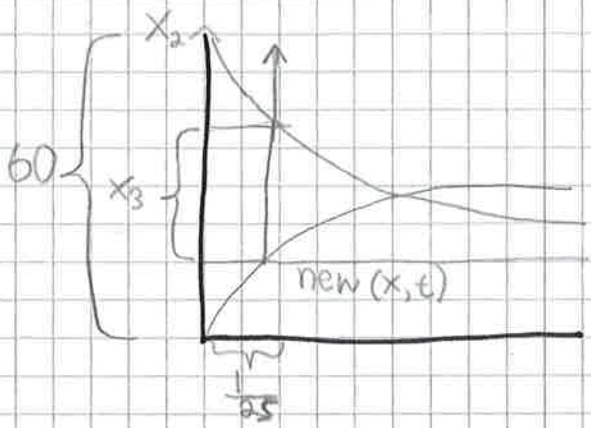
b)

$X_3 = X_2 - \frac{1}{2}s(80 + 40\,km/h)\ 43.3\,m$

$t^1_{stop} = 2.22\,s$

$t^2_{stop} = 4.44\,s$

crash?



$X_1(t) = X_2(t)$

$V_1 t - \frac{a}{2} t^2 = X_3 - V_2 t + \frac{a}{2} t^2$

$X_3 - (V_2 + V_1)t + at^2 = 0$

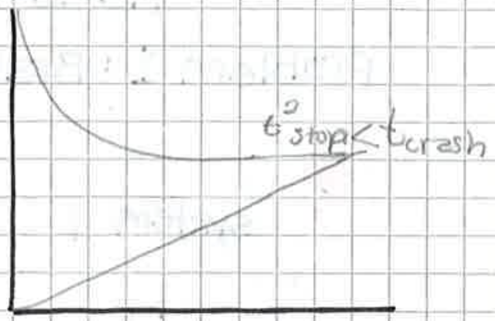
$t = \frac{V_2 + V_1}{2a}\left(1 \pm \sqrt{1 - \frac{4aX_3}{(V_1 + V_2)^2}}\right) = \underline{\underline{1.77\,s}}$

$V_{crash} = V_1(t_{cr}) - V_2(t_{cr})$

$V_1 + V_2 - 2at_{crash} = 56.3\,km/h$

c) $t^{2}_{stop} = \frac{V_2}{a}$

$$X^{2}_{final} = X_3 - \frac{V_2^2}{2a} > V_1 t_{crash} = \frac{V_1 V_2}{a}$$



$$X_3 - \frac{V_2^2}{2a} > \frac{V_1 V_2}{a}$$

$$X_3 > \frac{V_1 V_2}{a} + \frac{V_2^2}{2a}$$

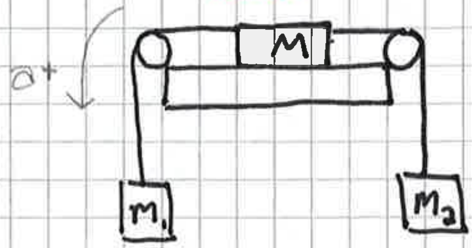
$$a > \frac{V_1 V_2 + V_2^2/2}{X_3} = \frac{(40 \cdot 80 + 80^2/2)/36^2}{43{,}3} = \underline{\underline{11{,}4 \ m/s^2}}$$

14.11.16

Hand-in 2 - Solutions

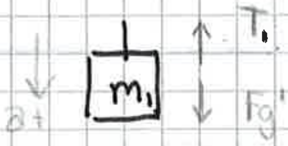
Problem 1 : Braking an elevator

25%



a)

system 1



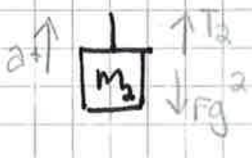
$m_1 a = m_1 g - T_1$

Boundary: $a_1 = a_2 = a_3$

$T_1 = T_1'$

$T_2 = T_2'$

system 2



$m_2 a_2 = T_2 - m_2 g$

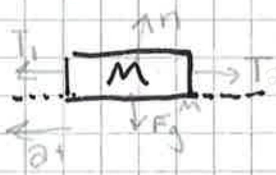
$m_1 a = mg - T_1$

$m_2 a = T_2 - m_2 g$

$Ma = T_1 - T_2$

$(m_1 + m_2 + M)a = (m_1 - m_2)g$

system 3



$$a = \frac{m_1 - m_2}{m_1 + m_2 + M} g$$

$Ma_3 = T_1' - T_2'$

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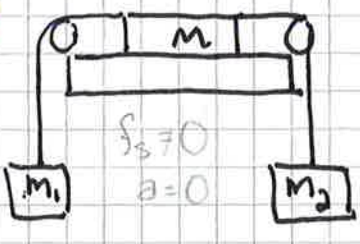
b)



$m_1 > m_2$

system 1	system 2	system 3
$0 = m_1 g - T_1$	$0 = T_2 - mg$	$0 = T_1 - T_2 - S_s =$
		$T_1 - T_2 - \mu_s \lvert n \rvert$
		→ limiting case

$m_1 g - m_2 g - \mu_s |n| = 0$

$(m_1 - m_2)g = \mu_s Mg \rightarrow (m_1 - m_2) \leq \mu_s M$

$\rightarrow (m_2 - m_1) \leq \mu_s M$

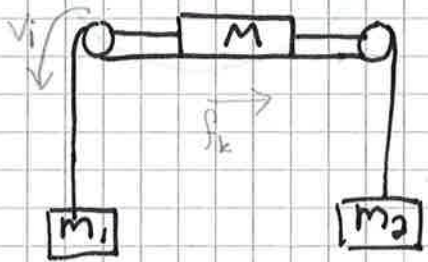
$|(m_1 - m_2)|$

100kg

c) $V_1 = 2m/s$ time to rest

$\mu_k = 0.7$ 1-D: $v(t) = V_1 - |a|t$

$m_2 = m_1 = 100kg$ $t_{stop} = \frac{V_1}{a}$



1. $m_1 a = m_1 g - T_1$
2. $m_2 a = T_2 - m_2 g$
3. $Ma = T_1 - T_2 - \mu_k |n| = T_1 - T_2 - \mu_k M g$

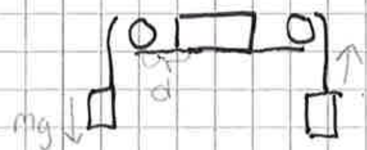
$(m_1 + m_2 + M)a = (m_1 - m_2 - \mu_k M)g$

$$a = \frac{m_1 - m_2 - \mu_k M}{m_1 + m_2 + M} g$$

$$= \frac{-70}{300} g$$

$$t_{stop} = \frac{2}{\frac{70}{300} g} = \underline{\underline{0.875 s}}$$

NB! Caveat (complicated way)



$W = \int F dx$

$\mu_k |n| d$

problem 2: Big Brother 25%

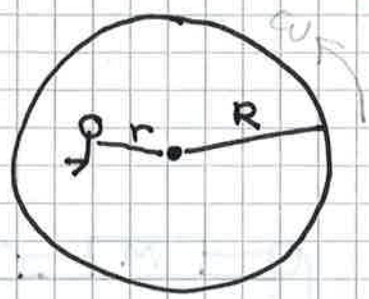
a) need F_c: $|F_c| = m\frac{v^2}{r} = mr\omega^2$

$|S_s| = \mu_s |n| = \mu_s mg$

$\rightarrow mr\omega^2 = \mu_s mg$

$\rightarrow \omega = \sqrt{\frac{\mu_s g}{r}}$ or less

$\omega = \sqrt{\frac{0.35 \cdot g}{1.5}} = \underline{1.51 \text{ rad/s}}$



b) now $\omega \rightarrow 2$ rad/s

$r \rightarrow$ to be safe?

$mr\omega^2 = \mu_s mg$

$r = \frac{\mu_s g}{\omega^2}$ or less

$\frac{0.35 \cdot g}{2^2} = 0.858 \text{ m}$

c) constant force $\rightarrow$ constant τ $\rightarrow$ constant α

$\vec{F} = \vec{F_c} + \vec{F_t}$

$mr\omega^2(t) + m\underbrace{a_t}_{mr\alpha}$

$|F| = |S_s|$

$|F| = \sqrt{(mr\omega)^2 + (mr\alpha)^2} = \mu_s mg$

largest at

largest α

$(mr\omega^2)^2 + (mr\alpha)^2 = (\mu_s mg)^2$

$r^2\alpha^2 = (\mu_s g)^2 - (r\omega^2)^2$

$\alpha_{max} = \sqrt{\left(\frac{\mu_s g}{r}\right)^2 - \omega^4} = \sqrt{\left(\frac{0.35 g}{0.5}\right)^2 - 2^4}$

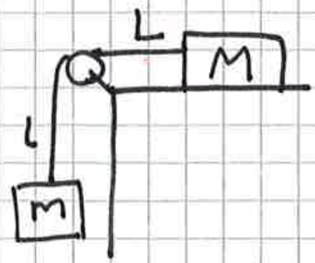
t_{stop}: $\omega(t) = \omega_i - |\alpha| t$

$t_{stop} = \frac{\omega_i}{\alpha_{max}}$ $\underline{0.359 \text{ s}}$

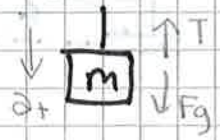
problem 3 : Chuck Norris 25%

a)

$m = 80kg$
$M = 200kg$
$L = 20m$
$l = 5m$



system 1



$ma = mg - T$

system 2

$Ma = T$

$Ma = T$

$\underline{ma = mg - T}$

$(m+M)a = mg$

$a = \frac{mg}{m+M} = \frac{80}{280}g \approx 2.80\, m/s^2$

$\frac{1}{2}at^2 = L$

$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2 \cdot 20}{a}} = \underline{\underline{3.78s}}$

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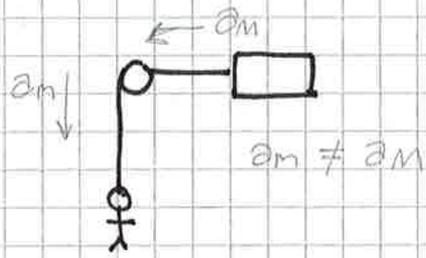
b) "boundary"



$\frac{1}{2}a_m t^2 = -l$

$\frac{1}{2}a_M t^2 = L$

$\frac{a_m}{a_M} = \frac{-l}{L} = \frac{-5}{20} = -\frac{1}{4}$

$a_m = -\frac{a_M}{4}$

$m a_m + M a_M = mg$

$m(-\frac{1}{4}a_M) + M a_M = mg$

$a_M = \frac{mg}{M - m/4} = \frac{80}{200-20}g$

$= 4.36\, m/s^2$

$a_m = \underline{\underline{1.09}}\, m/s^2$

system 1

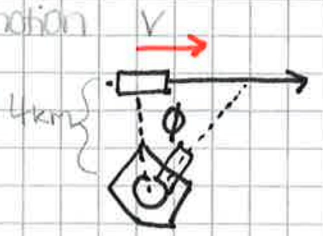
$m a_m = mg - T$

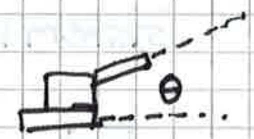
system 2

$M a_M = T$

Problem 4 : Tesla - Targeting 30%

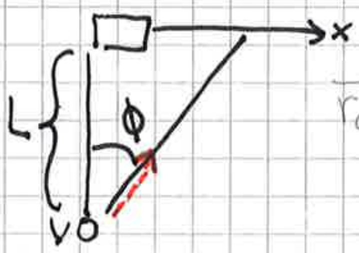
a) Kinematics + projectile motion





$\bar{r}_c(t)$

$\bar{r}_p(t)$



$$\bar{r}_c(t) = \begin{pmatrix} V_c t \\ L \\ 0 \end{pmatrix}$$

$$\bar{r}_p(t) = \begin{pmatrix} V_x t \\ V_y t \\ V_z t - \frac{g}{2} t^2 \end{pmatrix} \rightarrow \text{3 equations} \quad t, \theta, \phi$$

$V_z = V_p \sin\theta$

$V_{xy} = V_p \cos\theta$

$V_y = V_p \cos\theta \cos\phi$

$V_x = V_p \cos\theta \sin\phi$

Hit

for same t

$\bar{r}_c(t) = \bar{r}_p(t)$

$X: \quad V_c t = V_x^p t = V_p \cos\theta \sin\phi \, t$

$y: \quad L = V_y^p t = V_p \cos\theta \cos\phi \, t$

$z: \quad 0 = V_z^p t - \frac{g}{2} t^2 = V_p \sin\theta t - \frac{g}{2} t^2$

$\frac{x_c}{y_c} = \frac{x_p}{y_p}$

$\downarrow$

$\frac{V_c t}{L} = \frac{V_p t \cos\theta \sin\phi}{V_p t \cos\theta \cos\phi}$

$= \tan\phi$

$0 = \left(V_p \sin\theta - \frac{g}{2} t\right) t$

$t = \frac{2 V_p \sin\theta}{g}$

$A \boxed{\frac{2 V_c V_p}{gL}} \sin\theta = \tan\phi$

$\frac{2 \cdot 40 \cdot 800}{g \cdot 4000} \quad \frac{16}{g}$

$V_c t = V_p \cos\theta \sin\phi \, t$

$\frac{1}{20} = B \boxed{\frac{V_c}{V_p}} = \cos\theta \sin\phi$

$A \boxed{\frac{2 V_c V_p}{gL}} \sin\theta = \frac{\sin\phi}{\cos\phi}$ $\tan\phi$

$X = \sin\phi \rightarrow \cos\phi = \sqrt{1 - x^2}$

$Y = \sin\theta \rightarrow \cos\theta = \sqrt{1 - y^2}$

$B = X\sqrt{1 - y^2} \rightarrow B^2 = x^2(1 - y^2)$

$Ay = \frac{x}{\sqrt{1 - x^2}}$ $\quad A^2 y^2 = \frac{x^2}{1 - x^2}$

$(1 - x^2) A^2 y^2 = x^2$

$\downarrow$

$$A^2y^2 - x^2A^2y^2 = x^2$$

$$Ay^2 = x^2 + x^2A^2y^2$$

$$\frac{Ay^2}{1+A^2y^2} = x^2$$

$$\frac{B^2}{1-y^2} = \frac{A^2y^2}{1+A^2y^2}$$

$$B^2 + A^2B^2y^2 = A^2y^2 - A^2y^4$$

$$A^2y^4 + (A^2B^2 - A^2)y^2 + B^2 = 0$$

$$A^2(y^2)^2 + (A^2B^2 - A^2)y^2 + B^2 = 0$$

$$y^2 = z$$

$$A^2z^2 + A^2(B^2-1)z + B^2 = 0$$

$$z = \frac{A^2(1-B^2)}{2A^2}\left(1 \pm \sqrt{1 - \frac{4A^2B^2}{(A^2(B^2-1))^2}}\right.$$

$$A = \frac{16}{9}, \quad B = \frac{1}{20}$$

$$\sqrt{} \rightarrow y = \sin\theta$$

$$z_- = y_-^2 = 0.0307 \rightarrow \theta^- = 1.76^\circ$$

$$z_+ = y_+^2 = 0.998 \rightarrow \theta^+ = 86.6^\circ$$

$$\phi^- = 2.87^\circ$$

$$\phi^+ = 58.5^\circ$$

b) tim for impact

$t = \frac{2V_pD}{g} \sin\theta$

$\left(V_p^{-} t - \frac{g}{2} t^2 = 0\right)$

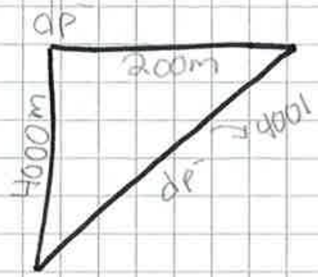
$t_{-} = 5.01\,s$

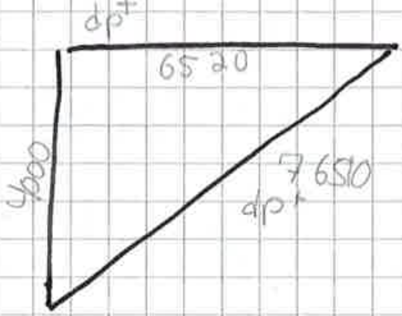
$t_{+} = 163\,s$

c) How far car

$d_c^{-} = V_c t^{-} = 200\,m$

$d_c^{+} = V_c t^{+} = 6520\,m$





d) projectile height

$z_p^{max}: V_z - gt = 0 \rightarrow t_{max} = \frac{V_z}{g}$

$z_p^{max}: = V_z t - \frac{g}{2} t^2 = \frac{V_z^2}{2g} = \frac{V_p^2 \sin^2\theta}{2g}$

$z_p^{max-} = \underline{\underline{30,7\,m}}$

$z_p^{max+} = \underline{\underline{32.5\,km}}$

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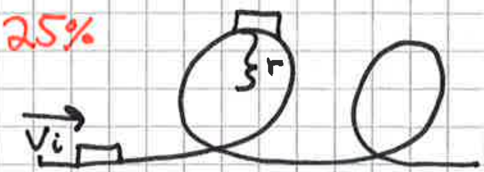
Hand-in 3 - Solutions

Problem 1: loopy-looper 25%

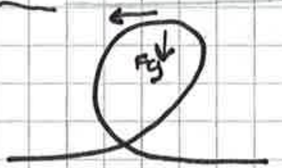


a) $K_i = \frac{1}{2} M v_i^2$ $U_g^i = 0$

$K_f = 0$ $U_g^f = Mg\,2r \rightarrow Mg\,2r = \frac{1}{2} M v_i^2$

$v_i = \sqrt{4rg}$

b) $K_i = \frac{1}{2} M v_i^2$ $K_f = \frac{1}{2} M v_f^2$



$F_g = F_c$

$V_g^i = 0$ $V_g^f = Mg\,2r$

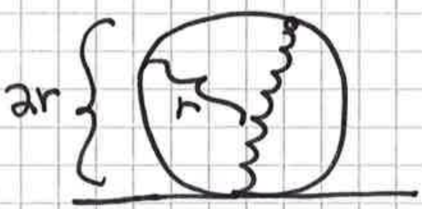
$Mg = M \frac{V_f^2}{r}$

$\rightarrow V_f^2 = rg$

$\frac{1}{2} M v_i^2 = \frac{1}{2} M rg + Mg\,2r \cdot 2 = 5rg$

$v_i = \sqrt{5rg}$

problem 2



12

$K_i = 0$

$V_g^i = mg\,2r$

$V_s^i = \frac{1}{2} k (2r)^2$

3

$K_3 = \frac{1}{2} m V_3^2$

$V_g^3 = mgr$

$V_s^3 = \frac{1}{2} k\,2r^2$

6

$K_6 = \frac{1}{2} m V_6^2$

$V_g^6 = 0$

$V_s^6 = 0$

a) $\frac{1}{2} m V_3^2 + mgr + \frac{1}{2} k\,2r^2 = mg\,2r + \frac{1}{2} k\,4r^2$

c) $V_3^2 = \frac{2(mgr + kr^2)}{m} \rightarrow V_3 = \sqrt{2gr + 2\frac{k}{m} r^2}$

b) $\frac{1}{2} m V_6^2 = mg\,2r + \frac{1}{2} k\,4r^2$

$V_6 = \sqrt{4rg + 4\frac{k}{m} r^2}$

problem 3 : Teeter toy 25%

a) $d_1(\theta)$

$d_2(\theta)$

$L\cos\theta = d$

$U_g = mgd_1 + mgd_2 = mg(d_1(\alpha) + d_2(\theta))$

$U_g(\theta) = mg(L\cos\theta - L_1 + L\cos\theta - L_2)$

$= mg(2L\cos\theta - (L\sin\beta + L\sin\gamma))$

$= mg\left[2L\cos\theta - L\left(\sin[90° - (\alpha - \theta)] + \sin[90° - (\alpha + \theta)]\right)\right]$

$d_1 = d - L_1$

$(L_1 = d + d_1)$

$L_2 = d - d_2$

$d_1 = L\cos\theta - L_1$

$d_2 = L\cos\theta \, L_2$

$180° = \gamma + \alpha + \alpha + \beta$

$90° = \theta + \alpha + \gamma$

$L_2 = L\sin\gamma$

$L_1 = L\sin\beta$

$\gamma = 90° - (\alpha + \theta)$

$\beta = 180° - (\gamma + 2\alpha)$

$\beta = 180° - (90° - (\alpha + \theta) + 2\alpha)$

$\beta = 90° + \theta - \alpha$

$= 90° - (\alpha - \theta)$

$\sin(90 - x) = \cos x$

$U_g(\theta) = mg\left[2L\cos\theta - L\left[\cos(\alpha - \theta) + \cos(\alpha + \theta)\right]\right]$

$\cos\alpha\cos\theta - \sin\alpha\sin(-\theta) + \cos\alpha\cos\theta$

$-\sin\alpha\sin\theta$

$+\sin\alpha\sin\theta$

$mg\left[2L\cos\theta - L\,2\cos\theta\cos\alpha\right]$

$U(\theta) = 2mg\cos\theta\left[L - L\cos\alpha\right]$

b)

Equilibrium:

$F = -\dfrac{dU(\theta)}{d\theta} = 0 : \dfrac{dU}{d\theta} = -2mg\sin\theta\left[L - L\cos\alpha\right]$

stable: $L - L\cos\alpha < 0$

$\dfrac{d^2U}{d\theta^2} > 0 \quad \dfrac{d^2U}{d\theta^2} = -2mg\cos\theta\left[L - L\cos\alpha\right] \overset{?}{>} 0$

Masses at $\theta = 0 \rightarrow$ below support part

Problem 4: Sleigh-Ride

a)

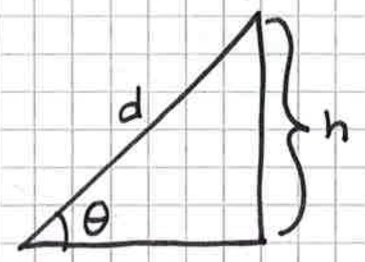
$W \rightarrow U_g + E_{int}$

$U_g \leftarrow mgh$

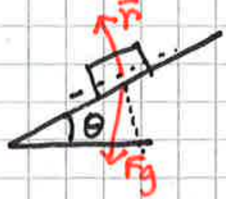
$E_{int} \leftarrow F_d = \int F dr$

$F \leftarrow \mu_k |n| \rightarrow \mu_k mg\cos\theta$

$d \rightarrow \frac{h}{\sin\theta}$



$h = d\sin\theta$



$F_g\cos\theta = |\vec{n}|$

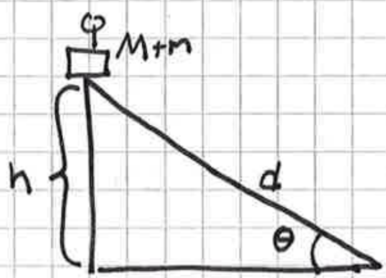
$W_{30°} = 9.20\,kJ$

$W_{15°} = 10.8\,kg$??

$W = mgh + \mu_k mgh\frac{\cos\theta}{\sin\theta} = mgh\left(1 + \frac{\mu_k}{\tan\theta}\right)$

$U_g = 7.84\,kJ$

b)



$U_g^i = (m+M)gh$

$K^i = 0$

$U_g^i = K^f + E_{int}$

$E_{int} \leftarrow F_k d = \mu_k |n| \frac{h}{\sin\theta}$

$(m+M)gh = \frac{1}{2}(m+M)V_f^2 + \frac{\mu_k(m+M)gh}{\tan\theta}$

$|\vec{n}| = mg\cos\theta = (m+M)g\cos\theta$

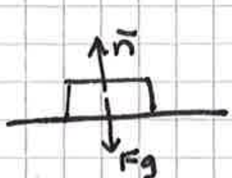
$gh = \frac{1}{2}V_f^2 + \frac{\mu_k gh}{\tan\theta}$

$V_f = \sqrt{2gh\left(1 - \frac{\mu_k}{\tan\theta}\right)}$

$V_f^{30} = 40.3\,m/s$

$V_f^{15} = 35.1\,m/s$

c)



$U_i^g = E_{int}$ (down hill) $+ E_{int}$ (Horizontal)

$\frac{\mu_k(m+M)gh}{\tan\theta}$

$F_k L$

$\mu_k |n| L$

$\mu_k (m+M)gL$

$(m+M)gh = \mu_k\frac{(m+M)gh}{\tan\theta} + \mu_k(m+M)gL \rightarrow L = \frac{h - \frac{\mu_k}{\tan\theta}h}{\mu_k}$

$= \frac{h\left(1 - \frac{\mu_k}{\tan\theta}\right)}{\mu_k}$

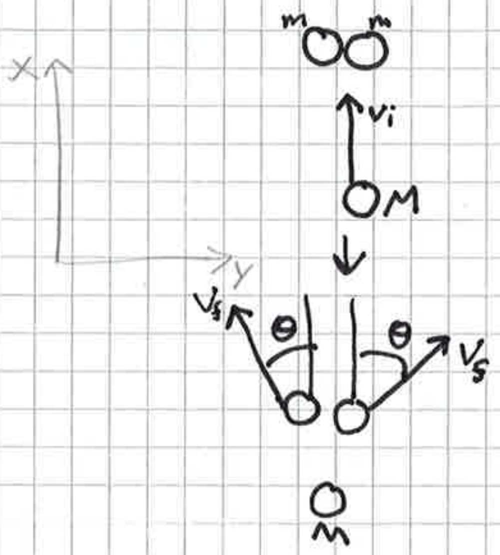
$L^{30} = 827\,m$

$L^{15} = 627\,m$

Hand-in 4 Solutions

18.11.16

Problem 1: Double Score 20%



a) Momentum:

$x \rightarrow M v_i + 0 = 2 m v_f \cos\theta$

$0 = m v_f \sin\theta + m v_f (-\sin\theta)$

Energy:

$\frac{1}{2} M v_i^2 = 2\left(\frac{1}{2} m v_f^2\right)$

$\rightarrow v_f = \sqrt{\frac{M}{2m}}\, v_i$

$\rightarrow \cos\theta = \frac{M}{2m}\frac{v_i}{v_f} = \frac{M}{2m}\sqrt{\frac{2m}{M}} = \sqrt{\frac{M}{2m}}$

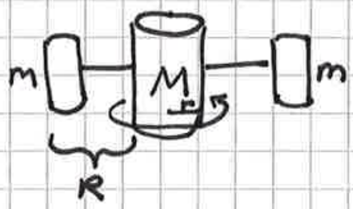
b) $\cos\theta = \sqrt{\frac{M}{2m}}$

$\rightarrow \frac{M}{2m} < 1 \; ; \; M \leq 2m$

M too big, M cannot be at rest afterwards.

Problem 2: Rotating Fitness 20%

a) Angular momentum



$$|\vec{L}| = I\omega$$

$$I_i \omega_i = I_s \omega_s \qquad T = 1\,s/rev$$

$$I_i \frac{2\pi}{T_i} = I_s \frac{2\pi}{T_s} \qquad \frac{2\pi}{\omega_i} = T_i$$

$$T_s = \frac{I_s}{I_i} T_i \qquad \omega_i = \frac{2\pi}{T_i}$$

$$\omega_s = \frac{2\pi}{T_s}$$

$r = 30\,cm$

$R = 75\,cm$

$m = 5\,kg$

$M = 80\,kg$

$T_i = 1\,s$

$$I_i = \frac{1}{2}Mr^2 + 2(mR^2)$$

$$I_s = \frac{1}{2}Mr^2 + 2(mr^2)$$

$$T_s = T_i \frac{\frac{1}{2}Mr^2 + 2mr^2}{\frac{1}{2}Mr^2 + 2mR^2} = T_i\left(\frac{1 + 4\frac{m}{M}}{1 + 4\frac{m}{M}\left(\frac{R}{r}\right)^2}\right) = \underline{\underline{0.488\,s}}$$

b)

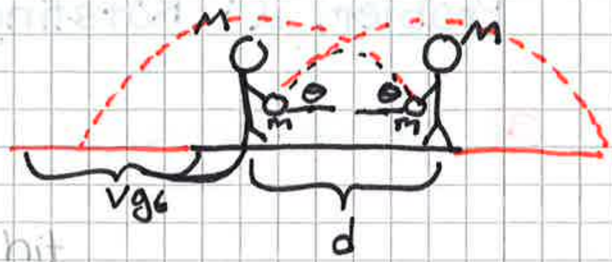
$$W = E_s - E_i = \frac{1}{2} I_s \omega_s^2 - \frac{1}{2} I_i \omega_i^2$$

$$= \frac{1}{2} I_i \omega_i^2 \left(\frac{I_s}{I_i}\frac{\omega_s^2}{\omega_i^2} - 1\right) = \frac{1}{2} I_i \omega_i^2 \left(\frac{I_i}{I_s} - 1\right)$$

$$\frac{I_s}{I_i}\left(\frac{I_i}{I_s}\right)^2$$

$$\frac{1}{2}\left(\frac{1}{2}Mr^2 + 2mR^2\right)\left(\frac{2\pi}{T_i}\right)^2\left(\frac{\frac{1}{2}Mr^2 + 2mR^2}{\frac{1}{2}Mr^2 + 2mr^2} - 1\right) = \underline{\underline{191\,J}}$$

Problem 3:



$\underline{\underline{\bar{P}}}$ $\quad MV_s = mV\cos\theta$

a) Projectile: y: $\quad V\sin\theta t - \frac{g}{2}t^2 = 0$ $\quad \underline{\underline{\text{hit}}}$

x: $\quad V\cos\theta\, t = d + V_s t$

$$V\sin\theta t - \frac{g}{2}t^2 = 0$$

$$V\sin\theta - \frac{g}{2}t = 0 \rightarrow t = \frac{2V}{g}\sin\theta$$

$$V\cos\theta\, t = d + V_s t$$

$$t = \frac{d}{V\cos\theta - V_s} \longrightarrow \frac{d}{V\cos\theta - \frac{m}{M}V\cos\theta}$$

$$MV_s = mV\cos\theta \rightarrow V_s = \frac{m}{M}V\cos\theta$$

$$t = \frac{d}{V\cos\theta\left(1 - \frac{m}{M}\right)} = \frac{2V\sin\theta}{g}$$

$$\frac{dg}{V^2\left(1-\frac{m}{M}\right)} = 2\sin\theta\cos\theta = \sin 2\theta$$

$$\theta = 14.8^\circ$$

$$\theta = 75.2^\circ$$

$d = 5m$
$g = 9.8\,m/s^2$
$V = 10\,m/s$
$m = 0.6$
$M = 75$

b) $d_f = d + 2V_s t$

$$= 2(d + V_s t) - d$$

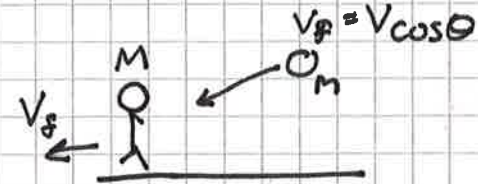
$$= 2V\cos\theta t - d$$

$$t = \frac{2V}{g}\sin\theta$$

$$= \frac{4V^2}{g}\sin\theta\cos\theta - d$$

Both angles: $\underline{\underline{5{,}08m}}$

c) $MV_s + mV\cos\theta = (M+m_s)V_{after}$



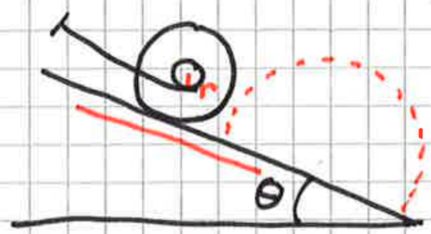
$$V_{after} = \frac{MV_s + mV\cos\theta}{M+m}$$

$$= \frac{2mV\cos\theta}{M+m} = 0{,}153 \text{ m/s}$$

$0{,}0405$ m/s

Problem 4: yo-yo

a) it must slip!



$s = r\phi$

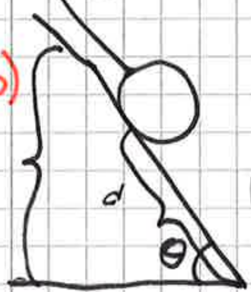
$s = R\phi$

$\neq$ if $r \neq R$

$$\phi = \frac{s}{r} \neq \frac{s}{R}$$

energy

b)



$h = d\sin\theta$

$U_g = (2M+m)g\,d\sin\theta$

$= \frac{1}{2}(2M+m)V_s^2 + \frac{1}{2}I_{CM}\omega_s^2$

$$I_{CM} = \frac{1}{2}mr^2 + 2\left(\frac{1}{2}MR^2\right)$$

$s = r\theta \rightarrow V_s = r\omega_s$

$$(2M+m)g\,d\sin\theta = \frac{1}{2}(2M+m)V_s^2 + \frac{1}{2}\frac{I_{CM}}{r^2}V_s^2$$

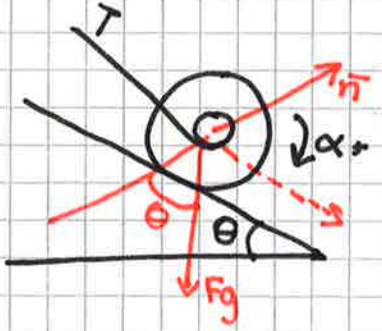
$$V_s = \sqrt{\frac{2g\,d\sin\theta}{1 + \frac{I_{CM}}{(2M+m)r^2}}}$$

c) $|\vec{n}| = (2M+m)g\cos\theta$

$(2M+m)a = (2M+m)g\sin\theta - T$

$\tau = Tr = I\alpha$

$\rightarrow T = \frac{I}{r^2}a$



$\rightarrow \left((2M+m) + \frac{I}{r^2}\right)a = (2M+m)g\sin\theta$

$a = \frac{g\sin\theta}{1 + \frac{I}{(2M+m)}} \rightarrow \alpha \rightarrow \frac{a}{r}$

$T = \frac{I}{r^2}a$

1-D Kinematics

$\frac{1}{2}at^2 = d$, $V = at$

$t = \sqrt{\frac{2d}{a}} \rightarrow V = \sqrt{2ad}$

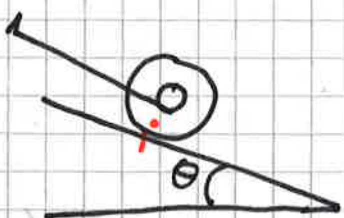
$V = \sqrt{2ad}$

$= \sqrt{\frac{2dg\sin\theta}{(1 + I/\quad)}}$

d) μ_k $\quad f_k = \mu_k|\vec{n}| = \mu_k(2M+m)g\cos\theta$

$E_{int} \cdot \mu_k(2M+m)g\cos\theta\, l$

$U_i = (2M+m)gd\sin\theta$



$= \frac{1}{2}(2M+m)V_f^2 - \frac{1}{2}I_{CM}\omega_f^2 + E_{int}$

$\mu_k(2M+m)g\cos\theta\left(\frac{R}{r}+1\right)d$

$V_f = \omega_f \cdot r$

$\frac{1}{2}(2M+m)gd\sin\theta - \mu_k(2M+m)gd\cos\theta\left(\frac{R}{r}-1\right)$

$V_f = \sqrt{\frac{2gd\left(\sin\theta - \mu_k\cos\theta\left(\frac{R}{r}-1\right)\right)}{1 + \frac{I_{CM}}{(2M+m)r}}}$

only spinning:

$l = R\phi$

only rolling

$L = 0$

V relative surface:

$R\omega - r\omega$

$= (R-r)\omega = \left(\frac{R}{r}-1\right)r\omega$

$l = \left(\frac{R}{r}-1\right)d$