(I)

EXAM PET 550 — Nat-Gas processing & Transportation (Continuation/resit)

August 20 th., 2015

SUGGESTED ANSWERS

EX1 a) Given: $du = Tds - pdv$

$a = u - Ts$

E.g: $\frac{da}{dv} = \frac{du}{dv} - \frac{d}{dv}(T \cdot s) = \frac{du}{dv} - T\frac{ds}{dv} - s\frac{dT}{dv}$

Also: $\frac{du}{dv} = T\frac{ds}{dv} - p\frac{dv}{dv}$ (since T is const. AND differentials can be treated as any algebraic item)

combined: $\frac{da}{dv} = T\frac{ds}{dv} - p\frac{dv}{dv} - T\frac{ds}{dv} - s\frac{dT}{dv}$

cancel

$\Rightarrow \frac{da}{dv} = -p \quad (T = \text{const.})$ $= 1$ pr. def. $= 0$ because $T = $ const

QED

b) Ideal gas: $p^* = \frac{RT}{v}$; function of v only at const. T

$$\Delta a^* = -\int_{v_1^*}^{v_2^*} p^*(v)\, dv$$

$$\Rightarrow \Delta a^* = -\int_{v_1^*}^{v_2^*} \frac{RT}{v}\, dv = -RT\left[\ln v\right]_{v_1^*}^{v_2^*}$$

$$= RT \ln\left(\frac{v_1^*}{v_2^*}\right)$$

c) $$(a^* - a) = \int_{v^*}^{\infty} \frac{RT}{v}\, dv - \int_{v}^{\infty} p\, dv$$

1: First term corresponds to $-RT\ln\left(\frac{v_1^*}{v_2^*}\right)$ with $v_1^* = v^*$ and $v_2^* = \lim_{v\to\infty} v$

$$\Rightarrow \int_{v^*}^{\infty} \frac{RT}{v}\, dv = RT\left(\lim_{v\to\infty} \ln v - \ln v^*\right)$$

2: Second term is given, i.e. since $da = -p\,dv$, then $\Delta a = -\int_{v_1}^{v_2} p(v)\,dv$, with $v_1 = v$ and $v_2 = \lim_{v\to\infty} v$

$$\Rightarrow -\int_{v}^{\infty} p\, dv = RT\left(\ln(v-\beta) - \lim_{v\to\infty} \ln(v-\beta)\right)$$

$$+ \alpha\left(\frac{1}{v} - \lim_{v\to\infty}\frac{1}{v}\right)$$

3: Regroup: $$(a^* - a) = RT\left(-\ln v^* + \ln(v-\beta)\right)$$

$$+ \frac{\alpha}{v} + RT \lim_{v\to\infty}\left(\ln v - \ln(v-\beta)\right) - \alpha \lim_{v\to\infty}\frac{1}{v}$$

$$\lim_{v \to \infty} \ln\left(\frac{v}{v-\beta}\right) \to 0 \quad \left(\text{because } \frac{v}{v-\beta} \to 1\right)$$

$$\lim_{v \to \infty} \frac{1}{v} \to 0$$

$\Rightarrow$ We are left with:

$$(a^* - a) = RT \ln\left(\frac{v-\beta}{v^*}\right) + \frac{\alpha}{v}$$

EX2 a) From the diagram: $X = 0.17 \Rightarrow Y = 0.35$

Mass- and component balance ($X = X_{C3} = 1 - X_{nC4}$)

$$F = V + L$$

$$F \cdot Z = V \cdot Y + L \cdot X \quad \Rightarrow \quad Z = \frac{V}{F} Y + \frac{L}{F} X$$

$$L = F - V \Rightarrow \frac{L}{F} = \frac{F-V}{F} = 1 - \frac{V}{F}$$

$$\Rightarrow Z = \frac{V}{F} \cdot Y + \left(1 - \frac{V}{F}\right) \cdot X = 0.7 \cdot 0.35 + (1 - 0.7) \cdot 0.17$$

$$= 0.296$$

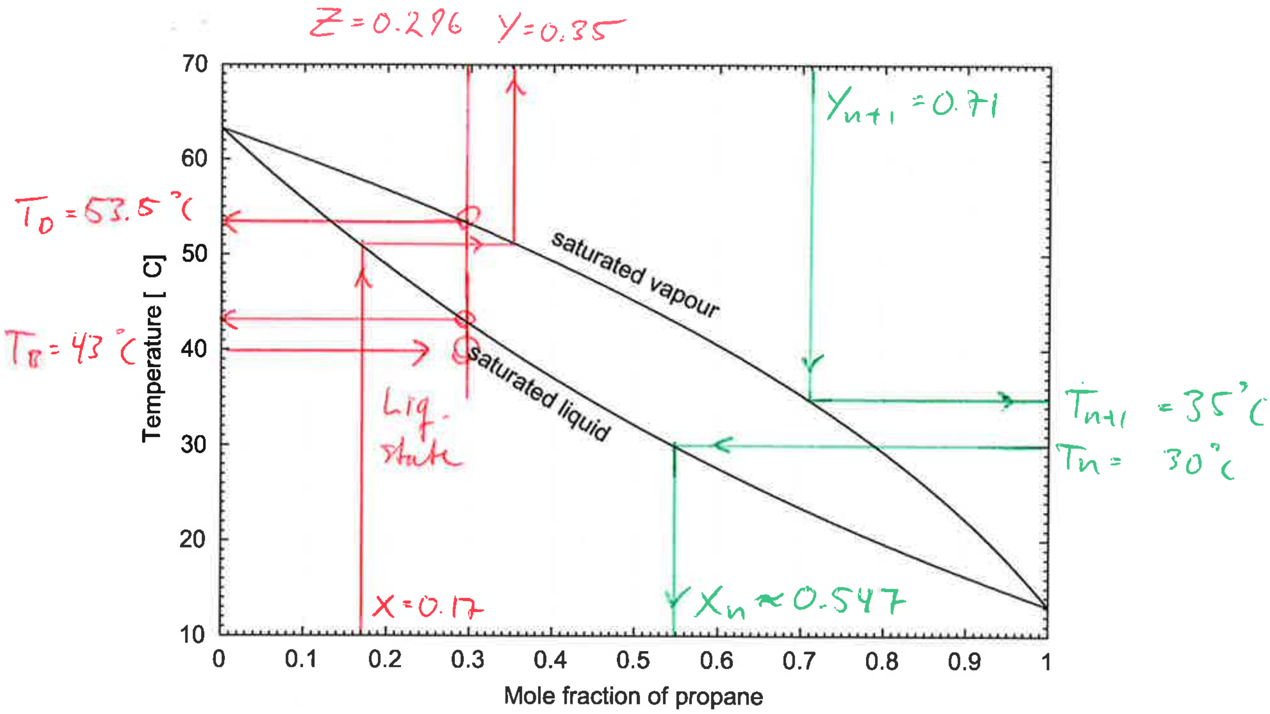


Figure 1: Binary phase diagram ($T - x - y$) for n-butane and propane at $p = 7.5$ bar.

b) I : $T_B = 43.0$ °C , $T_D = 53.5$ °C

II : Liquid state

III : More, since the "end points" (i.e. pure comp.)
– bubble point curves are shifted towards higher temperatures at higher pressure, the diagram will be shifted upwards by raising the pressure.

c) I Since the temperature at tray $n = 30$°C, and the liquid coming from there is at the boiling point, then from diagram $X_n \approx 0.547$

Mass- and component balance for the top-section:

$$V_{n+1} = L_n + D$$

$$V_{n+1} \cdot y_{n+1} = L_n \cdot x_n + D \cdot x_D \Rightarrow \frac{V_{n+1}}{D} y_{n+1} = \frac{L_n}{D} x_n + x_D$$

$$\frac{V_{n+1}}{D} = \frac{L_n + D}{D} = \frac{L_n}{D} + 1$$

$$L_n = L \Rightarrow \frac{L_n}{D} = R \Rightarrow (R+1)\, y_{n+1} = R x_n + x_D$$

$$\Rightarrow y_{n+1} = \frac{R}{R+1} x_n + \frac{1}{R+1} x_D = \frac{1.75}{2.75} \cdot 0.547 + \frac{0.995}{2.75}$$

$$= \underline{\underline{0.71}}$$

II This vapour is at the dew point at tray n+1, so from the diagram: $\underline{\underline{T_{n+1} \approx 35\,°C}}$

Ex. 3 a) $\rho u = \frac{\dot{m}}{A}$, $\rho = \frac{p}{ZRT}$

E.g.: $\rho u^2 = \frac{(\rho u)^2}{\rho} = \left(\frac{\dot{m}}{A}\right)^2 \cdot \frac{ZRT}{p}$

$$\Rightarrow -\frac{dp}{dL} = K \cdot \rho u^2 \Rightarrow dL = -\frac{1}{K \rho u^2} dp$$

$$= \underline{\underline{-\frac{A^2}{K\, ZRT\, \dot{m}^2}\, p\, dp}}$$

QED

VI

b) 1: Integrate:

$$\int_0^L dL = -\frac{A^2}{k Z R T \dot{m}^2} \int_{P_1}^{P_2} p\,dp$$

$$\Rightarrow L = -\frac{A^2}{k Z R T \dot{m}^2} \cdot \frac{1}{2}\left(P_2^2 - P_1^2\right)$$

2: Solve for $\dot{m}$:

$$\dot{m}^2 = -\frac{A^2 \cdot \left(P_2^2 - P_1^2\right)}{2 k Z R T L} \Rightarrow \underline{\dot{m} = \sqrt{\frac{\left(P_1^2 - P_2^2\right) A^2}{2 k Z R T L}}}$$

3: Insert data <u>in basic units</u>:

$$\dot{m} = \sqrt{\frac{\left[\left(137 \cdot 10^5\right)^2 - \left(85 \cdot 10^5\right)^2\right]\left(\frac{N}{m^2}\right)^2 \cdot \left(0.46\ m^2\right)^2}{2 \cdot 0.0065\ \frac{1}{m} \cdot 0.94 \cdot 455\ \frac{Nm}{kg\,K} \cdot 280.45\ K \cdot 540000\ m}}$$

$$\left(R = \frac{8.3144\ J/mole\ K}{0.63 \cdot 29\ g/mole} = 0.455\ \frac{J}{g\,K} = \underline{455\ \frac{J}{kg\,K}} \right.$$

$$\left. T = 7.3\,^\circ C + 273.15 = \underline{280.45\ K} \right)$$

$$\Rightarrow \dot{m} = \sqrt{29009\left(\frac{kg}{s}\right)^2} = \underline{\underline{170.3\ \frac{kg}{s}}}$$

c) 1. Need diameter for the Weymouth eq.

$$A = \frac{\pi}{4} D^2 \rightarrow D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \cdot 0.46}{\pi}} = \underline{0.765\ m}$$

2. Pressures in kPa:

$P_1 = 137$ bar $= 13700$ kPa

$P_2 = 85$ bar $= 8500$ "

$P_{sc} =$ 101.325 kPa

$$\Rightarrow q_{sc} = 1.185 \cdot 10^7 \cdot \frac{288.15}{101.325} \cdot \sqrt{\frac{(13700^2 - 8500^2) \cdot 0.765^{5.333}}{0.63 \cdot 540\,000 \cdot 280.45 \cdot 0.94}}$$

$$= 18.716.427\ Sm^3/d = \underline{\underline{18{,}72 \cdot 10^6\ Sm^3/d}}$$

d) Convert answer in c) to kg/s:

$$18.72 \cdot 10^6\ Sm^3/d \cdot 42\,300 \frac{kmol}{10^6\ Sm^3} = 791\,856 \frac{kmol}{d}$$

$$\cdot 18.27 \frac{kg}{kmol} = 14.467.209\ kg/d = \underline{167.4\ kg/s}$$

$$\left(M_g = \gamma \cdot M_a = 0.63 \cdot 29\ g/mol = 18.27 \frac{g}{mole} \left(\frac{kg}{kmole} \right) \right)$$

VIII

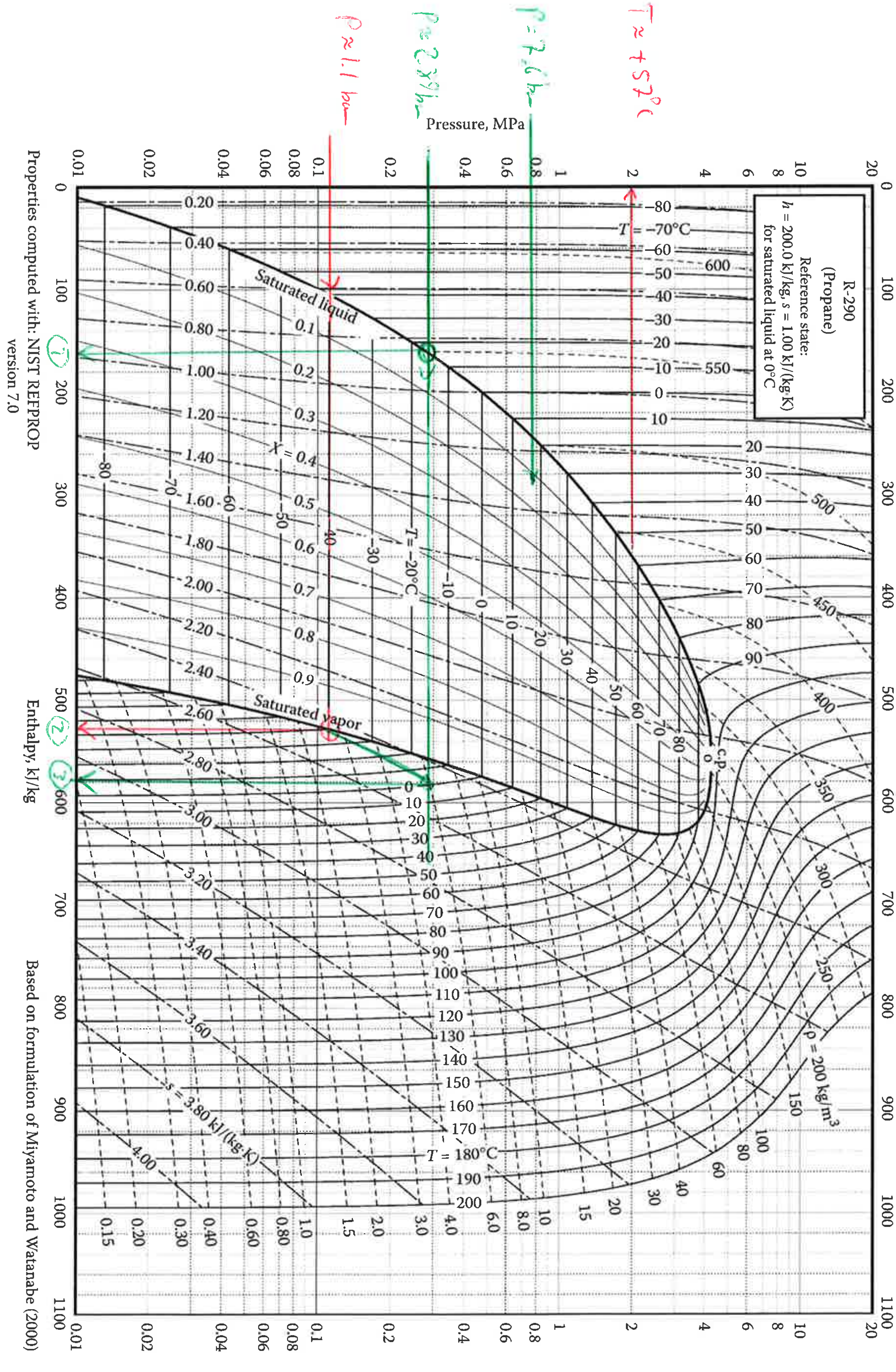
~ Deviation : $\frac{170.3 - 167.4}{167.4} = 0.017 = 1.7\%$

(might as well choose the other as reference)

Ex. 4 a)

I : $T_{min} \approx -40°C$

II : $P_{max} \approx 20$ bar



III: $\frac{P_c}{P_3} = \frac{P_3}{P_2} = \frac{P_2}{P_1} = R \Rightarrow P_3 = P_2 \cdot R$

$\Rightarrow \frac{P_c}{P_2 R} = \frac{P_2}{P_1} = R \Rightarrow P_2 = P_1 \cdot R$

$\Rightarrow \frac{P_c}{(P_1 R) \cdot R} = R \Rightarrow \frac{P_c}{P_1} = R^3 \rightarrow R = \sqrt[3]{\frac{P_c}{P_1}} = \sqrt[3]{\frac{20}{1.1}}$

$$= 2.63$$

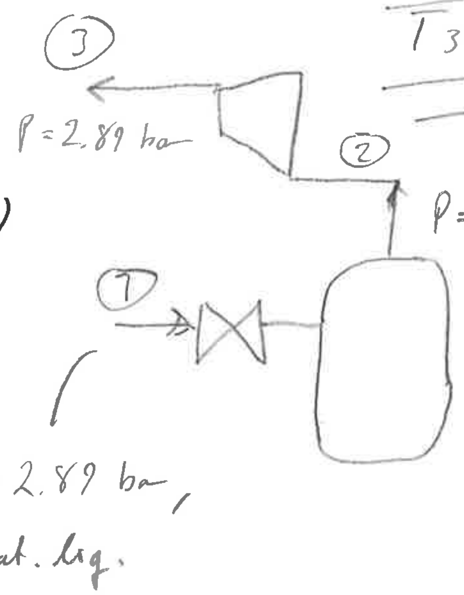
$\Rightarrow P_2 = 1.1 \cdot 2.63 = \underline{\underline{2.89 \text{ bar}}}$ (MP)

$P_3 = 2.89 \cdot 2.63 = \underline{\underline{7.60 \text{ bar}}}$ (HP)

Temperatures: $\underline{\underline{T_2 \approx -15\,°C}}$

$\underline{\underline{T_3 \approx +17\,°C}}$

b)



①: $h_1 \approx 160$ kJ/kg

②: $h_2 \approx 530$ "

③: $h_{3is} \approx 580$ " (if compression was isentropic)

Cold duty: $Q_0 = \dot{m}(h_2 - h_1) = 237 \frac{kg}{s} \cdot (530 - 160) \text{ kJ/kg}$

$$\underline{\underline{= 87\,690 \text{ kW}}}$$

X

Compression work, stage 1:

$$W_c = \dot{m}\,\frac{(h_{3is} - h_2)}{\eta_{is}} = 237 \cdot \frac{580 - 530}{0.85} = \underline{\underline{13941\ kW}}$$

c) 9% of cold duty: $\dot{Q}_{NG} = 0.09 \cdot 87690\ kW$

$$= \underline{7892.1\ kW}$$

$$\dot{Q}_{NG} = \dot{m}_{NG} \cdot \Delta h_{NG} \Rightarrow \dot{m}_{NG} = \frac{7892.1\ kW}{59\ kJ/kg}$$

$$= \underline{133.8\ kg/s}$$

$$= 133.8\,\frac{kg}{s} \cdot 3600\,\frac{s}{h} \cdot 24\,\frac{h}{d} \cdot 365\,\frac{d}{a} = 4.22 \cdot 10^9\,\frac{kg}{a}$$

$$= 4.22 \cdot 10^6\ tonnes/a = \underline{\underline{4.22\ mtpa}}$$