

# EXAM PET 550 - Natural Gas Processing and Transportation

①

August 25, 2016 (Cont.)

ANSWERS:

Ex 7 a) vdw EOS:  $p(v, T) = \frac{RT}{v-\beta} - \frac{\alpha}{v^2}$

$T = \text{const} \Rightarrow da = -p(v)dv$ ; to be integrated  
between  $v^L$  and  $v^V$  to obtain the difference  $(a^V - a^L)$ :

$$(a^V - a^L) = - \int_{v^L}^{v^V} p(v) dv = - \left[ RT \int_{v^L}^{v^V} \frac{1}{v-\beta} dv - \alpha \int_{v^L}^{v^V} \frac{1}{v^2} dv \right]$$

$$\int \frac{1}{x} dx = \ln x$$
$$\int x^n dx = \frac{1}{n+1} \cdot x^{n+1}$$

(Basic integration rules)

$$\Rightarrow \int_{v^L}^{v^V} \frac{1}{v-\beta} dv = \left[ \ln(v-\beta) \right]_{v^L}^{v^V} = (\ln(v^V-\beta) - \ln(v^L-\beta))$$
$$= \ln \frac{v^V-\beta}{v^L-\beta}$$

$$\int_{v^L}^{v^V} \frac{1}{v^2} dv = \frac{1}{-2+1} \cdot v^{-2+1} = \left[ -\frac{1}{v} \right]_{v^L}^{v^V} = \left( -\frac{1}{v^V} + \frac{1}{v^L} \right)$$

(Now watch the signs ...)

$$(a^V - a^L) = - \left( RT \cdot \ln \left( \frac{v^V-\beta}{v^L-\beta} \right) - \alpha \left( \frac{1}{v^L} - \frac{1}{v^V} \right) \right)$$
$$= \underline{RT \ln \left( \frac{v^L-\beta}{v^V-\beta} \right) + \alpha \left( \frac{1}{v^L} - \frac{1}{v^V} \right)} \quad \text{QED}$$

b) At phase equilibrium T and p are the same in both phases:

$$h = a + Ts + pv \Rightarrow (h^V - h^L) = (a^V - a^L) + T(s^V - s^L) + p(v^V - v^L)$$

We already have  $a^V - a^L$

$$\Rightarrow \frac{\partial}{\partial T} (a^V - a^L) = R \ln \left( \frac{v^L - \beta}{v^V - \beta} \right) = -(s^V - s^L)$$

$$\Rightarrow (h^V - h^L) = RT \ln \left( \frac{v^L - \beta}{v^V - \beta} \right) + \alpha \left( \frac{1}{v^L} - \frac{1}{v^V} \right) + T \cdot \left( -R \ln \left( \frac{v^L - \beta}{v^V - \beta} \right) \right) + p(v^V - v^L)$$

*cancel*

$$\Rightarrow (h^V - h^L) = \alpha \left( \frac{1}{v^L} - \frac{1}{v^V} \right) + p(v^V - v^L)$$

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c) Need to calculate  $\alpha$  first:

$$\alpha = \frac{27 \cdot (8.3144 - 369.8)^2}{64 \cdot 42.5 \cdot 10^5} \left[ \frac{\left( \frac{J}{mol \cdot K} - K \right)^2}{\frac{N}{m^2}} = \frac{Nm^4}{mol^2} \right]$$

$$= 0.9384 \frac{Nm^4}{mol^2}$$

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$$(h^V - h^L) = 0.9384 \cdot \left( \frac{1}{1.199 \cdot 10^{-4}} - \frac{1}{3.528 \cdot 10^{-3}} \right) \left[ \frac{Nm^4}{mol^2} \cdot \frac{mol}{m^3} = \frac{Nm}{mol} \right]$$
$$\left[ \frac{N}{m^2} \cdot \frac{m^3}{mol} = \frac{Nm}{mol} \right]$$

$$+ 4.89 \cdot 10^5 \cdot (3.528 \cdot 10^{-3} - 1.199 \cdot 10^{-4}) = 9723.4 \frac{Nm}{mol}$$

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Ex 2 a)

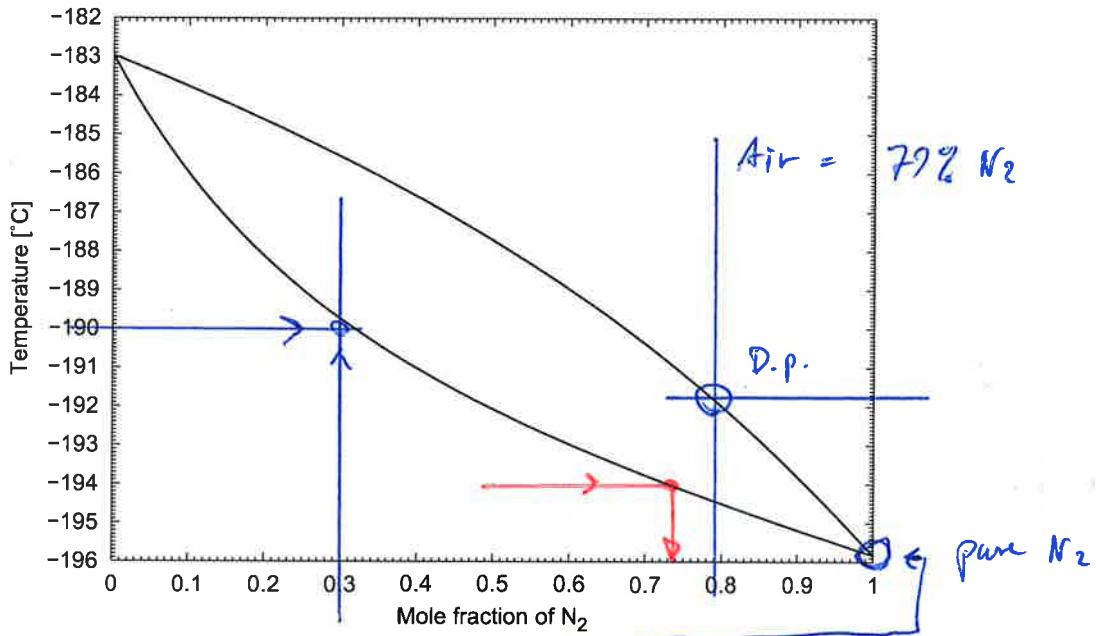


Figure 1: Binary phase diagram for N<sub>2</sub> and O<sub>2</sub> at p = 1 atm (= 1.01325 bar)

I Nitrogen - Boils at a lower T than Oxygen at the same pressure

II -191.8°C

III Single phase liquid (but just outside of the phase envelope)

b) Mass:  $V_{n+1} = L_n + D$

$\begin{matrix} \uparrow & & \uparrow & \nearrow \\ \text{in} & & \text{out} & \\ \downarrow & & \downarrow & \searrow \end{matrix}$

Component:  $V_{n+1} \cdot Y_{n+1} = L_n \cdot X_n + D \cdot X_0$

$\Rightarrow (L_n + D) Y_{n+1} = L_n \cdot X_n + D \cdot X_0$

$$\text{Using } \frac{L_n}{D} = R \rightarrow L_n = R \cdot D$$

(4)

$$\Rightarrow (R \cdot D + D) y_{n+1} = R \cdot D \cdot x_n + D \cdot x_D$$

$$\Rightarrow (R+1) y_{n+1} = R x_n + x_D$$

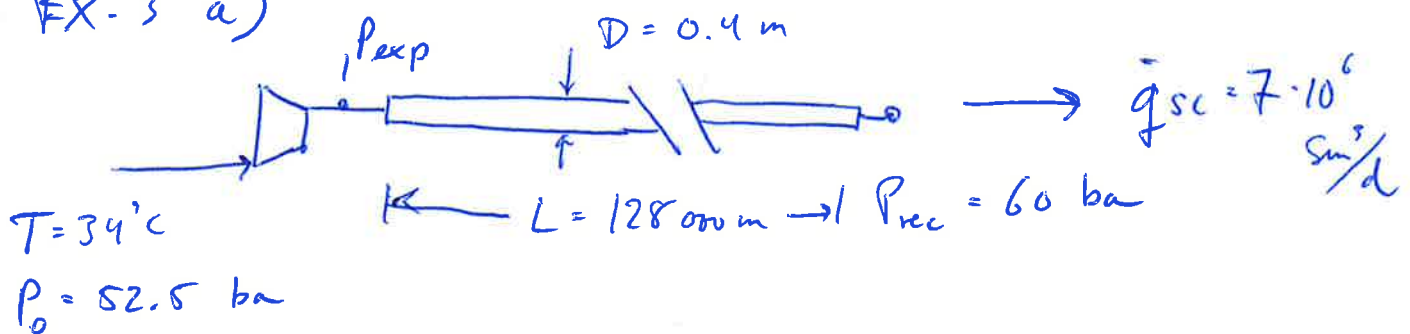
$$\Rightarrow y_{n+1} = \frac{R}{R+1} x_n + \frac{1}{R+1} x_D$$

c) The liquid coming from tray n is in equilibrium with its vapour at  $T = -194^\circ\text{C}$

$$\Rightarrow x_n \approx 0.735$$

$$\rightarrow y_{n+1} = \frac{1.5}{2.5} \cdot 0.735 + \frac{1}{2.5} \cdot 0.999 = \underline{\underline{0.841}}$$

EX-3 a)



$$\text{Gas flow eqn: } \dot{q}_{\text{sc}} = 1.149 \cdot 10^6 \left( \frac{T_{\text{sc}}}{P_{\text{sc}}} \right) \sqrt{\frac{(P_1^2 - P_2^2) D^5}{f_D \gamma L T_m z}}$$

$$(P_1 = P_{\text{exp}}, P_2 = P_{\text{rec}})$$

(5)

Solve for  $P_1$ :

$$\left[ \frac{\dot{q}_{sc}}{1.149 \cdot 10^6} \cdot \left( \frac{P_{sc}}{T_{sc}} \right) \right]^2 = \frac{(P_1^2 - P_2^2) D^5}{f_D \gamma L T_m z}$$

$$\Rightarrow P_1 = \sqrt{\frac{f_D \cdot \gamma L T_m z}{D^5} \cdot \left[ \frac{\dot{q}_{sc}}{1.149 \cdot 10^6} \cdot \left( \frac{P_{sc}}{T_{sc}} \right) \right]^2 + P_2^2}$$

Use prescribed units:

$$P_1 [\text{kPa}] = \left\{ \frac{0.018 \cdot 0.61 \cdot 128000 \cdot 279.15 \cdot 0.9}{(0.4)^5} \cdot \left[ \frac{7 \cdot 10^6}{1.149 \cdot 10^6} \cdot \frac{101.325}{288.15} \right]^2 + 60000^2 \right\}^{1/2}$$

$$= \underline{13937 \text{ kPa}} = \underline{139.4 \text{ bar}} = P_{Exp.}$$

b) Mass flow:

$$\dot{m} = 7 \cdot 10^6 \frac{\text{Sm}^3}{d} \cdot 42300 \frac{\text{kmol}}{10^6 \text{Sm}^3} \cdot (0.61 \cdot 29) \frac{\text{kg}}{\text{kmol}}$$

$$\cdot \frac{1}{3600 \cdot 24} \frac{d}{s} = \underline{\underline{60.6 \text{ kg/s}}}$$

c) Isentropic work in mass units:

$$w_{is} = \frac{R}{\gamma \cdot M_n} \cdot T_1 \cdot \frac{\gamma}{\gamma - 1} \cdot \left( \left( \frac{P_{exp}}{P_0} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right)$$

$$= \frac{8.3144 \frac{\text{J}}{\text{mol} \cdot \text{K}}}{0.61 \cdot 29 \text{ g/mol}} \cdot \underbrace{(34 + 273.15) \text{ K}}_{307.15} \cdot \frac{1.23}{0.23} \cdot \left( \left( \frac{139.4}{52.5} \right)^{\frac{0.23}{1.23}} - 1 \right)$$

$$= \underline{154.7 \frac{\text{J}}{\text{g}} \left( \frac{\text{kJ}}{\text{kg}} \right)}$$

Power (isentropic):

$$P = \dot{m} \cdot w_{is} = 60.6 \frac{\text{kg}}{\text{s}} \cdot 154.7 \frac{\text{kJ}}{\text{kg}} = \underline{\underline{9373 \text{ kW}}}$$

Ex. 4 a) I: -40°C (approximately)

II: With an approach of 5K, the lowest temperature will be -35°C  
⇒ 17.5 bar (approximately)

b) From the  $\log(p) - h$  diagram for ethylene:

$$\left. \begin{aligned} h_c &= 180 \text{ kJ/kg} \\ h(30^\circ\text{C}) &= 640 \text{ kJ/kg} \end{aligned} \right\} \Delta h = 640 - 180 = \underline{\underline{460 \text{ kJ/kg}}}$$

must be handled by the propane circuit

c) Compression from saturated vapor @  $-100^\circ\text{C}$  (ethylene)

$$\left. \begin{aligned} h_v &= 490 \text{ kJ/kg} \text{ (sat. vapor)} \\ h_{is} &= 660 \text{ "} \end{aligned} \right\} \Delta h_{is} = 660 - 490 = \underline{\underline{170 \text{ kJ/kg}}}$$

$$\eta_{is} = \frac{\Delta h_{is}}{\Delta h_{comp.}} \Rightarrow \Delta h_{comp.} = \frac{\Delta h_{is}}{\eta_{is}} = \frac{170}{0.86} = \underline{\underline{197.7 \text{ kJ/kg}}}$$

Outlet temp: Abs. enthalpy out of compression will be:  $490 + 197.7 = \underline{\underline{687.7 \text{ kJ/kg}}}$

From the diagram:  $T_{out} \approx 60^\circ\text{C}$



Attachment 2b:  $\log(p)$ - $h$  diagram for ethylene (for Exercise 4)

