PET565 PART B: EXERCISE SET 4

Goals

- Become familiar with the analytical solution for tracer flow (advection + diffusion / dispersion)

Important notes

The analytical solution is also implemented in a Matlab code ('tra_ads.m') which accounts for advection, dispersion and linear adsorption.
The analytical solution for advection and dispersion at a fixed boundary concentration c^{inj} and initial concentration c_i is given by equation (3.66), repeated below:

$$c(x,t) = c_i + \frac{c^{inj} - c_i}{2}\Big[erfc(\frac{x - vt}{\sqrt{4D_L t}}) + exp(\frac{vx}{D_L})erfc(\frac{x + vt}{\sqrt{4D_L t}})\Big]$$

1. Exercises

1.1. Assume a brine with concentration $c_0 = 10$M is injected into a core of 0.05 m length, containing brine with initial concentration $c_i = 5$M. The dominant mechanisms are dispersion and advection. Assume a dispersion coefficient $D_L = 1e - 4\text{m}^2/\text{hr}$ and water velocity of 0.05m/hr.

- Sketch the concentration profile after 0.5 hrs using equation (3.66). You can use Excel or the table with the $erfc$ function or the Matlab code.
- Sketch the produced concentration at the outlet position $x = 0.05$ m as a function of injected pore volumes.
- Now let the dispersion coefficient be reduced to $D_L = 1e - 5\text{m}^2/\text{hr}$ (this makes dispersion less significant). What is the effect on the concentration profile?
- Now assume there is no advection by setting $v = 0$, and only diffusion. Show that equation (3.66) can be reduced to equation (3.58).
- In the case where only diffusion is present, plot the concentration profile after 5hrs using $D_L = 1e - 5\text{m}^2/\text{hr}$ (a representative diffusion coefficient). What is the difference compared to when advection is present?
- Calculate the time at which the concentration $c = 7.5$ reaches the outlet position $x = 0.05$m for this case, using equation (3.58).

Answers.

- Example: let $x = 0.03$ m. See figure 1

$$\begin{aligned} c(x,t) &= 5 + \frac{10-5}{2}\Big[erfc(\frac{0.03 - 0.05\cdot 0.5}{\sqrt{4\cdot 1e-4\cdot 0.5}}) + \exp(\frac{0.05\cdot 0.03}{1e-4})erfc(\frac{0.03 + 0.05\cdot 0.5}{\sqrt{4\cdot 1e-4\cdot 0.5}})\Big] \\ &= 5 + 2.5\Big[erfc(\frac{0.005}{\sqrt{2e-4}}) + \exp(\frac{0.0015}{1e-4})erfc(\frac{0.055}{\sqrt{2e-4}})\Big] \\ &= 5 + 2.5\Big[erfc(0.354) + \exp(15)erfc(3.89)\Big] \\ &= 5 + 2.5\Big[0.617 + 0.123\Big] \\ &= 6.85 \end{aligned} \tag{1}$$

- See figure 1

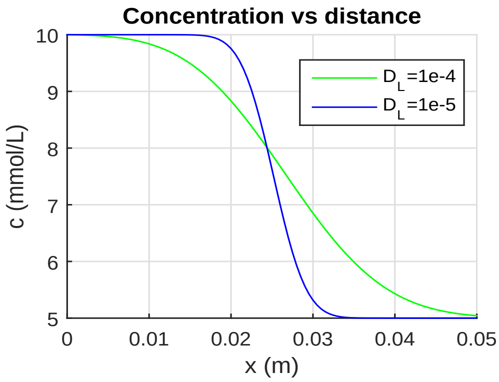

FIGURE 1. Concentration distribution at $t = 0.5$h. Larger dispersion coefficient D_L leads to smoother distribution of the injected species.

- The analytical solution for advection and dispersion at a fixed boundary concentration is given by

$$c(x,t) = c_i + \frac{c^{inj} - c_i}{2}\left[erfc(\frac{x - vt}{\sqrt{4D_L t}}) + exp(\frac{vx}{D_L})erfc(\frac{x + vt}{\sqrt{4D_L t}})\right] \quad (2)$$

By setting $v = 0$ we obtain:

$$c(x,t) = c_i + \frac{c^{inj} - c_i}{2}\left[erfc(\frac{x}{\sqrt{4D_L t}}) + exp(0)erfc(\frac{x}{\sqrt{4D_L t}})\right] \quad (3)$$

$$= c_i + (c^{inj} - c_i)erfc(\frac{x}{\sqrt{4D_L t}}) \quad (4)$$

- See figure 2.
- We can apply the analytical solution for diffusion (equation 3.58). The time is the only unknown.

$$c = c_i + (c_0 - c_i)erfc(\frac{x}{\sqrt{4D_L t}}) \quad (5)$$

$$erfc(\frac{x}{\sqrt{4D_L t}}) = \frac{c - c_i}{c^{inj} - c_i} = \frac{7.5 - 5}{10 - 5} = 0.5 \quad (6)$$

From the table (Fig 4) we see that

$$erfc(\beta) = 0.5 \quad if \quad \beta = 0.48 \quad (7)$$

$$\beta = \frac{x}{\sqrt{4D_L t}} \quad (8)$$

$$t = \frac{x^2}{4D_L\beta^2} = \frac{0.05^2}{4 \cdot 1e - 5 \cdot 0.48^2} = 271h \quad (9)$$

1.2. A brine is injected with 2 species (SCN^- and SO_4^{2-}) both having concentration c^{inj} = 10mM into a core of 0.05 m length, and the initial concentrations are c_i = 0mM. The dominant mechanisms are dispersion and advection. The SO_4^{2-} species can adsorb, while SCN^- is a tracer. Assume a dispersion coefficient $D_L = 1e - 4\text{m}^2/\text{hr}$ and water velocity of 0.05m/hr. The porous cross section is 5 cm^2.

- Assume the adsorbing species SO_4^{2-} has a linear isotherm $q = kc$. Plot the concentration profiles along the core after 1 hr assuming $k = 0.05, 0.25$ and 0.5.
- Plot the produced concentration time profiles for the same cases during 2 injected pore volumes.
- Assume the case $k = 0.5$ corresponds to a water-wet core with residual oil, while the case $k = 0.05$ is for a strongly oil-wet core. Calculate the adsorbed amount of moles in each case after flooding the cores by

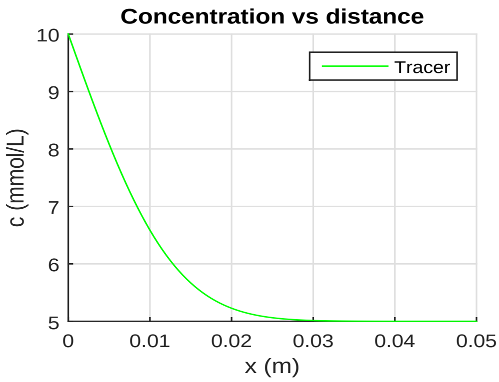

FIGURE 2. Concentration distribution at $t = 5$h. No advection, only diffusion, by setting $v = 0$, while $D_L = 1e - 5\text{m}^2/\text{hr}$.

  - Integrating the difference in concentration time profiles,
  - and by calculating the amount of adsorbed species $q(c)$ in the core for a given concentration.
- Calculate the wettability index of the core.

Answers.

- See Figure 3 (left).
- See Figure 3 (right).
- Let t_1 and t_2 be times when both concentrations are stable and equal to initial and injected values, respectively.

$$N_{ads} = A_{por} v_w \int_{t_1}^{t_2} (c_{scn} - c_{so}) dt \tag{10}$$

Calculate the profiles and integrate numerically to get

$$N_{ads} = 0.13 \text{ mmol for k=0.5} \tag{11}$$

$$N_{ads} = 0.013 \text{ mmol for k=0.05} \tag{12}$$

See also figure 4.

- Before flooding the core has adsorbed $q(c^{init}) = 0$ mmol/L pore, while at equilibrium the core has adsorbed $q(c^{inj} : k) = c^{inj} k$ mmol / L pore. The adsorbed moles are

$$N_{ads} = A_p L(q(c^{inj}) - q(c^{init})) = 0.025[L]10[\text{mmol/L}]k \tag{13}$$

$$N_{ads} = 0.125 \text{ mmol (for k=0.5)} \tag{14}$$

$$N_{ads} = 0.0125 \text{ mmol (for k=0.05)} \tag{15}$$

- The wettability index:

$$WI = N_{ads}^{mixed-wet} / N_{ads}^{water-wet} = 0.013/0.13 = 0.1 \tag{16}$$

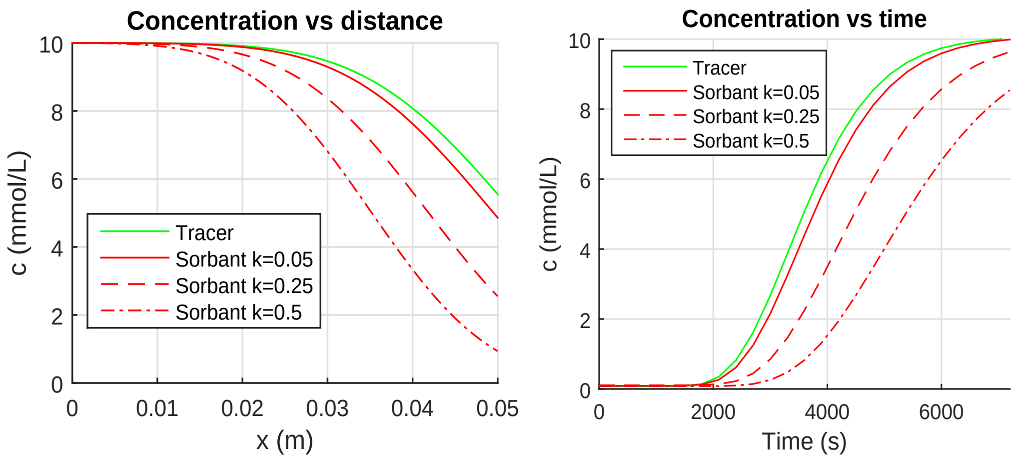

FIGURE 3. Concentration profile (left) after 1 PV (1 hr) and effluent profile during 2PV injection (left).

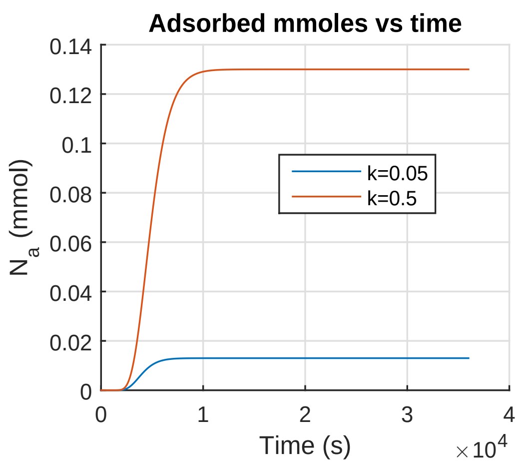

FIGURE 4. Adsorbed moles with time, based on integrating concentration profile difference.

$$\operatorname{erf}(\beta) = \frac{2}{\sqrt{\pi}} \int_0^{\beta} e^{-\epsilon^2}\, d\epsilon$$

$$\operatorname{erf}(-\beta) = -\operatorname{erf} \beta$$

$$\operatorname{erfc}(\beta) = 1 - \operatorname{erf}(\beta)$$

β	erf (β)	erfc (β)
0	0	1.0
0.05	0.056372	0.943628
0.1	0.112463	0.887537
0.15	0.167996	0.832004
0.2	0.222703	0.777297
0.25	0.276326	0.723674
0.3	0.328627	0.671373
0.35	0.379382	0.620618
0.4	0.428392	0.571608
0.45	0.475482	0.524518
0.5	0.520500	0.479500
0.55	0.563323	0.436677
0.6	0.603856	0.396144
0.65	0.642029	0.357971
0.7	0.677801	0.322199
0.75	0.711156	0.288844
0.8	0.742101	0.257899
0.85	0.770668	0.229332
0.9	0.796908	0.203092
0.95	0.820891	0.179109
1.0	0.842701	0.157299
1.1	0.880205	0.119795
1.2	0.910314	0.089686
1.3	0.934008	0.065992
1.4	0.952285	0.047715
1.5	0.966105	0.033895
1.6	0.976348	0.023652
1.7	0.983790	0.016210
1.8	0.989091	0.010909
1.9	0.992790	0.007210
2.0	0.995322	0.004678
2.1	0.997021	0.002979
2.2	0.998137	0.001863
2.3	0.998857	0.001143
2.4	0.999311	0.000689
2.5	0.999593	0.000407
2.6	0.999764	0.000236
2.7	0.999866	0.000134
2.8	0.999925	0.000075
2.9	0.999959	0.000041
3.0	0.999978	0.000022

FIGURE 5. Table for evaluation of the error and complementary error function