

EXAM PET 550 - Nat. gas processing & transportation

①

May 10th, 2017

(ANSWERS)

Ex 1

a) Simplifying assumptions: I: $v^L \ll v^V$

$$\Rightarrow v^V - v^L \approx v^V$$

$$\text{II: } v^V \approx v^* = \frac{RT}{P}$$

(Liq. volume negligible)

(Vapour = ideal gas)

$$\Rightarrow \left(\frac{dp}{dT}\right)_{\text{sat}} = \frac{(s^V - s^L)}{v^*} = \frac{P(s^V - s^L)}{RT}$$

b) $dh = T ds + v dp$ $\Rightarrow dh = T ds$
const. P

$$\Delta h = \int dh = \underline{T} \int ds \Rightarrow \Delta h = T \Delta s$$

const. T, outside of integral

$$\Rightarrow \text{Between saturated states: } \underline{(h^V - h^L) = T(s^V - s^L)}$$

c) $\left(\frac{dp}{dT}\right)_{\text{sat}} = \frac{P \cdot (s^V - s^L)}{RT} = \frac{P \cdot (h^V - h^L)}{RT \cdot T}$

$$\Rightarrow \frac{dp}{P} = \frac{(h^V - h^L)}{R} \frac{dT}{T^2} \Rightarrow \int_{P_0}^P \frac{dp}{P} = \frac{(h^V - h^L)}{R} \int_{T_0}^T \frac{dT}{T^2}$$

(2)

$$\int \frac{dP}{P} = \ln P \quad , \quad \int \frac{dT}{T^2} = -\frac{1}{T}$$

$$\Rightarrow [\ln P]_{P_0}^P = \frac{(h^v - h^L)}{R} \cdot \left[-\frac{1}{T} \right]_{T_0}^T$$

$$\Rightarrow \underline{\underline{\ln\left(\frac{P}{P_0}\right) = \frac{(h^v - h^L)}{R} \cdot \left(\frac{1}{T_0} - \frac{1}{T}\right)}} \quad \text{QED}$$

$$d) \quad \frac{1}{T_0} - \frac{1}{T} = \frac{R}{(h^v - h^L)} \cdot \ln\left(\frac{P}{P_0}\right)$$

$$\Rightarrow T(P) = \left\{ \frac{1}{T_0} - \frac{R}{(h^v - h^L)} \cdot \ln\left(\frac{P}{P_0}\right) \right\}^{-1}$$

$$(T \text{ must be in K}) : T_0 = -42.1 + 273.15 = 231.05 \text{ K}$$

$$\Rightarrow T(5 \text{ bar}) = \left\{ \underbrace{\frac{1}{231.05}}_{4.3281 \cdot 10^{-3}} - \frac{8.3144}{18781} \cdot \underbrace{\ln\left(\frac{5}{1.01325}\right)}_{1.5963} \right\}^{-1}$$

$$\qquad \qquad \qquad \underbrace{\hspace{10em}}_{7.0668 \cdot 10^{-4}}$$

$$\underline{\underline{T(5 \text{ bar}) = 276.14 \text{ K} = 2.99^\circ \text{C}}}$$

Ex. 2

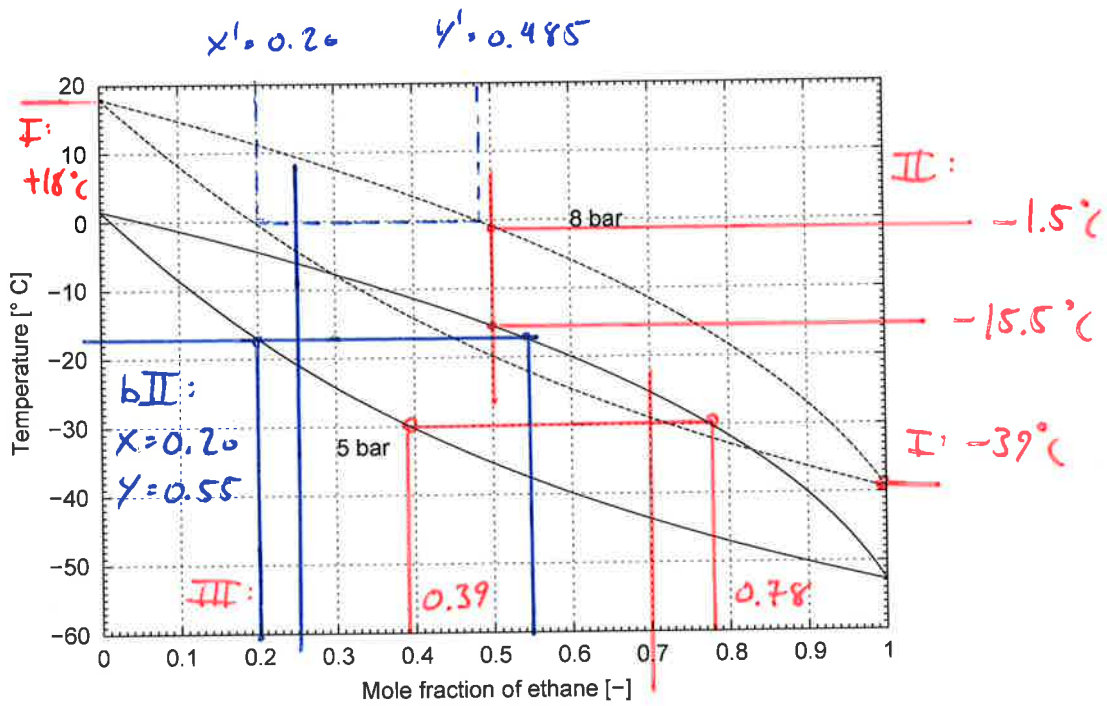


Figure 1: Binary phase diagram ($T - x - y$) for ethane and propane at $p = 5$ bar (solid curves) and $p = 8$ bar (dotted curves).

a) I: Ethane -39°C , Propane $+18^\circ\text{C}$

II: -1.5°C (8 bar), -15.5°C (5 bar)

III: $X \approx 0.39$, $Y \approx 0.78$

b) I: $z = \frac{N_{C2}}{N_{C2} + N_{C3}}$

$N_{C2} = \frac{m_{C2}}{Mw_{C2}} = \frac{3.1 \text{ g}}{30.07 \text{ g/mole}} = 0.10309 \text{ mole}$

$N_{C3} = \frac{13.6 \text{ g}}{44.09 \text{ g/mole}} = 0.30846 \text{ mole}$

$\Rightarrow z = \frac{0.10309}{0.10309 + 0.30846} = 0.25049 \approx \underline{\underline{0.25}}$

QED

II (From figure 7): $X = 0.20$, $Y = 0.55$

III ("Lever rule"): $Z = \alpha \cdot Y + (1 - \alpha) X$

$\rightarrow \alpha = \frac{Z - X}{Y - X} = \frac{0.25 - 0.2}{0.55 - 0.2} = 0.143$

c) Qualitative answer: X stays the same, Y diminishes, which means that α increases

Quantitative answer: Y is now ≈ 0.485
 $\rightarrow \alpha = \frac{0.25 - 0.2}{0.485 - 0.2} = 0.175$

EX. 3

a) Cold duty: $Q_0 = \dot{m} \cdot \Delta h_{Evap.} = \dot{m} \cdot (h_A - h_0)$

From Attachment 2: $h_0 \approx 305 \text{ kJ/kg}$, $h_A \approx 550 \text{ kJ/kg}$

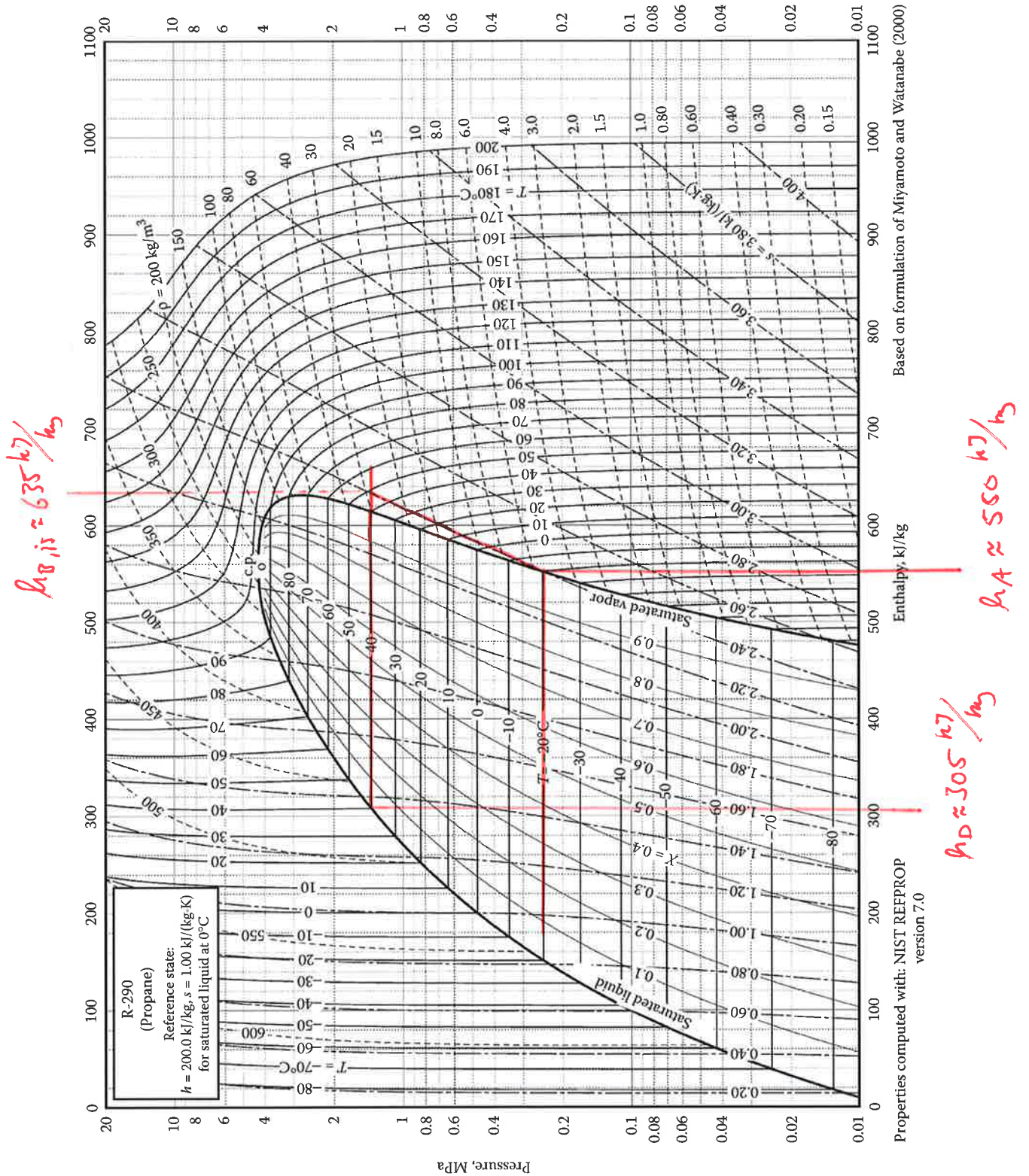
$\Rightarrow \dot{m} = \frac{9 \text{ kW}}{(550 - 305) \text{ kJ/kg}} = 0.0367 \text{ kg/s}$

b) Compressor work: $W_c = \dot{m} \cdot (h_B - h_A)$
 $= \dot{m} \frac{(h_{B,is} - h_A)}{Z_{is}}$

From Att. 2: $h_{B,is} \approx 635 \text{ kJ/kg}$

$\Rightarrow W_c = 0.0367 \text{ kg/s} \cdot \frac{635 - 550}{0.77} = 4.05 \text{ kW}$

Attachment 2: $\log(p)$ - h diagram for propane (for Exercise 3)



(5)

Coefficient of performance: $\epsilon_0 = \frac{Q_0}{W_c}$

$$= \frac{9}{4.05} = \underline{\underline{2.22}}$$

c) Lowering condensation temperature:

1: h_D will decrease, which means that Δh_{Evap} will increase, thus requiring less mass flow for the same cold duty

2: $h_{g,15}$ will decrease, thus $h_g - h_f$ will decrease, provided z_{15} stays the same

— Both 1 & 2 will lead to less compressor work, which improves ϵ_0 .

Ex 4

a) For a general refrigeration plant, $W_c = Q_c - Q_0$

* Theoretically $Q_0 = h_{in} - h_{LWR}$, which can be read off the diagram:

$$Q_0 = -75.33 - (-90.46) = \underline{\underline{15.13 \text{ kJ/mole}}}$$

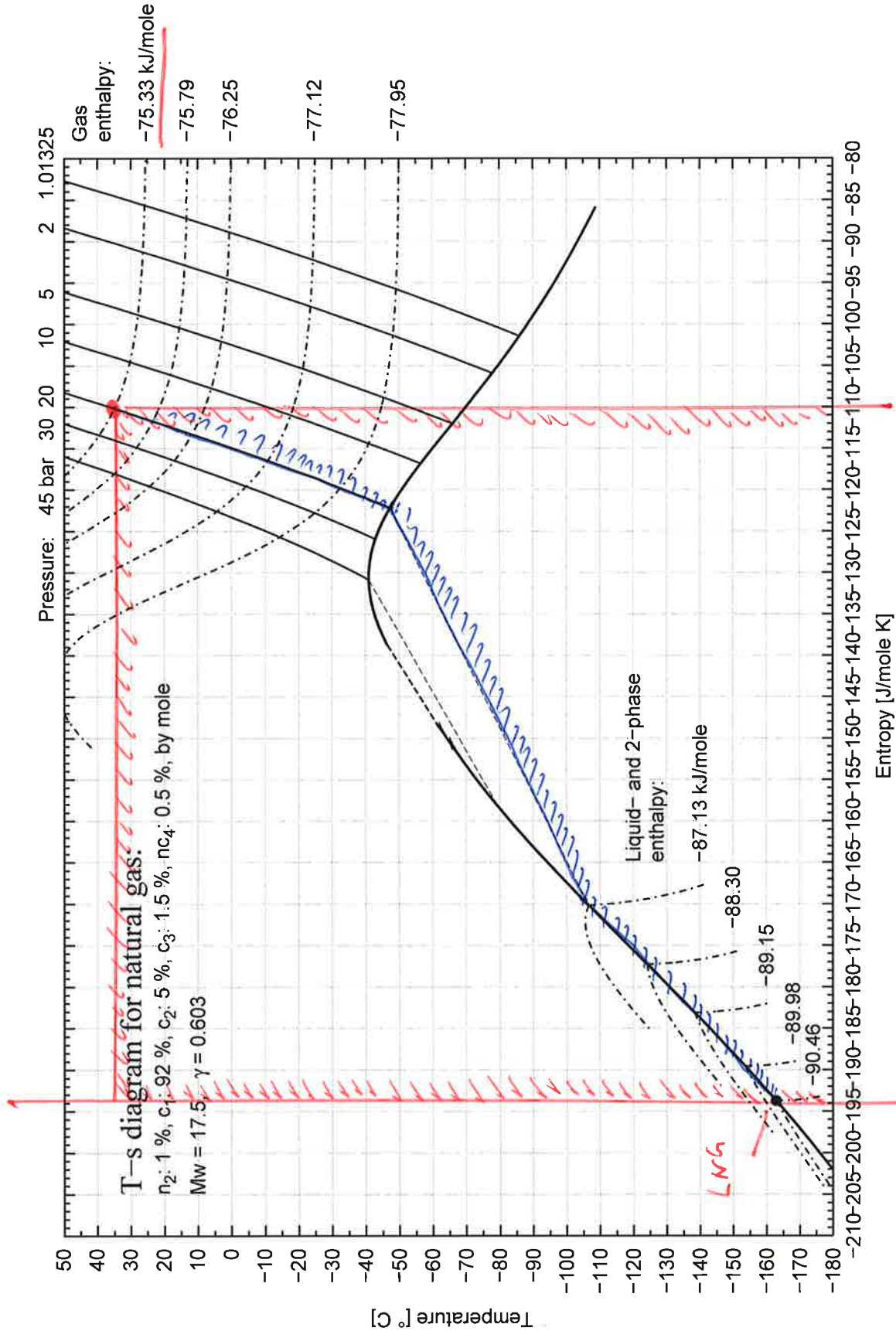
* Theoretically $Q_c = T_c \cdot \Delta S = T_c \cdot (S_{in} - S_{LWR})$;

Refrigerant has the same properties as the gas, all

5a

Attachment 3: T-s diagram for natural gas (for Exercise 4)

$w_{in} (20 \text{ bar}, 35^\circ\text{C}) = -75.33 \text{ kJ/mole}$



$S_{in} \approx -110 \text{ J/mole K}$

$S_{LN2} \approx -90.46 \text{ kJ/mole K}$

$S_{LN2} \approx -194 \text{ J/mole K}$

heat exchange towards ambient takes place at constant temperature, disregarding temp. differences.

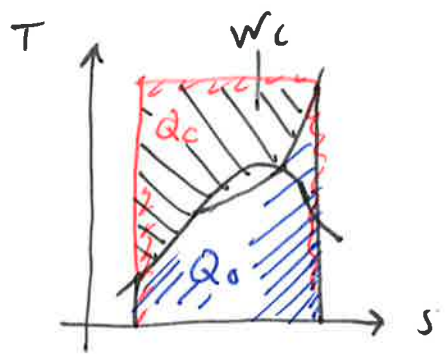
$$(T_c = T_{amb})$$

From the diagram: $S_{in} \approx -110 \text{ J/mol K}$

$$S_{out} \approx -194 \text{ "}$$

$$\rightarrow Q_c = \underbrace{(35 + 273.15)}_{308.15 \text{ K}} \cdot \underbrace{(-110 - (-194))}_{84 \text{ J/mol K}} = \underline{25885 \text{ J/mol}}$$

$$\Rightarrow W_c (\text{theoretical min}) = 25.89 - 15.13 \approx \underline{\underline{11 \text{ kJ/mol}}}$$



b) One-stage compression from 20 - 200,

Theoretical compression work:

$$W_{15} = 8.3144 \cdot 308.15 \cdot \frac{1.28}{0.28} \cdot \left[\left(\frac{200}{20} \right)^{\frac{0.28}{1.28}} - 1 \right] = 7669 \text{ J/mol}$$
$$= \underline{\underline{7.67 \text{ kJ/mol}}}$$

c) Compressor work must depend on pressure drop over the pipeline:

$$\dot{q}_{sc} = 1.149 \cdot 10^6 \cdot \left(\frac{T_{sc}}{P_{sc}}\right) \cdot \sqrt{\frac{(P_1^2 - P_2^2) D^5}{f_D \gamma L T_m Z}}$$

$$\rightarrow \frac{(P_1^2 - P_2^2) D^5}{f_D \gamma L T_m Z} = \left[\frac{\dot{q}_{sc}}{1.149 \cdot 10^6} \cdot \left(\frac{P_{sc}}{T_{sc}}\right) \right]^2$$

$$\Rightarrow P_{EXP} = P_1 = \sqrt{\frac{f_D \gamma L T_m Z}{D^5} \cdot \left[\frac{\dot{q}_{sc}}{1.149 \cdot 10^6} \cdot \left(\frac{P_{sc}}{T_{sc}}\right) \right]^2} + P_2^2$$

$$= \left\{ \frac{\overbrace{0.018 \cdot 0.603 \cdot 900000 \text{ m} \cdot 283.15 \text{ K} \cdot 7}^{(f_D \gamma L T_m Z)} \cdot \left[\frac{14.8 \cdot 10^6}{1.149 \cdot 10^6} \cdot \left(\frac{101.325 \text{ kPa}}{288.15 \text{ K}}\right) \right]^2}{(0.6)^5} \right\}^{1/2} + \underbrace{(6000 \text{ kPa})^2}_{3.6 \cdot 10^7}$$

$\underbrace{3.5571 \cdot 10^7}_{12.881} \cdot \underbrace{0.35164}_{4.5295^2 = 20.516} = 27673 \text{ kPa}$

$= 277 \text{ bar}$

$$\Rightarrow W_c = 8.3144 \cdot 308.15 \cdot \frac{1.28}{0.28} \cdot \left[\left(\frac{277}{20}\right)^{\frac{0.28}{1.28}} - 1 \right] = 9101 \text{ J/mole}$$

Pipeline still more favourable in terms of energy use at this stage...