



FACULTY OF SCIENCE AND TECHNOLOGY

DATE: September 2, 2015

SUBJECT: PET 565 – Core scale modeling and interpretation

TIME: 4 hours

AID: No printed or written means allowed. Definite basic calculator allowed.

THE EXAM CONSISTS OF 6 PROBLEMS ON 3 PAGES

REMARKS: You may answer in English or Norwegian. Exercises 1 - 3 (part A) and exercises 4 - 6 (part B) are given equal weight.

Problem 1.

- (a) We focus on the model for spontaneous imbibition

$$\phi \partial_t S_w + \partial_x \left(K \frac{\lambda_w \lambda_o}{\lambda_t} \partial_x P_c(S_w) \right) = 0, \quad \lambda_i = \frac{k_{ri}}{\mu_i}, \quad i = w, o \quad (1)$$

where $\lambda_t = \lambda_w + \lambda_o$ is the total mobility. K and ϕ are, respectively, absolute permeability and porosity. k_{ri} and μ_i represent, respectively, relative permeability and viscosity.

- (i) Formulate the mass balance equations as well as Darcy's law for water and oil.
- (ii) Assume countercurrent flow (i.e., $u_w = -u_o$) and use this to find an expression for p_{wx} expressed in terms of P_{cx} , where p_w is water pressure and $P_c = p_o - p_w$ is capillary pressure.
- (iii) Finally, demonstrate how to obtain (1).
- (b) We consider a vertical flow domain. Include gravity in Darcy's law and derive the corresponding water-oil model for the combined effect of imbibition and gravity.

Problem 2. The Buckley-Leverett model for injection of water in an oil reservoir takes the form in dimensionless variables

$$\begin{cases} S_t + f(S)_x = 0, & x \in [0, 1] \\ S(x=0, t) = 1, \end{cases} \quad (2)$$

where S represents the water saturation.

- (a) Give a description (without mathematical derivation) of how to construct the solution after a time $t > 0$ before the front has reached the producer at $x = 1$ for a typical choice of the fractional flow function $f(S)$. Make plots to illustrate the solution.
- (b) Explain why the solution constructed by using the method of characteristics becomes unphysical. Give a mathematical explanation of the physical correct solution.
- (c) Derive (by mathematical calculations) an expression for the oil recovery $R(t)$ for the solution discussed in (a) for all times.

Problem 3.

- (a) Consider the conservation law

$$(*) \quad u_t + f(u)_x = 0, \quad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \quad u(x, t=0) = \phi(x).$$

- Explain by using the *method of characteristics* why a general solution of (*) and (**) takes the form

$$(***) \quad u(x, t) = \phi\left(x - f'(u(x, t))t\right)$$

- (b) Based on (***) compute an expression for u_x .
- Explain under what circumstances u_x might blow up (i.e., becomes infinitely large) when we assume that f is convex ($f'' > 0$)
- Verify by direct calculation that (***) satisfies (*) and (**) subject to the condition

$$1 + \phi'(x_0) f''(\phi(x_0)) t \neq 0.$$

(c) Generally, we must consider *weak solutions* of the model problem composed of (*) and (**).

- Give the mathematical description of a weak solution in terms of an integral equality.
- Why do we need to study weak solutions?

(d) Consider (*) with $f(u) = u^2$ and

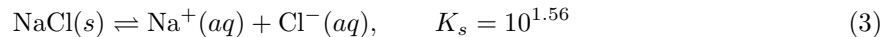
$$\phi(x) = \begin{cases} 2, & 0 \leq x < 1; \\ 0, & \text{otherwise} \end{cases}$$

- Consider the two Riemann problems, one at $x = 0$ and the other at $x = 1$. Compute solutions in terms of shock wave and/or rarefaction wave. Make figures that show the characteristics.
- In particular, compute the time T_c when the two waves will start interacting.

(e) For the problem discussed in (d), compute the solution for $t > T_c$

- either by using mathematical relations that characterize the behaviour of the two interacting waves (Rankine-Hugoniot condition, etc)
- or by using the "Equal-Area Rule"

Problem 4. Assume activity coefficients are 1 in this task. Halite, NaCl, dissolves according to the reaction



- Write the equilibrium condition for this reaction. Calculate the number of moles NaCl that can dissolve in 1 L of water. In a similar way complexes can form according to



- Write the equilibrium conditions for this system and the mass balances for Na and Cl.
- Calculate the number of moles NaCl that can dissolve in 1 L of water when accounting for the formation of complexes.

Problem 5.

- Explain how adsorption tests can be used to measure wettability properties

A species adsorbs according to $q = c^3$ where c is concentration and q (mol/L pore) is the adsorption isotherm (amount adsorbed species as function of brine concentration).

- Assume a core contains $c = 0$ initially and is flooded with $c = 1$. Consider the time when 1 pore volume (PV) has been flooded. Sketch the concentration profile along the core.
- Assume a core contains $c = 1$ initially and is flooded with $c = 0$. Consider the time when 1 PV has been flooded. Sketch the concentration profile along the core.
- In the last situation, how many mols of species are still stored on the surface after injecting 1PV? Assume the pore volume is 0.04 L.

Problem 6. A chalk core is flooded with 1 mol/L MgCl₂ brine. It is assumed that calcite CaCO₃ can dissolve, while magnesite MgCO₃ precipitates in a substitution-like manner and that ion exchange of Ca and Mg occurs at the surface. The formation water contains Ca in the brine and on the surface, but not Mg or Cl.

- Write down relevant reactions, rates and equilibrium conditions for the mentioned chemistry.
- Write down transport partial differential equations for the species in the brine (Ca, Mg, Cl), on the surface (Ca, Mg) and the minerals (calcite, magnesite). Also write down initial conditions and boundary conditions.

- Sketch the expected concentration profiles for Ca, Mg, Cl as measured at the outlet vs injected pore volumes. What is the role of the different mechanisms you have included in the system?

APPENDIX A. FORMULAS

Davies formula (at 25°C):

$$\log_{10} \gamma_i = -0.5085 Z_i^2 \left(\frac{\sqrt{I}}{1 + \sqrt{I}} - 0.3I \right) \quad (5)$$

Retardation formula

$$R_c = 1 + \frac{dq}{dc}, \quad (\text{broadening front}) \quad (6)$$

$$R_f = 1 + \frac{\Delta q}{\Delta c}, \quad (\text{sharpening front}) \quad (7)$$

pH

$$pH = -\log_{10}([H^+]) \quad (8)$$