University of
Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

DATE: May 18, 2016

SUBJECT: PET 565 – Core scale modeling and interpretation

TIME: 4 hours

AID: No printed or written means allowed. Definite basic calculator allowed.

THE EXAM CONSISTS OF 5 PROBLEMS ON 3 PAGES

REMARKS: You may answer in English or Norwegian. Exercises 1 - 2 (part A) and exercises 3 - 5 (part B) are given equal weight.

Problem 1.

(a) Consider the conservation law

$$(*) \qquad u_t + f(u)_x = 0, \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x, t = 0) = \phi(x).$$

- We assume that $f'' > 0$. The expression

$$(***) \qquad u(x,t) = \phi\Big(x - f'(u(x,t))t\Big)$$

is then a general solution of (*) with (**), subject to the condition

$$1 + \phi'(x_0) f''(\phi(x_0))t \neq 0.$$

Verify this by performing a direct calculation (i.e. putting (***) into (*) and (**)).

(b) Based on (***) compute an expression for u_x.
- Explain the concept of breaking time for the equation (*). Can you say something about the behavior of the characteristics at breaking time?

(c) Consider a shock wave moving to the left with speed $s = -\frac{\Delta x}{\Delta t}$. Given that u_L and u_R are the solution values to the left and the right of the shock, respectively, the Rankine-Hugoniot condition states that

$$s = \frac{f(u_R) - f(u_L)}{u_R - u_L}.$$

- Show how to obtain this expression by integrating equation (*) over the region $R = [x_1, x_1 + \Delta x] \times [t_1, t_1 + \Delta t]$.

(d) Consider (*) with $f(u) = \frac{1}{3}u^2$ and

$$\phi(x) = \begin{cases} 2, & 0 \leq x < 2; \\ 0, & \text{otherwise} \end{cases}$$

- Consider the two Riemann problems, one at $x = 0$ and the other at $x = 2$, respectively. Compute solutions in terms of shock waves and/or rarefaction waves.
- Compute the time T_c when the two waves will start to interact.

(e) Utilize the Rankine-Hugoniot condition in order to compute the solution for $t > T_c$, for the problem discussed in (d).

Problem 2.

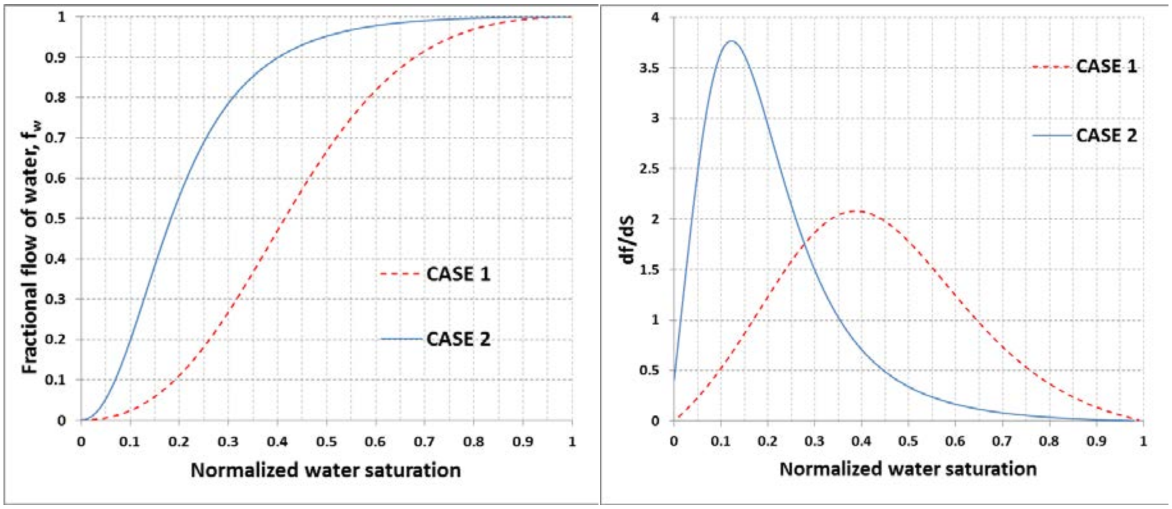

Fig. 1. Fractional flow curves vs. normalized water saturation for CASE 1 and CASE 2 to the left and corresponding df/dS curves to the right.

The Buckley-Leverett (B-L) equation for water-oil transport (horizontal flow) is given by:

$$\frac{\partial S}{\partial t}+\frac{\partial f(S)}{\partial x}=0 \qquad x\in[0,1] \qquad (*)$$

where S is normalized water saturation, t is dimensionless time and x is normalized length.

The equation (*) can further be developed to (**) below.

$$\frac{\partial S}{\partial t}+f'(S)\frac{\partial S}{\partial x}=0 \qquad (**)$$

a) Show how (**) can be derived from (*). What is the quantify f'(S) representing?

b) Sketch the general steps involved for determining the water saturation shock front height and the water saturation profile behind the front ($S > S_f$) for a system with a given fractional flow curve.

c) Determine the time when water breakthrough occurs for CASE 1 (called TB1) and CASE 2 (called TB2)?

d) Let water be injected into two different core samples having fractional flow curve characteristics corresponding to CASE 1 and CASE 2 above. What is the expected water-cut exactly at the time when water is breaking through at the outlet of the core sample for CASE 1 and CASE 2? We assume that water-cut is equal to water fractional flow value meaning that we assume $B_w = B_o$ which should be a reasonable assumption in a laboratory test using dead oil and low pressures during in the experiments.

e) Calculate oil recovery for both cases at the time when water breakthrough occurs for CASE 1 i.e., at TB1.

Problem 3. A brine contains 0.15 mol/L Na_2SO_4 consisting of the ions Na^+ and SO_4^{2-}.

a) Calculate the ionic strength.

In the following we consider dissolution of calcite $CaCO_3$ in this brine, according to

$$CaCO_3 \rightleftharpoons Ca^{2+} + CO_3^{2-}, \qquad K_{calcite} = 10^{-8.5} \tag{1}$$

The dissolved species are assumed not to affect the ionic strength. The carbon content is assumed to be mainly in form of CO_3^{2-}.

b) Calculate activity coefficients of Na^+, Ca^{2+}, CO_3^{2-} and SO_4^{2-}.

c) Without accounting for complexes, how many moles $CaCO_3$ per L can be dissolved in this brine?

Problem 4. This is a continuation of Problem 3, considering how the formation of complexes influences the solubility of calcite in the given brine. The complex $CaSO_4^0$ can form according to

$$CaSO_4^0 \rightleftharpoons Ca^{2+} + SO_4^{2-}, \qquad K_{caso} = 10^{-2.5} \tag{2}$$

a) Write down the following 5 equations:
  - Equilibrium of the aqueous complex reaction (2) according to the law of mass action
  - 2 mass balance equations for Ca^{2+} and SO_4^{2-} species
  - Equilibrium of the calcite dissolution reaction (1)
  - The relation between the concentrations of CO_3^{2-} and Ca^{2+}

  Specify which variables are unknown.

PS: Express the equations using total concentrations C_i, concentrations of free species m_i and concentrations of complexes n_{ij}. You do NOT have to solve the equations.

Problem 5. A core contains c=0 mol/L of a species initially and is then injected with a brine containing c=1 mol/L. The species adsorbs according to an isotherm $q(c) = c^3$. Assume a retardation model can be applied.

a) Sketch the concentration profile along the core after flooding 1 pore volume.

b) How many pore volumes must be flooded before the core stops adsorbing more species?

c) After the core is flooded sufficiently with the 1 mol/L brine, the core is cleaned by flooding a brine with $c = 0$ mol/L. How many pore volumes of the new brine must be injected to remove all the species from the core?

Formulas.

$$I = 1/2 \sum_i m_i Z_i^2 \tag{3}$$

$$\log_{10} \gamma_i = -0.51 Z_i^2 \Big(\frac{\sqrt{I}}{1 + \sqrt{I} - 0.3I} \Big) \tag{4}$$

$$R_c = 1 + \frac{dq}{dc}, \qquad \text{(broadening front)} \tag{5}$$

$$R_f = 1 + \frac{\Delta q}{\Delta c}, \qquad \text{(sharpening front)} \tag{6}$$