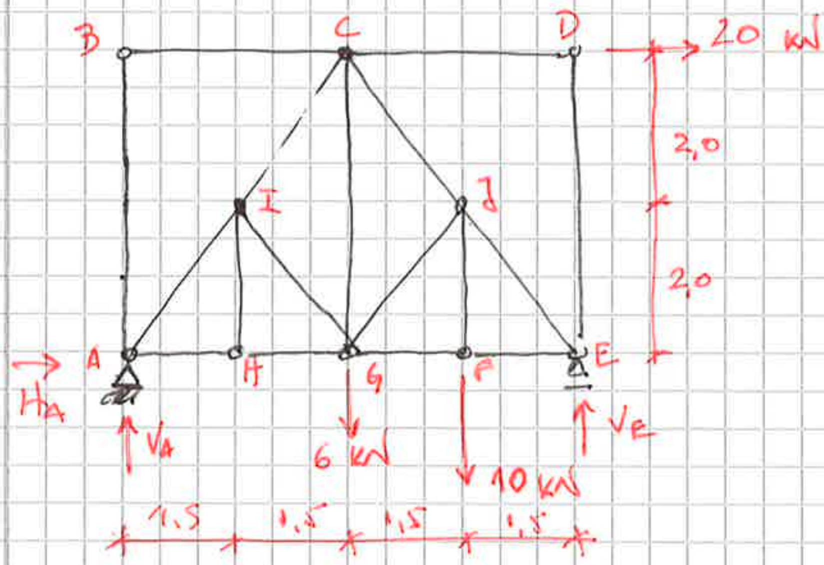
BYG 140 – KONSTRUKSJONSMEKANIKK 1

SOLUTIONS – 18. MAY 2017



(a) Support reactions

$$\sum F_H = 0 \rightarrow H_A + 20 = 0 \rightarrow \boxed{H_A = -20 \text{ kN}}$$

$$\sum F_V = 0 \rightarrow V_A + V_E - 6 - 10 = 0$$

$$\sum M_A = 0 \rightarrow -6 \times 3 - 10 \times 4,5 - 20 \times 4 + 6 V_E = 0$$

$$\boxed{V_E = 23,83 \text{ kN}}$$

$$\boxed{V_A = -7,833 \text{ kN}}$$

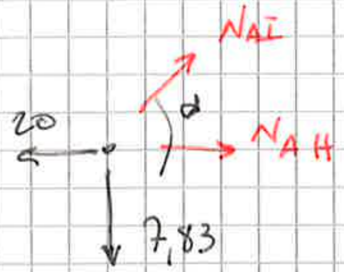
(b) Zero force members: $\overline{AB}$; $\overline{BC}$; $\overline{IH}$; $\overline{IG}$

Joint B: Zero force applied, just 2 members
→ $\overline{AB}$ and $\overline{BC}$ → zero force members

Joint H: 2 colinear members, no force at joint, 3rd member is zero force
→ $\overline{IH}$ zero force member.

Joint I: the same reasoning as joint H
→ $\overline{IG}$ → zero force member

(c) $N_{AH} = ?$ $N_{AI} = ?$



$\cos\alpha = 3/5$

$\sin\alpha = 4/5$

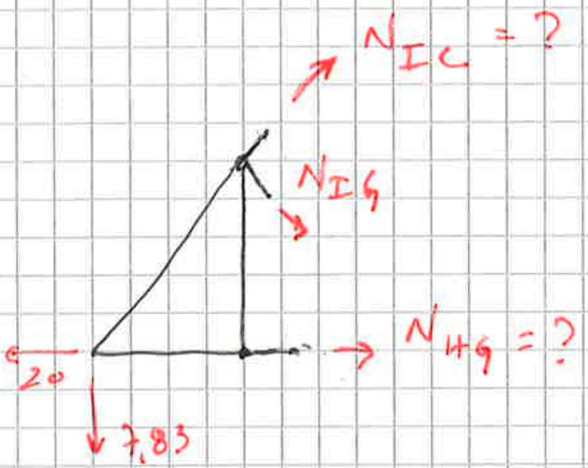
$\Sigma F_H = 0 \rightarrow \quad -20 + N_{AH} + N_{AI}\cos\alpha = 0$

$\Sigma F_V = 0 \rightarrow \quad -7{,}83 + N_{AI}\sin\alpha = 0$

$$\boxed{N_{AI} = +9{,}791 \text{ kN}}$$

$$\boxed{N_{AH} = 14{,}125 \text{ kN}}$$

(d) Using the method of sections



$\Sigma F_V = 0$

$\Sigma F_H = 0$

$\Sigma M_A = 0$

$\Sigma M_A = 0 \qquad N_{IG} = 0 \longrightarrow$ Previously known

$\Sigma F_V = 0 \qquad -7{,}83 + N_{IC}\sin\alpha = 0$

$\Sigma F_H = 0 \qquad -20 + N_{IC}\cos\alpha + N_{HG} = 0$

$N_{IC} = +9{,}791$ kN $\qquad N_{HG} = +14{,}125$

(e) $\sigma_{All} = 200$ MPa

$$\sigma = \frac{N}{A} \rightarrow A \geq \frac{N}{\sigma_{All}} = \frac{9{,}791}{200 \times 10^3} = 48{,}96 \times 10^{-6} \ m^2$$

$$A = 48{,}96 \ mm^2$$

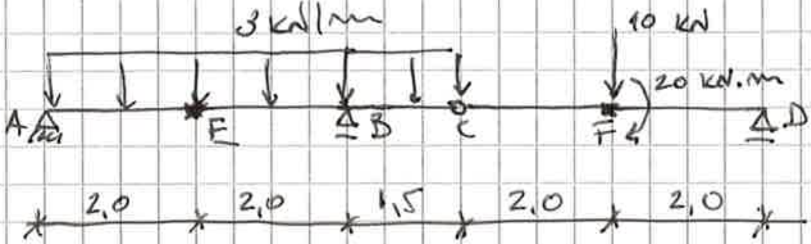
(f)

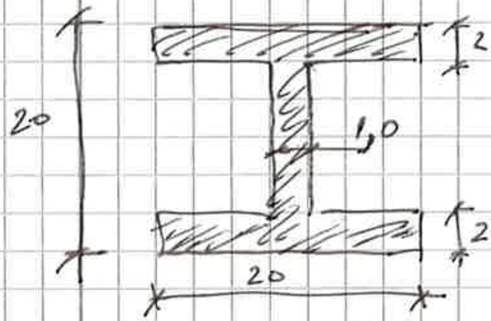
$$\delta = \frac{NL}{EA} = \frac{9{,}791 \times 2{,}5}{200 \times 10^6 \times 48{,}96 \times 10^{-6}} =$$

$$= 2{,}5 \times 10^{-3} \ m = \underline{2{,}5 \ mm} \quad \text{(elongated)}$$

(g) Since FH is a zero member in this structure for the load conditions in existance, the structure will behave the same way as before

Problem 2

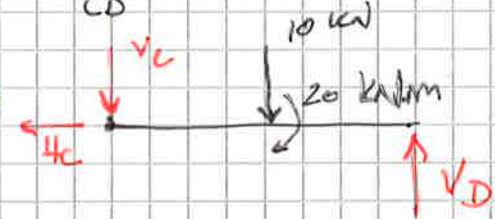




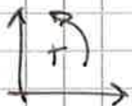
a) Free body diagrams

$\overline{ABC}$

$\overline{CD}$



From $\overline{CD}$



$$\sum M_C = 0 \rightarrow$$

$$-10 \times 2 - 20 + 4 V_D = 0 \rightarrow \boxed{V_D = 10}$$

$$\sum F_V = 0 \rightarrow V_C = 0$$

$$\sum F_H = 0 \rightarrow H_C = 0$$

From $\overline{ABC}$

$$\sum M_A = 0 \rightarrow 4 V_B - 3 \times \frac{5,5 \times 5,5}{2} = 0 \rightarrow \boxed{V_B = 11,34 \text{ kN}}$$

$$\sum F_H = 0 \rightarrow \boxed{H_A = 0}$$

$$\sum F_V = 0 \rightarrow V_A + 11,34 - 3 \times 5,5 = 0 \rightarrow V_A = 5,16 \text{ kN}$$

b) Internal forces $\underline{E}$ and right of $\underline{F}$

E

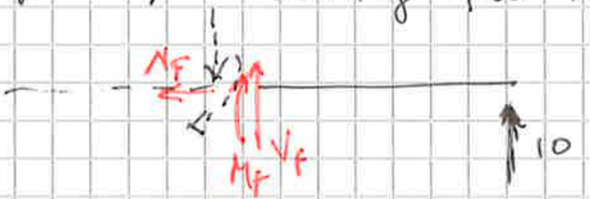
$\Sigma M_E = 0 \rightarrow -5{,}16 \times 2 + 3 \times \frac{2^2}{2} + M_E = 0$

$\boxed{M_E = 4{,}32 \text{ kN.m}}$

$\Sigma F_V = 0 \rightarrow 5{,}16 - V_E - 3 \times 2 = 0 \Rightarrow \boxed{V = -0{,}84 \text{ kN}}$

$\Sigma F_H = 0 \rightarrow \boxed{N_E = 0}$

Right of F → Starting fro right the force and the moment are not there



$\Sigma F_V = 0 \rightarrow \boxed{V_F = -10 \text{ kN}}$ $\quad \Sigma F_H = 0 \rightarrow \boxed{N_F = 0}$

$\Sigma M_F = 0 \rightarrow -M_F + 10 \times 2 = 0 \rightarrow \boxed{M_F = 20 \text{ kN m}}$

c) Area and moment of Inertia

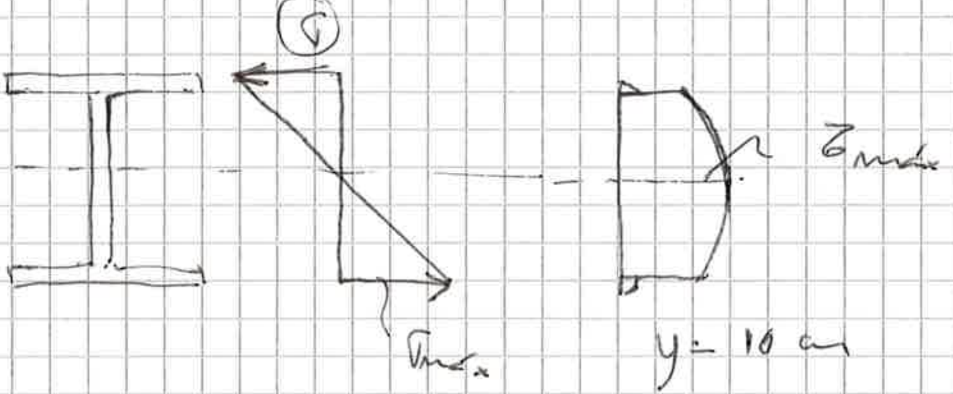
Area: $2 \times (2 \times 20) + 16 \times 1 = \boxed{96 \text{ cm}^2 = 9{,}6 \times 10^{-3} \text{ m}^2}$

Inertia: being symmetrical, the centroid is at mid height.

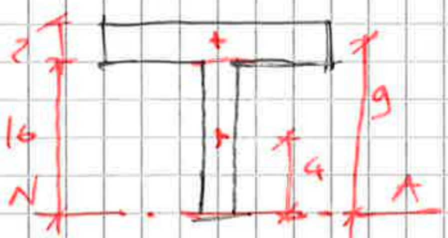
$$I = \frac{20 \times 20^3}{12} - 2 \times \frac{16^3 \times 9{,}5}{12} = 6848 \text{ cm}^4 = 68{,}48 \times 10^{-6} \text{ m}^4$$

d)

$$\sigma = -\frac{M \times y}{I} \qquad \tau = \frac{V\,Q}{I\,t}$$



Computation of Q



$$Q = 20 \times 2 \times 9 + 8 \times 1 \times 4 =$$
$$= 360 + 32 = 392 \text{ cm}^3$$
$$= 0{,}392 \times 10^{-3} \text{ m}^3$$

E

$$\sigma_{max} = -\frac{4{,}32 \times 10^{3} \times (-0{,}10)}{68{,}48 \times 10^{-6}} = 6{,}31 \times 10^{6} \text{ Pa} = \boxed{6{,}31 \text{ MPa}}$$

Right of F

$$\sigma_{max} = -\frac{20 \times (-0{,}10)}{68{,}48 \times 10^{-6}} = \boxed{29{,}21 \text{ MPa}}$$

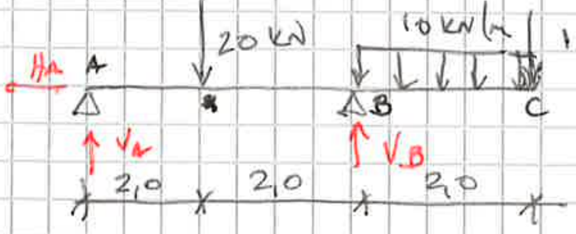
E

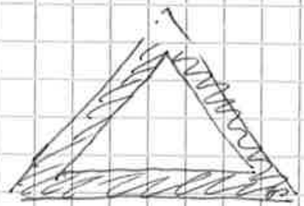
$$\tau = \frac{-0{,}84 \times 0{,}392 \times 10^{-3}}{68{,}48 \times 10^{-6} \times 0{,}01} = -480{,}8 \frac{\text{kN}}{\text{m}^2} = \boxed{-0{,}48 \text{ MPa}}$$

Right of F

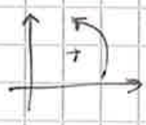
$$\tau = \frac{-10 \times 0{,}392 \times 10^{-3}}{68{,}48 \times 10^{-6} \times 0{,}01} = \boxed{-5{,}724} \text{ MPa}$$

Question 3





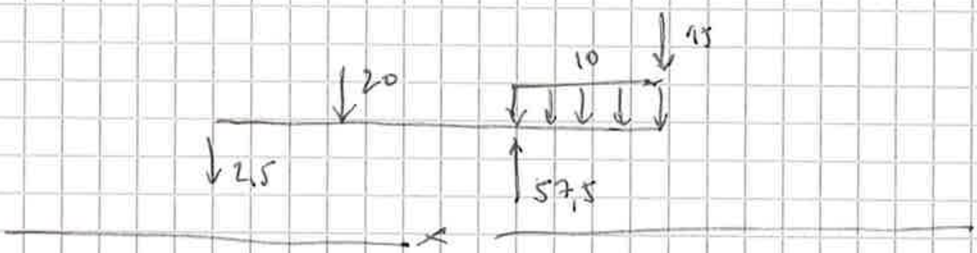
a) Support reactions and FBD



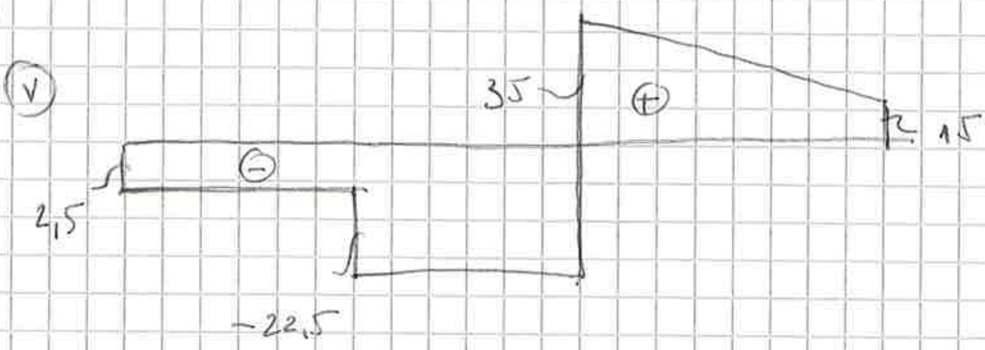
$\Sigma F_V = 0$ $\quad V_A + V_B - 20 - 10\times 2 - 15 = 0 \rightarrow \boxed{V_A = -2,5 \text{ kN}}$

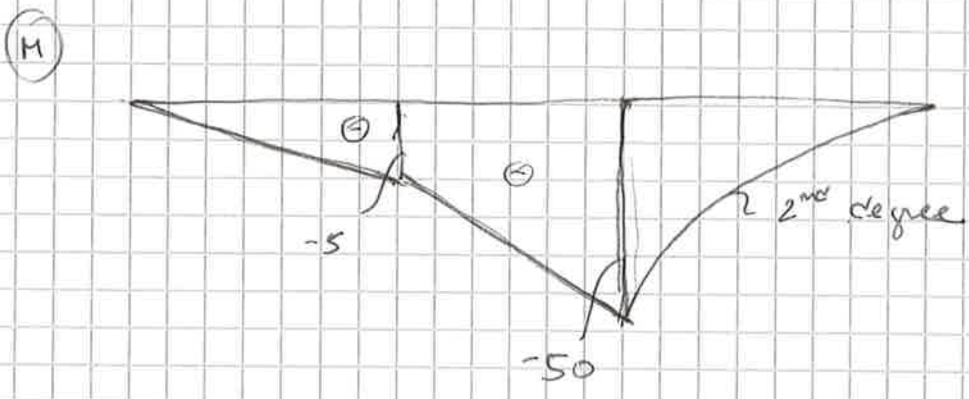
$\Sigma M_A = 0$ $\quad 4V_B - 20\times 2 - 10\times 2\times 5 - 15\times 6 = 0 \quad \boxed{V_B = 57,5 \text{ kN}}$

$\Sigma F_H = 0$ $\quad \boxed{H_A = 0}$

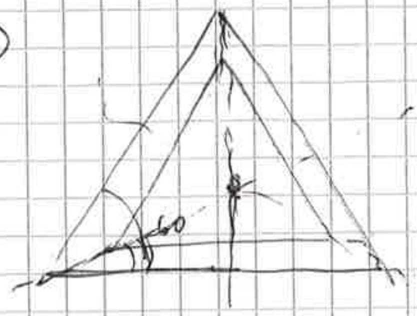


b)



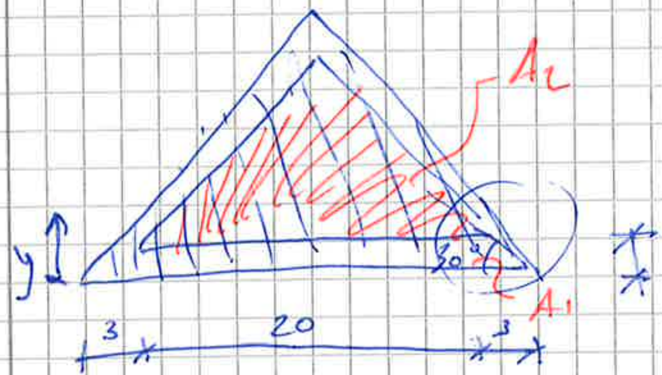


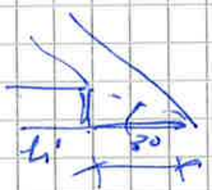
c)



Centroids lay allways, by definition, at any axis of symmetry. Therefore the centroid is located at 1/3 of the height.

Computation:





$$tg\,30 = \frac{h'}{3,0}$$

$$h' = 3,0\ tg\,30 = 1,7321\ cm$$

$$h_1 = 26\ \sin 60 = 22,517\ cm$$

$$h_2 = 20\ \sin 60 = 17,32\ cm$$

Region	Area	$\tilde{y}$	$A \times \tilde{y}$	
1	$\frac{26,0 \times 22,517}{2}$	$\frac{22,517}{3}$	2197,1	7,506
2	$-\frac{20 \times 17,32}{2}$	$1,7321 + \frac{17,32}{3}$	-1300	7,506
	$\Sigma A_i = 119,52$		$\Sigma A_i \tilde{y}_i = 897,1$	

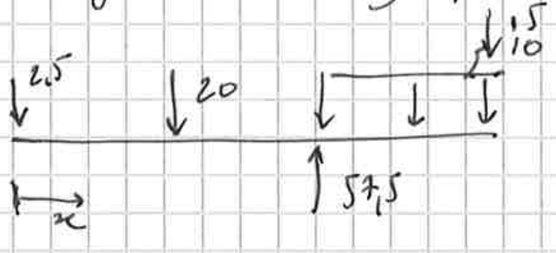
$$\bar{y} = \frac{\Sigma A_i \tilde{y}_i}{\Sigma A_i} = \frac{897,1}{119,5} = \boxed{7,506\ cm}$$

QED

$$I = \frac{b_1 h_1^3}{36} - \frac{b_2 h_2^3}{36} = \frac{22{,}517^3 \times 26}{36} - \frac{17{,}32^3 \times 20}{36} =$$

$$= \underline{5358{,}7 \text{ cm}^4} = 53{,}59 \times 10^{-6} \text{ m}^4$$

d) Using singularity functions



$$M(x) = -2{,}5\,x - 20\,\langle x-2{,}0\rangle + 57{,}5\,\langle x-4\rangle - \frac{10\,\langle x-4\rangle^2}{2}$$

$$EI \cdot \frac{d^2 v}{dx^2} = M(x) \longrightarrow EI \cdot \frac{dv}{dx} = \int M(x)\,dx =$$

$$= -2{,}5\,\frac{x^2}{2} - 10\,\langle x-2\rangle^2 + 57{,}5\,\frac{\langle x-4\rangle^2}{2} - 5\,\frac{\langle x-4\rangle^3}{3} + C_1$$

$$EI \cdot v(x) = -2{,}5\,\frac{x^3}{6} - 10\,\frac{\langle x-2\rangle^3}{3} + 57{,}5\,\frac{\langle x-4\rangle^3}{6} -$$

$$- 5\,\frac{\langle x-4\rangle^4}{12} + C_1 x + C_2$$

Boundary conditions

$$x = 0 \longrightarrow v = 0 \longrightarrow C_2 = 0$$

$$x = 4 \longrightarrow v = 0$$

$$-2{,}5 \times \frac{4^3}{6} - 10 \times \frac{2^3}{3} + 4\,C_1 = 0$$

$$C_1 = 11{,}33$$

So, Rotation at A equals

$$\frac{dv}{dx}(x=0) = \frac{C_1}{EI} = \frac{13,33}{200\times10^{6} \times 53,59\times10^{-6}} =$$

$$= 1,244 \times 10^{-3} \text{ rad} =$$

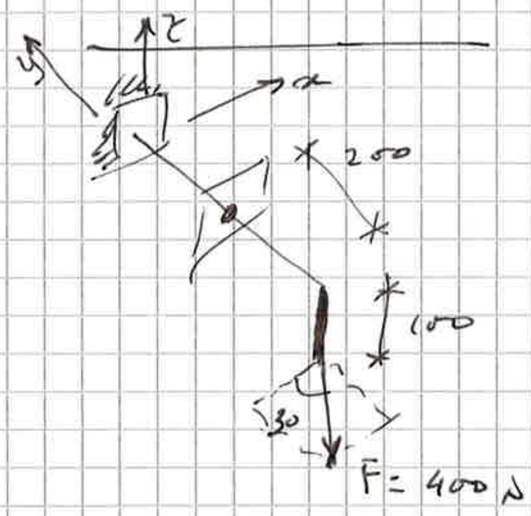
$$= 0,071^{\circ}$$

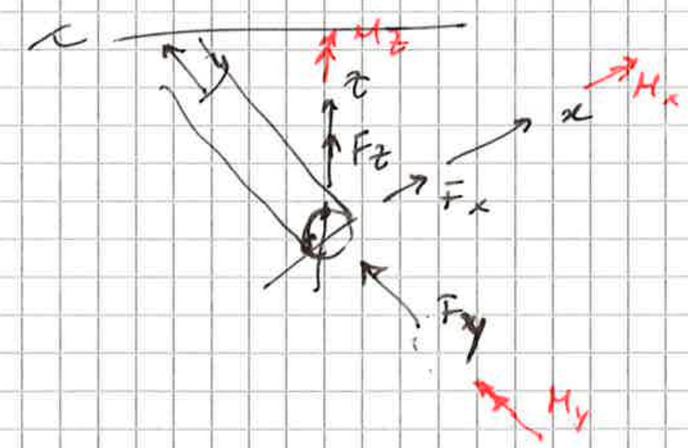
e)

Vertical displacement at point C

$$EI \cdot v(x=6) = -2,5 \times \frac{6^3}{6} - 10 \times \frac{4^3}{3} + 57,5 \times \frac{2^3}{6} - 5 \times \frac{2^4}{12} + 6 \times 13,33$$

$$v(x=6) = -\frac{153,33}{EI} = -1,43 \times 10^{-2} \text{ m} = 1,43 \text{ cm}$$





a)

Internal forces at ED

$$F_x = -400 \times \cos 30 = 346,41 \text{ N}$$

$$F_y = -400 \times \sin 30 = 200 \text{ N}$$

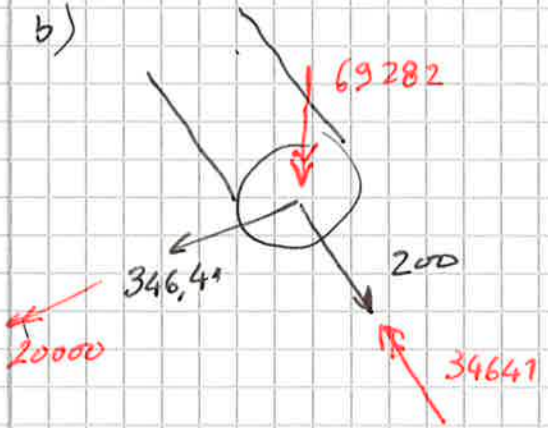
$$F_z = 0$$

$M_x = -200 \times 100 = -20000$ N.mm

$M_y = 346{,}41 \times 100 = 34641$ N.mm

$M_t = -346{,}41 \times 200 = 69282$ N.mm

b)



$A = \pi R^2 = \pi \times 15^2 = 706{,}86$ mm^2

$J = \frac{\pi R^4}{2} = \frac{\pi \times 15^4}{2} = 79522$ mm^4

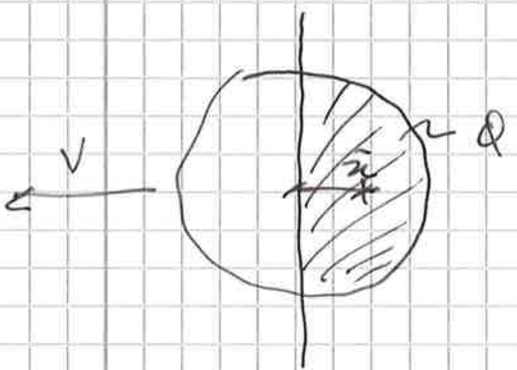
$I = \frac{\pi R^4}{4} = \frac{\pi \times 15^4}{4} = 39761$ mm^4

At point D:

$\sigma = +\frac{200}{A} - \frac{20000 \times 15}{I} = \underline{-7{,}26 \text{ MPa}}$

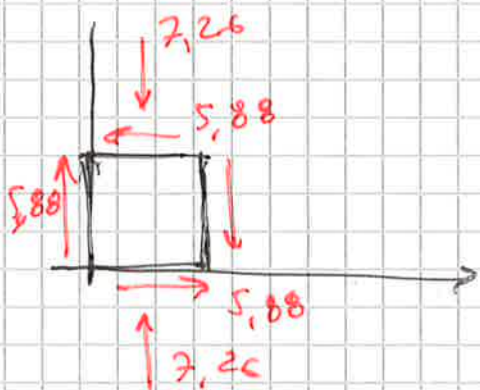
$\tau = \frac{T \times R}{J} - \frac{V\,Q}{I\,t} = \frac{34641 \times 15}{79522} - \frac{346{,}41 \times 2250}{39761 \times 30} =$

$= \underline{5{,}88 \text{ MPa}}$



$Q = A \times \bar{x} = \frac{\pi R^2}{2} \times \frac{4R}{3\pi} =$

$= \frac{2}{3} R^3 = 2250$ mm^3



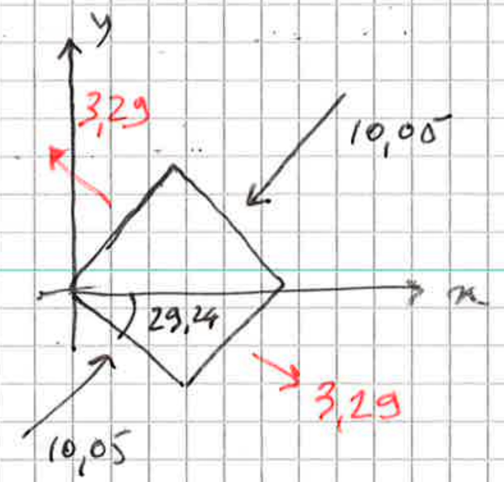
$\sigma_x = 0 \quad \sigma_y = -7,21 \quad \tau_{xy} = -5,88$

$$\tan 2\theta_p = \frac{-5,88}{7,21/2} = -1,631$$

$$\theta_p = -29,24°$$

$$\sigma_{1,2} = \frac{0-7,21}{2} \pm \sqrt{\left(\frac{7,21}{2}\right)^2 + \left(-5,88\right)^2} =$$

$\sigma_1 = 3,29$ MPa $\quad \sigma_2 = -10,50$ MPa



d) $$\tau_{máx} = \frac{\sigma_{máx} - \sigma_{mín}}{2} =$$

$$= \frac{3,29 - (-10,05)}{2} = 6,67 \text{ MPa}$$

e)

$$\varepsilon_z = \varepsilon_x = \left[0 + \left(-0,30 \times (-7,21)\right)\right] \times \frac{10^6}{200 \times 10^9} = +10,82 \times 10^{-6} \text{ m/m}$$

$$\varepsilon_y = \frac{-7,21 \times 10^6}{200 \times 10^9} = -36,05 \times 10^{-6} \text{ m/m}$$

$$\gamma_{xy} = \frac{-5,88 \times 10^6}{200 \times 10^9} = -29,4 \times 10^{-6} \quad \gamma_{xz} = \gamma_{yz} = 0$$

f) If the Diameter decreases:

$$\sigma = \frac{M \times y}{I} \qquad \tau = \frac{VQ}{It}$$

$$\tau = \frac{T \times r}{J} \qquad \sigma = \frac{N}{A}$$

In all cases, if D decreases the Inertia, Area and J decrease also more intensily than the Diameter

So the stresses must <u>increase</u>