University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: **PET 510 – Computational Reservoir and Well Modeling**

DATE: **24 november, 2015**

TIME: **4 hours**

AID: **Basic calculator is allowed**

THE EXAM CONSISTS OF 5 PROBLEMS ON 6 PAGES AND APPENDIX A - D

REMARKS:
You may answer in English or Norwegian. Exercises 1 and 2 (part A) and exercises 3-5 (part B) are given equal weight.

COURSE RESPONSIBLE: Steinar Evje and Kjell-Kåre Fjelde

Problem 1.

(a) Consider the linear transport equation

$$(*) \qquad u_t + e^{-t} u_x = 0, \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x, t = 0) = \phi(x).$$

Compute the solution $u(x,t)$ and verify that it satisfies (*) and (**).

(b) Make a sketch of the characteristics in the $x - t$ coordinate system. Use this sketch and explain by words the transport process described by (*).

(c) Now we choose $\phi(x) = \exp(-x^2)$.
Make a sketch of the solution when time goes to infinity.

(d) Formulate a discrete scheme based on the upwind principle for solving the problem in (a). We consider the domain $[-L, L]$ and assume a grid of M cells with cell center $x_1, \ldots, x_M$. The scheme allows us to compute $\{u_j^{n+1}\}_{j=1}^{j=M}$ where we assume that $u_0^{n+1} = u_{M+1}^{n+1} = 0$.
Demonstrate how we can show the estimate

$$\sum_{j=1}^{M} |u_j^{n+1}| \leq \sum_{j=1}^{M} |u_j^n|$$

under an appropriate condition on $\frac{\Delta t}{\Delta x}$.

(e) Compute the solution as in (a) but where we have added a source term such that we now consider

$$(*)' \qquad u_t + e^{-t} u_x = x + e^{-t}, \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

Check that the new solution satisfies $(*)'$.

Problem 2.

(a) In the following we consider a horizontal 1D reservoir.
- State the single-phase porous media mass balance equation in 1D (without source term) and identify the various variables (rock and fluid).
- Introduce Darcy's law and derive an equation for the pressure where it is assumed that $\phi = \phi(p)$, $\rho = \rho(p)$, and permeability and viscosity are constant.
- Assuming a weakly compressible rock (compressibility c_r is small) we get a linear relation for $\phi(p)$.

$$\phi(p) = \phi_0[1 + c_r(p - p_0)],$$

where p_0 and ϕ_0 are reference pressure and porosity. Use this together with the assumption that the fluid is incompressible and show that we then can obtain a pressure equation of the form

$$(*) \qquad p_t = \kappa p_{xx}, \qquad x \in \mathbb{R} = (-\infty, +\infty),$$

and identify the constant parameter $\kappa > 0$.

(b) Verify that

$$(**) \qquad p(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{x}{2\sqrt{\kappa t}}} e^{-\theta^2}\, d\theta$$

satisfies (*).
What must be the initial condition corresponding to the solution (**) ?
(**Hint**: use that $\int_{-\infty}^{\infty} e^{-\theta^2} d\theta = \sqrt{\pi}$)

(c) Next, consider (*) with $\kappa = \frac{1}{9}$ on the spatial domain $[-1,1]$. We assume the boundary conditions

$$p(x=-1,t) = 0, \qquad p(x=1,t) = 1.$$

Divide the domain into 6 cells with points x_1, x_2,..., x_6 located at the center of each cell. Formulate a discrete version of (*) based on an explicit time discretization for cells $j = 2, \dots, 5$.
Set

$$p_1^{n+1} = 0, \qquad p_6^{n+1} = 1$$

consistent with the boundary condition.

(d) Use the scheme defined in (c) and compute the numerical solution after a time $T = 1$ by using two time steps, i.e., $\Delta t = 1/2$ with initial data as given in (b).

Exam Part B – Solving Nonlinear Equations & Modelling of Well Flow

There are 11 questions in total. Some formulas, equations and Matlab codes are found in Appendixes. This part constitutes 50 % of exam.

Exercise 3 – Bisection method

a) The quadratic $(x-0.4)(x-0.6)=x^2-x+0.24$ has zeros at x = 0.4 and x = 0.6. Why can we not choose a = 0 and b = 1 as our initial search interval to pick out one of the roots using the bisection method ?

b) We will consider the function $f(x)=e^x-x-2$. Write down the necessary matlab code to plot the roots of this function. Here you should be careful with choosing the correct x interval such that the roots are shown on the graph.

c) Consider the equation $e^x-x-2=0$. We want to pick out the largest root. The root shall be found with an accuracy such that $|f(x_3)|< ftol=0.05$. Fill out the following table until a satisfactory solution is found.

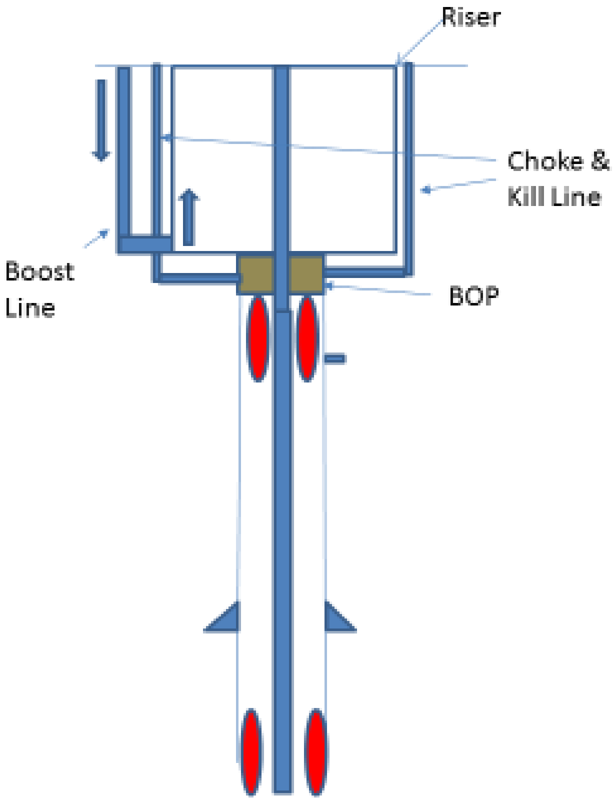
Iteration	x1	x2	x3	f(x1)	f(x2)	f(x3)
1						
2						
3						
etc						

d) Explain how you would change the code in Appendix B to solve the problem given in c)

Exercise 4 - Well pressures & Cuttings transport

We are considering a vertical well that is 5000 meter deep in total. The mudweight is 1.7 sg and we use a waterbased mud (gas kicks will migrate in static well).

The BOP is situated at the sea bottom with a riser connecting it to surface. The riser has an inner diameter of 19 inches. The distance from the rig floor to the BOP is 1000 meters. A 9 5/8" casing with inner diameter 8.5 inches has been run to 4000 m and is hanging in the wellhead just below the BOP. We have continued drilling an 8 ½" hole.



The drillpipe has an outer diameter of 5.5 inches and runs from the rig down to bottom of the well.

a) In the annulus below the BOP, we must have that the annular flow velocity must be at least 150 ft/min. What must the minimum flowrate in liters per minute be then ?

b) Let us assume that a flowrate of 1000 lpm was adequate for hole cleaning in the annulus below the BOP. The riser has a much larger diameter than the rest of the well. Hence, in order to ensure proper cuttings transport also in the riser, a boost line provides additional flow in the riser. What must the flowrate from the boost line into the riser be in liters per minute to also ensure that the cuttings is transported in the riser ?

For the remaining questions, we do not consider the use of the boost line.

c) During circulation with 1000 lpm we have that the annular friction is 25 bars. Calculate the ECD (equivalent circulating density) at bottom in sg.

d) If a kick is taken, BOP has to be closed and the kick will at a later stage be circulated out of the well through the chokeline using a specific kill rate of e.g. 500 liters per minute. The chokeline has a quite small inner diameter, typical 3 inches.

In order to do this safely one need to approximate the friction in the chokeline. In operations, they estimate this by the following operation at regular intervals.

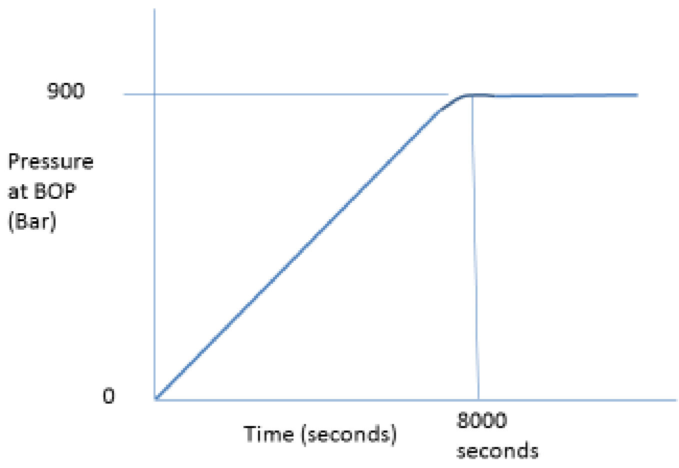
1) Pump mud up the riser with 500 lpm and monitor pump pressure, e.g Ppump,riser = 220 bar
2) Close BOP and pump up through chokeline with 500 lpm and monitor pump pressure, e.g. Ppump,choke =230 bar
3) Then they say that chokeline friction is equal to Ppump,choke - Ppump,riser= 10 bar

Explain what kind of assumption they make when doing this!

e)

A kick has been taken at 5000 meters. The BOP was shut in and kept closed and the kick migrated to just beneath the BOP. The pressure at the BOP was monitored and the following pressure build up was seen in the figure below (next page). We assume that the mud is incompressible. Try to answer the following question.

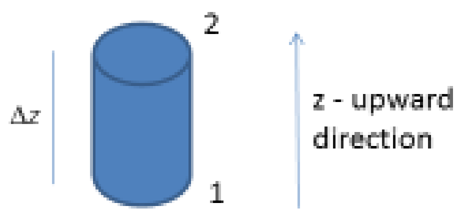
What is the pressure of the zone that induced the kick and what is the S value in the gas slippage model?



Exercise 5 – Conservation laws

a) What kind of a model should be used to simulate the pressure build up seen in Exercise 4 (steady state or transient). Explain why!

b) The following ordinary differential equation $\frac{\partial}{\partial z} p = -(\rho_l g + \frac{\Delta p_{fric}}{\Delta z})$ can be integrated across a vertical well segment and we obtain the following formula $p_1 = p_2 + \rho_l g \Delta z + \Delta p_{fric}$. When will p_1 have smallest value when comparing upward and downward flow. Explain why!



Appendix A – Some Units & Formulas

1 inch =2.54 cm = 0.0254 m

1 feet = 0.3048 m

1 bar = 100000 Pa

1 sg = 1 kg/l (sg - specific gravity)

$M = Q \cdot \rho$ M massrate (kg/s), Q Volumerate (m^3/s), ρ density (kg/m^3)

$Q = v \cdot A$ Q Volumerate (m^3/s), v velocity m/s. A area m^2

$p = \rho \cdot h \cdot 0.0981$ p (bar), ρ density (sg), h – vertical depth (m)

$\frac{P \cdot V}{T} = C$, from Ideal gas law, NB T is in Kelvin and the relation to Celsius is $K = {}^{\circ}C + 273,15$

$P \cdot V = C$, Boyles law (temperature is assumed constant)

Appendix B

Main.m

```
% Main program that calls up a routine that uses the bisection
% method to find a solution to the problem f(x) = 0.
% The search intervall [a,b] is specified in the main program.
% The main program calls upon the function bisection which again calls upon
% the function func.

% if error = 1, the search intervall has to be adjusted to ensure
% f(a) x f(b)<0

% Specify search intervall, a and b will be sent into the function
% bisection
 a = 4.0;
 b = 5.0;

% Call upon function bisection which returns the results in the variables
% solution and error.
  [solution,error] = bisection(a,b);

 solution % Write to screen.
 error   % Write to screen.
```

Bisection.m

```
function [solution,error] = bisection(a,b)

% The numerical solver implemented here for solving the equation f(x)= 0
% is called Method of Halving the Interval (Bisection Method)

% You will not find exact match for f(x)= 0. Maybe f(x) = 0.0001 in the
end.
% By using ftol we say that if abs(f(x))<ftol, we are satisfied. We can
% also end the iteration if the search interval [a,b] is satisfactory
small.
% These tolerance values will have to be changed depending on the problem
% to be solved.


 ftol = 0.01;

% Set number of iterations to zero. This number will tell how many
 % iterations are required to find a solution with the specified accuracy.

  noit = 0;

  x1 = a;
  x2 = b;
```

```
  f1 = func(x1);
  f2 = func(x2);


%   First include a check on whether f1xf2<0. If not you must adjust your
%   initial search intervall. If error is 1 and solution is set to zero,
%   then you must adjust the search intervall [a,b].


 if (f1*f2)>=0
     error = 1;
     solution = 0;
 else
 % start iterating, we are now on the track.
     x3 = (x1+x2)/2.0;
     f3 = func(x3);

     while (f3>ftol | f3 < -ftol)
        noit = noit +1 ;

        if (f3*f1) < 0
           x2 = x3;
        else
           x1 = x3;
        end

        x3 = (x1+x2)/2.0;
        f3 = func(x3);
        f1 = func(x1);

     end
     error = 0;
     solution = x3;
     noit % This statement without ; writes out the number of iterations to
the screen.
 end
```

func.m

```
function f = func(x)


  f = x^2-4*x+2;
```

Appendix C

```
% Program where the Larsen Cuttings Transport Model is implemented

%  First specify all input parameters:

do = 8.5; % Outerdiameter (in) ( 1 in = 0.0254 m)
di = 5; % Innerdiameter   (in)
rop = 33 % Rate of Penetration - ROP ft/hr (1 ft = 0.3048m)
pv = 15 % Plastic viscosity (cP)
yp = 16 % Yield point (lbf/100ft2)
dcutt = 0.1 % Cuttings diameter (in) (1 inch = 0.0254 m)
mw = 10.833 % Mudweight (ppg - pounds per gallon) 1 ppg = 119.83 kg/m3.
rpm = 80 % rounds per minute
cdens = 19 % cuttings density (ppg - pounds per gallon)
angstart = 50 % Angle with the vertical

% vcut - Cuttings Transport Velocity (CTF in Larsens paper)
% vcrit - Critical Transport fluid velocity (CTFV) in Larsens paper. This
% is the minimum fluid velocity required to maintain a continously upward
% movement of the cuttings.
% vslip - Equivalent slip velocity (ESV) defined as the velocity difference
% between the cuttings and the drilling fluid
% vcrit = vcut+vslip
% All velocities are in ft/s.
% ua - apparent viscosity

% It should be noted that the problem is nested. Vcrit depends on vslip
% which again depends on an updated/correct value for vcrit. An iterative
% approch on the form  x(n+1) = g(x(n)) will be used.

for i = 1:8

ang(i)=angstart+i*5
vcut = 1/((1-(di/do)^2)*(0.64+18.16/rop));

vslipguess = 3;
vcrit = vcut + vslipguess;

% Find the apparent viscosity (which depends on the "guess" for vcrit)
ua = pv+ (5*yp*(do-di))/vcrit

% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
if (ua <= 53)
 vslip = 0.0051*ua+3.006;
else
 vslip = 0.02554*(ua-53)+3.28;
end

%Now we have two estimates for vslip that can be compared and updated in a
% while loop. The loop will end when the vslip(n+1) and vslip (n) do not
% change much anymore. I.e the iterative solution is found.
n=1;
while (abs(vslip-vslipguess))>0.01
 vslipguess = vslip;
 vcrit = vcut + vslipguess;
% Find the apparent viscosity (which depends on the "guess" for vcrit)
  ua = pv+ (5*yp*(do-di))/vcrit;
```

```
% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
  if (ua <= 53)
   vslip = 0.0051*ua+3.006;
  else
   vslip = 0.02554*(ua-53)+3.28;
  end
  n=n+1;
  vslip  % Take away ; and you will se how vslip converges to a solution
end % End while loop

%
% Cuttings size correction factor: CZ = -1.05D50cut+1.286
 CZ = -1.05*dcutt+1.286
% Mud Weight Correction factor (Buoancy effect)
  if (mw>8.7)
  CMW = 1-0.0333*(mw-8.7)
  else
  CMW = 1.0
  end

% Angle correction factor

CANG = 0.0342*ang(i)-0.000233*ang(i)^2-0.213

vslip = vslip*CZ*CMW*CANG;   % Include correction factors.

% Find final minimum velocity required for cuttings transport (ft/s).

vcrit = vcut + vslip

vcritms = vcrit*0.3048 % Velocity in m/s

Q = 3.14/4*((8.5*0.0254)^2-(5*0.0254)^2)*vcritms % (m3/s)
Q = Q*60*1000  % (lpm)

yrate(i)=Q
end

plot(ang,yrate)
```

Appendix D – Steady State Model for Two Phase Flow

Conservation of liquid mass

$$\frac{\partial}{\partial z}(A\rho_l \alpha_l v_l) = 0$$

Conservation of gas mass

$$\frac{\partial}{\partial z}(A\rho_g \alpha_g v_g) = 0$$

Conservation of momentum.

$$\frac{\partial}{\partial z} p = -(\rho_{mix} g + \frac{\Delta p_{fric}}{\Delta z})$$

Gas slippage model (simple):

$v_g = K v_{mix} + S$ (K=1.2)

Liquid density model (simple)

$\rho_l(p) = \rho_{lo} + \frac{(p - p_O)}{a_L^2}$, assume water: $\rho_{lo} = 1000$ kg/m^3, $p_O = 100000 Pa$, $a_L = 1500$ m/s

Gas density model (simple)

$\rho_g(p) = \frac{p}{a_g^2}$, ideal gas: $a_g = 316$ m/s.

Friction model

The friction model presented here is for a Newtonian fluids like water. The general expression for the frictional pressure loss gradient term is given by:

$$\frac{\Delta p_{fric}}{\Delta z} = \frac{2 f \rho_{mix} v_{mix} abs(v_{mix})}{(d_{out} - d_{in})} \quad \text{(Pa/m)}$$

A - (m^2)

ρ_i - phase densities (kg/m^3), liquid – > i=l, gas ->i =g

v_i - phase velocities (m/s)

p - pressure (Pa)

g – gravity constant 9.81 m/s^2

α_i - phase volume fractions taking values between 0 and 1. $\alpha_l + \alpha_g = 1$.

$\rho_{mix} = \alpha_l \rho_l + \alpha_g \rho_g$ - mixture density

$v_{mix} = \alpha_l v_l + \alpha_g v_g$ - mixture velocity