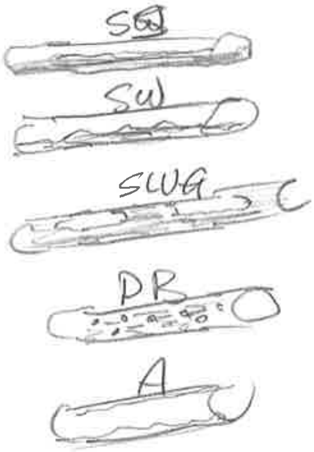
7/12 - 2016

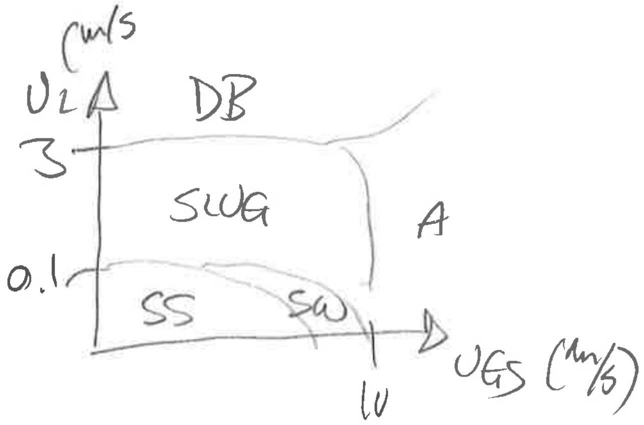
SOLUTIONS PET 505 - 2016
MULTIPHASE FLOW PART -

(1)

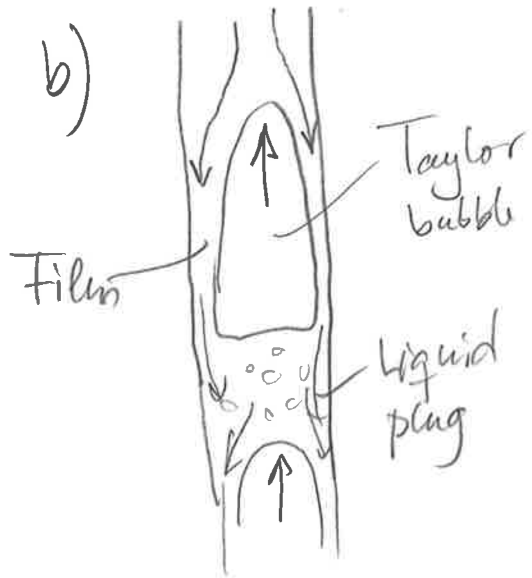
PROBLEM 1

a)





b)



$\Rightarrow$ Yes, the bubble can move upwards, since the film is flowing counter to the bubble – downwards

[As in lecture Nov. 9th]

c) Ideal gas $P \cdot V = n \cdot R \cdot T \Rightarrow P = \frac{n}{V} \cdot R \cdot T$

mol/volum ~ density

$$\Rightarrow \rho_G(P,T) = \rho_{G\,ref} \cdot \frac{P}{P_{ref}} \cdot \frac{T_{ref}}{T}$$

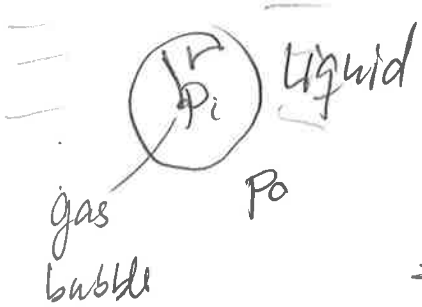
$T_{ref} = 25C$

$P_{ref} = 1 \text{ bar}$

$P = 5 \text{ bar}, \; T = 80°C = (80 + 273.15)$

$\Rightarrow \rho_G(P,T) = \underline{5.07} \text{ kg/m}^3$

When do we get an impact of surface tension, i.e. what diameter? (or what "critical" radius)



$$\Delta p = p_i - p_o = \frac{2\cdot\sigma}{r_{crit}} \Rightarrow r_{crit} = \frac{2\cdot\sigma}{\Delta p}$$

$$\Rightarrow r_{crit} = \frac{2\cdot 72\cdot 10^{-3}\,N/m}{100\cdot 10^{-3}\,bar} = \frac{144\cdot 10^{-3}\,N/m}{10^{4}\,\frac{N}{m^2}}$$

10^{5} / 10^{4}

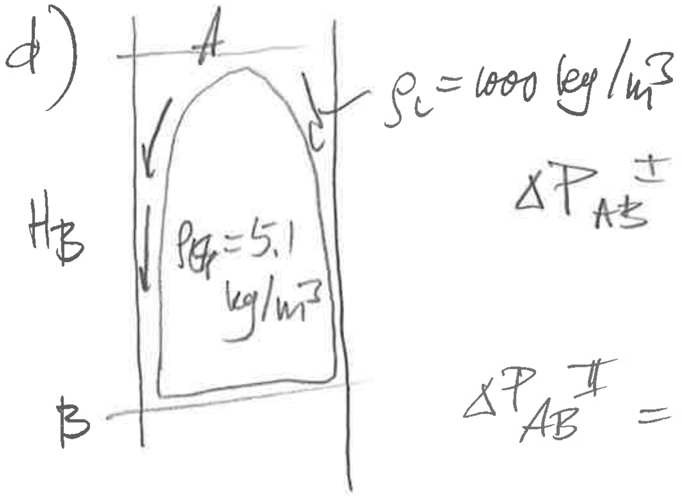
10^{4} Pa

$$= 144\cdot 10^{-7}\,m = 144\cdot 10^{-4}\,mm = 1.44\cdot 10^{-2}\,mm$$

$$= 1.44\cdot 10^{-5}\,m = \underline{14.4\,\mu m}$$

(= VERY SMALL!)

d)



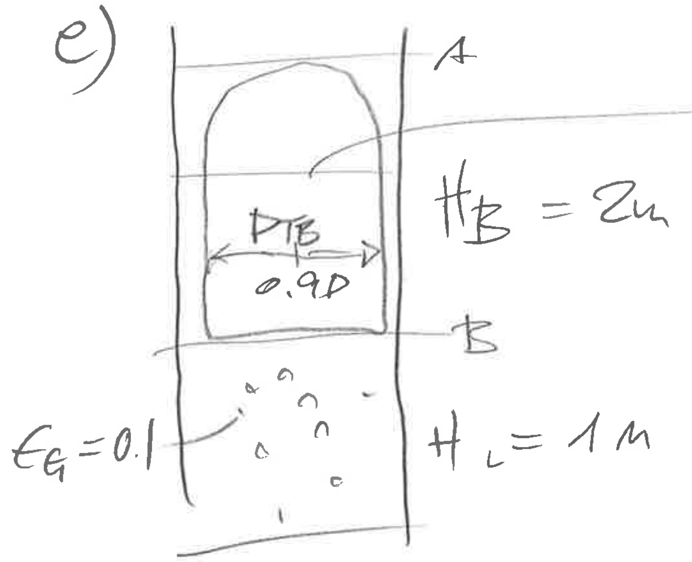
$$\Delta P_{AB}^{I} = \rho_G \cdot g \cdot H_B = 5.1\cdot 9.81\cdot 2$$
$$= \underline{100\ Pa} = 10^{-3}\ bar$$

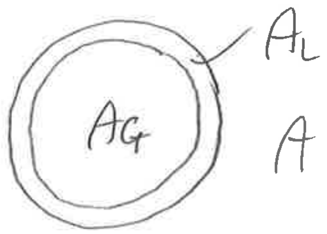
$$\Delta P_{AB}^{II} = \rho_L \cdot g \cdot H_B = 1000\cdot 9.81\cdot 2$$
$$= \underline{19620\ Pa} = 0.196\ bar$$

$\Rightarrow \underline{\Delta P_{AB}^{II} \sim 200\cdot \Delta P_{AB}^{I}}$ $\Leftrightarrow$ PARADOX, since if P_A is well defined to be $P_A = 5$ bar, then the pressure P_B should also be well defined. But here we have two different P_B values.

The solution to the paradox is that friction in the down flowing film balances the hydrostatic "drive".

e)





$$A = A_L + A_G$$

$$\epsilon_G = \frac{A_G}{A} = \frac{\pi \cdot D_{TB}^2/4}{\pi \cdot D^2/4} = \left(\frac{D_{TB}}{D}\right)^2 = (0.9)^2 = \underline{0.81}$$

i) So, $\epsilon_{GTB} = \frac{A_G}{A} = \underline{\underline{0.81}}$

ii) The homogeneous model says that

$$\Delta P_{hyd} = \rho_{mix} \cdot g \cdot H_B = \Big[\underbrace{\rho_L \cdot (1-\epsilon_G) + \rho_G \cdot \epsilon_G}_{194.1}\Big] \cdot 9.81 \cdot 2 = \underline{3.8 \cdot 10^3 \ Pa}$$

$\left[\sim 5.1 \cdot \Delta P_{AB}^{II} \text{ comparison}\right]$

iii) We could get the total ΔP_{ABC} by adding the extra hydrostatic contribution

$$\Delta P_{BC} = \rho_{mix} \cdot g \cdot H_L = (\rho_L \cdot 0.9 + \rho_G \cdot 0.1) \cdot 9.81 \cdot 1$$

$$= (1000 \cdot 0.1 + 500 \cdot 0.9) \cdot 9.81 \cdot 1 = 1.03 \cdot 10^3$$

$$\Rightarrow \Delta P_{AC} = \Delta P_{AB} + \Delta P_{BC} = 3.8 \cdot 10^3 + 1.03 \cdot 10^3 = 4.8 \cdot 10^3 \ Pa$$

$$\left.\begin{array}{c}\text{AVERAGE PRESSURE GRADIENT}\\ \text{OVER THE SLUG}\end{array}\right\} = \frac{\Delta P_{AC}}{H_B + H_L} = \frac{4.8 \cdot 10^3 \ Pa}{3 \ m} = 1.6 \cdot 10^3 \ \frac{Pa}{m}$$

PROBLEM 2

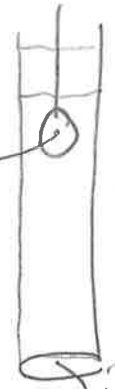
a)

i) 

$\rho_L = 1000$

$H_{liq} = 10\,m$

$$\Rightarrow P_{bottom} = \rho_L \cdot g \cdot H_{liq} = 1000 \cdot 9.8 \cdot 10 = 9.8 \cdot 10^4\,Pa$$
$$= 0.98\,bar$$

ii) 

$$\text{New } H_{liq}^{*} = H_{liq} + V_{ball} = H_{liq} + \frac{V_{ball}}{A} = 10 + \frac{3.8 \cdot 10^{-4}\,m^3}{7.9 \cdot 10^{-3}\,m^2}$$

Or: $\Delta H_{liq} = \dfrac{V_{ball}}{A}$ Better $\Rightarrow 0.484 \cdot 10^{-1}\,m$
$$= 4.84 \cdot 10^{-2}\,m$$

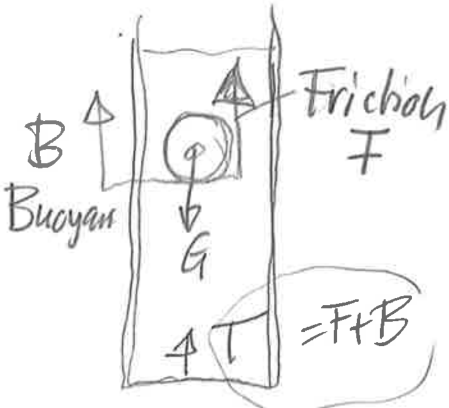
$$A = \frac{\pi D^2}{4} = \frac{\pi \cdot 10^{-2}}{4} = 7.9 \cdot 10^{-3}\,m^2$$

$$V_{Ball} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot \left(\frac{D}{2}\right)^3$$
$$= \frac{4}{3}\pi \cdot \frac{D^3}{8} = \frac{\pi D^3}{6} = \frac{\pi \cdot (0.09)^3}{6}$$
$$= 3.8 \cdot 10^{-4}\,m^3$$

$$\Delta p = \rho_L \cdot g \cdot \Delta H_{liq}$$
$$= 10^3 \cdot 9.81 \cdot 4.84 \cdot 10^{-2}$$
$$= 47.5 \cdot 10\,Pa$$
$$= 4.75 \cdot 10^2\,Pa \quad [10^2\,Pa = 1\,mbar]$$
$$= 4.75\,mbar$$

b) After some time the fall velocity is so high that friction + Buoyancy balances the gravitational force

$\Rightarrow$ Sum of forces $= 0$ $\Rightarrow$ constant fall velocity



Total force on Ball: $T = G - (F + B)$

The "counter" forces $F + B$ is set up by the fluid. To keep the fluid non-accelerated also, the bottom

%

takes the counter forces against the liquid.
⇒ So, yes, the pressure at the bottom now increases compared to a ii)!

c) $\boxed{f_M = 4 \cdot f_F}$ $\left(\frac{dP}{dx}\right)_F = \frac{4}{D} \cdot f_F \cdot \frac{1}{2}\rho U^2$

$\left(\frac{dP}{dx}\right)_M = \frac{1}{D} \cdot f_M \cdot \frac{1}{2}\rho U^2$

d) Testing the friction factors for some cases, making a table

	Re 5000	10 000	10^5	
Drew, Koo, McAdam a: $f_1 = 0.0056 + 0.5 \cdot Re^{-0.32}$	$3.84 \cdot 10^{-2}$	$3.18 \cdot 10^{-2}$	$1.82 \cdot 10^{-2}$	⇒ Moody
b) $f_2 = 0.052 \cdot Re^{-0.21}$	$8.7 \cdot 10^{-3}$	$7.5 \cdot 10^{-3}$	$4.6 \cdot 10^{-3}$	~ Nesbler Fanning
c) $f_3 = -1.8 \cdot \log_{10}\left(\frac{6.9}{Re}\right)$	5.1482 $3.77 \cdot 10^{-2}$	5.69 $3.09 \cdot 10^{-2}$	9.29 $1.78 \cdot 10^{-2}$	Fail!
d) $f_4 = 0.36 \cdot Re^{-0.2}$	$6.55 \cdot 10^{-2}$	$5.71 \cdot 10^{-2}$	$3.6 \cdot 10^{-2}$	Fail?
Dukler FANNING $= 0.046 \cdot Re^{-0.2}$	$8.4 \cdot 10^{-3}$	$7.3 \cdot 10^{-3}$	$4.6 \cdot 10^{-3}$	

a) is Moody b) Fanning c) & d) are wrong!