2017 5

Exercise 1.

a) We consider the general conservation law

$$(*) \qquad u_t + f(u)_x = 0, \qquad u(x, t=0) = u_0(x), \qquad x \in \mathbb{R},$$

where $f(u)$ is a nonlinear function.
What is a *weak solution* of (*)? Derive the mathematical formulation by starting with (*).
Why do we need to consider weak solutions? What is a potential challenge with weak solutions?

b) Assume that we consider a solution of (*) with a discontinuity $(u_l(t), u_r(t))$ where u_l refers to the left and u_r to the right value of u at the discontinuity. Can you formulate an expression which characterizes the speed $s(t)$ of the moving discontinuity (Rankine-Hugoniot condition)?

$S = \frac{f(u_L) - f(u_R)}{u_L - u_R}$ eller $\iint_R (u + f(u)_x)\,dx\,dt$

Make a sketch of the discontinuity in $x - t$ coordinate system and derive this expression.

c) Consider (*) with $f''(u) > 0$ and $u_0(x)$ that contains a single jump (u_l, u_r) located at $x = 0$ with $u_l < u_r$. What kind of solution should we seek for this riemann problem? Can you derive the general form of the solution?

d) In the following we assume that $f(u) = -u(1-u)$. We consider the initial data u_0 given by

$$(**) \qquad u_0(x) = \begin{cases} 1, & 0 < x < 1; \\ 0, & \text{otherwise.} \end{cases}$$

$x=0$ og $x=1$ 2

Compute the solution of this problem for the time period before the initial jump at $x = 0$ starts interacting with the jump located at $x = 1$. Sketch the solution at time $T = 1/2$.

e) Now, we consider the initial data u_0 given by

$$(***) \qquad u_0(x) = \begin{cases} 1, & 0 < x < 1; \\ 0.5, & \text{otherwise.} \end{cases}$$

$x=0$ og $x=1$ 2

Compute the solution of this problem. Make a sketch of the solution at time $T = 1$ and $T = 3$.

Exercise 2. The Buckley-Leverett model for injection of water in a horizontal 1D oil reservoir takes the form in dimensionless variables

$$(A) \qquad \begin{cases} S_t + f(S)_x = 0, & x \in [0,1] \\ S(x=0,t) = 1, & \end{cases}$$

where S represents the water saturation.

a) Describe how the fractional flow function $f(S)$ is expressed in terms of water and oil mobility functions $\lambda_w(S)$ and $\lambda_o(S)$, and define these. Make a sketch of a typical fractional flow function $f(S)$ when using Corey relative permeability functions. What is the impact of water and oil viscosity μ_w and μ_o on the fractional flow function $f(S)$. Illustrate this in the figure where you sketch $f(S)$.

b) Give a description (without mathematical derivation) of how to construct the solution after a time $t > 0$ before the front has reached the producer at $x = 1$ for a typical choice of the fractional flow function $f(S)$ given in a). Make plots to illustrate the solution.

c) Explain why the solution constructed by using the method of characteristics becomes unphysical. Give a mathematical explanation of the physical correct solution. Explain why the discontinuity satisfies the Rankine-Hugoniot condition.

d) Derive (by mathematical calculations) an expression for the oil recovery $R(t)$ for the solution discussed in b) before and after water breakthrough.

e) Formulate the mass balance equations for water and oil in 1D, impose natural assumptions, and derive the conservation law (A) that expresses the Buckley-Leverett model.

In part B, all 9 exercises 3a-3f and 4a-c count equally. If anything is unclear, state your assumptions.

Exercise 3. Albite ($NaAlSi_3O_8$) is a plagioclase feldspar mineral which dissolves in water according to the following reaction:

$$NaAlSi_3O_8 + 4H^+ \rightleftharpoons Al^{3+} + Na^+ + 2H_2O + 3SiO_2 \tag{1}$$

with solubility constant $K_{albite} = 10^{2.66}$.

a) Write the equation that describes equilibrium for this process (hint: use that K_{albite} is the equilibrium constant in the law of mass action).

b) Assuming that m denotes the amount of mol/L dissolved albite. Express, as functions of m, the concentrations of Na^+, Al^{3+} and SiO_2, denoted by m_{na}, m_{al}, m_{sio2}.

c) Given that pH=7, calculate the amount of albite (mol per L) that can dissolve in pure water based on a) and b). State your assumptions. $\gamma_i = 1$

d) Now consider a brine of 0.5 mol/L NaCl. Calculate the ionic strength of the brine. Also, calculate the activity coefficients of Na^+, Cl^-, H^+, Al^{3+} and SiO_2. $\gamma \neq 1$ $\gamma = 1$

e) Still assuming pH=7, but considering the 0.5 mol/L NaCl brine: how much albite will dissolve in this case? Hint: assume the Na+ concentration does not change with albite dissolution in this case. State your assumptions. $\gamma_i = 1$

f) Explain the different solubility of albite in the two brines. Would the solubility be different in 0.5 mol/L KCl brine?

Exercise 4. A core contains brine with concentration 1 mol/L of a chemical species. The core is then flooded with a brine with concentration 2.5 mol/L. The species adsorbs to the core surface according to an isotherm given by

$$q = 3c^2 \tag{2}$$

where q is the adsorbed species in mol/L and c is the concentration in mol/L. In the following assume that the retardation model can be applied.

a) Sketch the concentration profile along the core after 5 pore volume of brine has been flooded, and after 10 pore volumes have been flooded.

b) The effluent concentration is defined as the concentration evaluated at the end position of the core. Show that the effluent concentration is described by

$$t_c(PV) = R_c \tag{3}$$

where t_c is the time in pore volumes of concentration c to reach the outlet of the core (valid for concentrations between injected and initial values).

c) Plot the effluent concentration (y-axis) as a function of injected pore volumes (x-axis) from 0 to 20 pore volumes.

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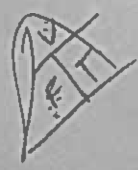

Formulas. Ionic strength:

$$I = 1/2 \sum_i m_i z_i^2 \tag{4}$$

Davies equation:

$$\log \gamma_i = -A(T) z_i^2 \left(\frac{\sqrt{I}}{1+\sqrt{I}} - 0.3I \right), \qquad A(25°C) \approx 0.51 \tag{5}$$

pH:

$$pH = -log([H^+]) \tag{6}$$

Retardation:

$$R_c = 1 + dq/dc, \quad R_f = 1 + \Delta q / \Delta c \tag{7}$$

$Na^+ + \frac{1}{2} CaX_2 \rightleftharpoons NaX + \frac{1}{2} Ca^{2+}$