University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: **PET 510 – Computational Reservoir and Well Modeling**

DATE: **28 November, 2017**

TIME: **4 hours**

AID: **Basic calculator is allowed**

THE EXAM CONSISTS OF 6 PROBLEMS ON 6 PAGES AND APPENDIX A - E

REMARKS:
You may answer in English or Norwegian. Exercises 1 and 2 (part A) and exercises 3-6 (part B) are given equal weight.

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Problem 1.

(a) In the following we consider a horizontal 1D reservoir.
- State the single-phase porous media mass balance equation in 1D (without source term) and identify the various variables (rock and fluid).
- Assuming a weakly compressible rock (compressibility c_r is small) we get a linear relation for $\phi(p)$.

$$\phi(p) = \phi_0[1 + c_r(p - p_0)],$$

where p_0 and ϕ_0 are reference pressure and porosity. Use this together with the assumption that the fluid is incompressible and show that we can obtain a pressure equation of the form

$$(*) \qquad p_t = \varepsilon p_{xx}, \qquad x \in \mathbb{R} = (-\infty, +\infty),$$

and identify the constant parameter $\varepsilon > 0$.

(b) Setting $\varepsilon = 1$ in (*) we know that

$$(**) \qquad p(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{x}{2\sqrt{t}}} e^{-\theta^2} d\theta$$

satisfies (*) with initial data equal to Heaviside function

$$p(x, t = 0) = \begin{cases} 0, & x < 0; \\ 1, & x > 0. \end{cases}$$

- Make use of (**) combined with an appropriate rescaling of x and derive an expression for the solution of (*) with $\varepsilon > 0$.
- Sketch the solution for a fixed time T and two different values of ε in order to indicate the impact from ε on the solution.

(c) We now consider the pressure equation (*) on the domain $x \in (-1, 1)$. In addition, we introduce a source term of the form $-K(p - p^*)$ where p^* is a known, constant pressure

$$(***) \qquad p_t = \varepsilon p_{xx} - K(p - p^*), \qquad x \in (-1, 1), \qquad K > 0 \text{ (constant)}$$
$$p_x(-1, t) = p_x(+1, t) = 0.$$

Demonstrate how we can derive a stability estimate for the pressure p in (***) in terms of an estimate of $\int_0^1 (p - p^*)^2 dx$. What does this stability estimate tell us?

(d) Set $\varepsilon = 2/5$ and $K = p^* = 1$ and introduce a discrete scheme for (***). Consider an initial pressure $p_0(x)$

$$p_0(x) = \begin{cases} -x, & x < 0; \\ +x, & x \geq 0. \end{cases}$$

Consider a grid of 5 cells on the domain $x \in (-1, 1)$ corresponding to

$$x_1 = -\frac{4}{5}, \quad x_2 = -\frac{2}{5}, \quad x_3 = 0, \quad x_4 = \frac{2}{5}, \quad x_5 = \frac{4}{5}.$$

Make use of the discrete scheme and compute a numerical solution after 1 time step where $\Delta t = 1/5$. Check stability condition. Try to give a brief physical interpretation of the resulting pressure solution.

Problem 2.

(a) Consider the linear transport equation

$$(*) \qquad u_t + \left(\frac{x}{2-t}\right)u_x = q(x,t,u), \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x,t=0) = \phi(x).$$

Set $q(x,t,u) = 0$.
- For what time interval is the model well-defined?
- Compute the solution $u(x,t)$ by using the method of characteristics.
- Make a sketch of characteristics and explain by words the transport process reflected by a characteristic.
- Verify by direct computation that your solution $u(x,t)$ satisfies (*) and (**).

(b) Consider (*) with $q(x,t,u) = x$.
- Compute the solution $u(x,t)$ by using the method of characteristics. Verify that your solution satisfies (*) and (**)

(c) Consider the solution in (a) with initial data $\phi(x) = 1 - x^2$.
- Make a sketch of the solution in the interval $[-2,+2]$ at time $t = 0$ and $t = 1$.
- What happens with the solution as $t \to 2$?
- Describe briefly by words the transport process represented by this solution.

(d) Consider the solution in (b) with initial data $\phi(x) = 1 - x^2$.
- Make a sketch of the solution in the interval $[-2,+2]$ at time $t = 0$ and $t = 1$.
- What happens with the solution as $t \to 2$?
- Describe briefly by words the transport process represented by this solution.

(e) Now, consider the transport equation

$$u_t + \frac{1}{2}u_x = 0, \qquad x \in [0,1]$$

with initial data and boundary data

$$u(x,t=0) = 0, \qquad u(x=0,t) = 1.$$

- Describe the characteristics for this model and make a plot of some of them for $x \in [0,1]$. Compute the general solution and make a sketch of the solution $u(x,t=1/2)$.

(f) Consider a scheme for the model in (e) given in the form

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + \frac{1}{2\Delta x}(U_{j+1/2}^n - U_{j-1/2}^n) = 0.$$

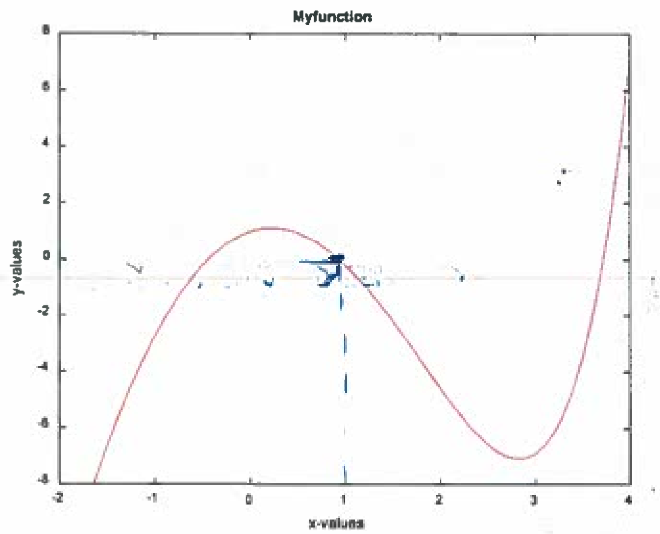
based on upwind discretization. Use it to compute numerical solutions for a grid of 6 cells with cell centers x_1, x_2, ... , x_6. Compute the solution at time $t = 1/2$ by using 2 timesteps. For the first cell, set $U_{1/2} = 1$ to take into account the left boundary condition. Make a figure where you include exact solution and numerical approximate solution computed from the scheme.

Exam Part B – Solving Nonlinear Equations & Modelling of Well Flow

There are 11 questions in total. Some formulas, equations and Matlab codes are found in Appendixes. This part constitutes 50 % of exam.

Exercise 3 – Bisection & Iterative method

a) We are given the function $f(x) = e^x - 3x^2$. Write down a matlab code that produces a figure identical to the one shown below. (In matlab we write the exponential function as exp(x)).



b) We want to pick out the middle root and the starting points are given. Use a table similar to the one shown below. The solution shall be found with an accuracy such that the uncertainty in the x value of the root is less than 0.05. Also find out what ftol should have been in this case (if we had used a criteria on the y value of the root instead).

Iteration	X1	X2	X3	f(x1)	f(x2)	f(x3)	(x2-x1)/2
1	0	2	1	1	-4.61	-0.28	1
2							
3							
Etc							

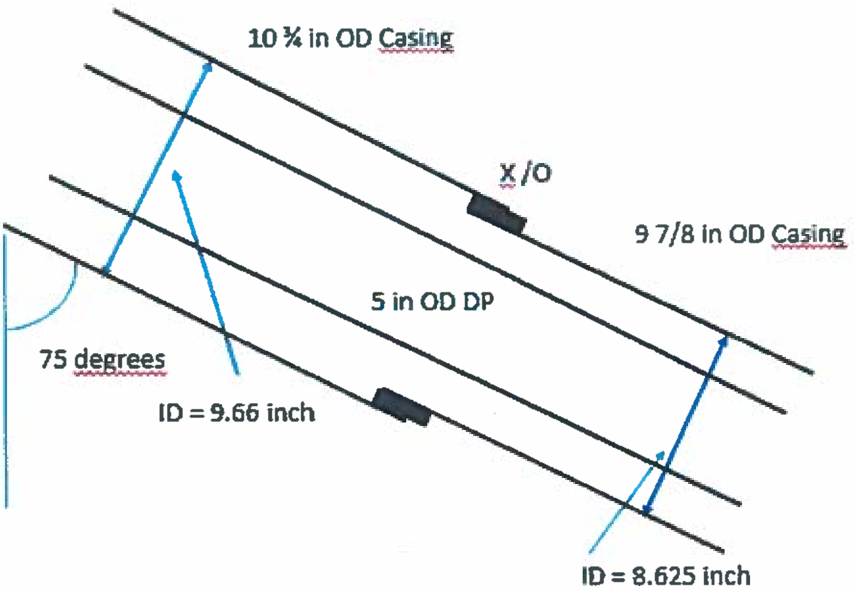
c) We observe that the function has a local minimum value near x = 3. Explain how you can modify the code in Appendix B to calculate and display to computer screen the x and y value of this local minimum. (ftol can be kept unchanged).

d) As seen in 3a, $f(x) = e^x - 3x^2 = 0$ has three roots. We could alternatively tried to use reformulate the problem as $x = g(x)$ and used the iterative approach. We have obviously the two arrangements: $x = +\sqrt{(e^x/3)}$ and $x = -\sqrt{(e^x/3)}$. If we use $x_o = 0$, these two forms will converge to the roots near 1.0 and -0.5. However they will not work for the largest root near 4.0 even if we make an initial guess very close to the exact solution.

Can you find an alternative formulation for picking out the largest root? Choose an appropriate value for x_o and find an approximation of the root satisfying the accuracy requirement $|x_{n+1} - x_n| < xtol = 0.01$ Finally, show how could we be guaranteed to have a solution in the last case.

Exercise 4 – Cuttings Transport

Here, one has just started to drill the 8 ½ inch hole. The inclination is around 75 degrees. A 9 7/8 inch casing has been set at bottom but we have used a crossover to 10 ¾ casing a bit further up in the inclined section. The 10 ¾ casing extends to surface. The inner diameters of the different casings are shown on the figure below. A drillpipe with outer diameter 5 inch is used.



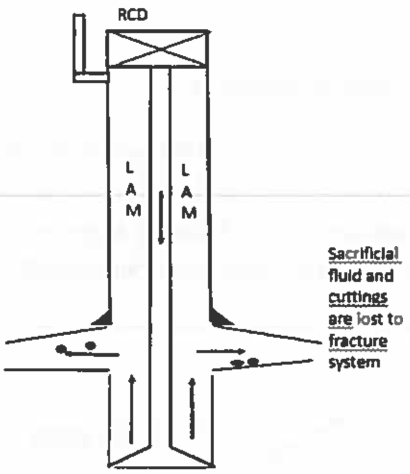
a) We have used the Larsen model to evaluate cuttings transport and found that the minimum cuttings transport velocity to avoid formation of cutting beds is 250 ft/min. What is the minimum flowrate in lpm that you will recommend for this section?

b) Will the minimum required flowrate become higher or lower in the following two cases ? 1) Inclination is 87 degrees instead of 75. 2) The ROP (rate of penetration) is increased.

Exercise 5 - Well pressures

When drilling in carbonate structures, we can experience huge mud losses into fractures and possible caves. The picture below shows a Karst structure at Svalbard and we can find these formations in the Barents sea.

Billefjorden Svalbard



Pressurized mud cap drilling arrangement.

In order to drill such formations, we can adopt a pressurized mud cap drilling arrangement. The annulus of the well is sealed with a rotating control device (RCD) on top of annulus. There shall be a certain pressure just below the RCD. In the annulus above the fractures and below the RCD, there will be a static light annular mud (LAM). Drilling is performed with seawater (1.03 sg) and seawater is circulated down the drillstring, up the lower part of the annulus and injected into the fracture system along with the cuttings during drilling. The fracture system can both receive fluids (seawater, cuttings etc) and produce fluids (e.g. introducing a kick into the well).

The fracture in this case will be positioned at 3000 meter and it has a pressure of 1.4 sg. It means that if the wellpressure is slightly above that it will receive fluids and if it is slightly below it can produce fluids (pore pressure/fracture pressure is in some sense the same here)

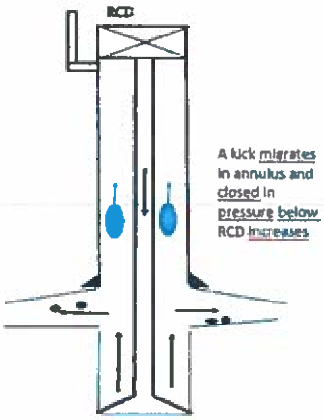
a) We want to design the static LAM fluid such that there will be 10 bar pressure just below the RCD (RCD pressure) and the well pressure at 3000 meter shall balance the 1.4 sg fracture pressure. Show by calculation that the LAM density must be 1.37 sg.

b) The bit is at 3500 meter. We circulate seawater. What will the ECD in sg be at bottom of the well if the friction in the annulus is 12 bar?

If a kick enters the annulus and starts to migrate upwards, an increase in RCD pressure will be seen. The RCD pressure acts like a kick indicator (See figure below)

We will first assume that the LAM fluid is incompressible, no LAM fluid is allowed to escape downwards into the fracture, no temperature change of the kick and no expansion of casing.

c) What will the maximum pressure that we possibly can experience at the RCD become? and how long time will you estimate it will take before the kick reaches the RCD ?



d) In practice, some LAM fluid will be forced downwards and into the fractures when the kick migrates. How will that impact on the pressure build up and the speed of the kick and explain why? (qualitative explanation/not numbers)

Exercise 6 – Two phase flow model and conservation laws

a) In Appendix E, a copy of a part of the two-phase flow model implemented in matlab is presented. The function wellpressure was responsible for performing all calculations from bottom to top in a well based on a guessed bottomhole pressure and known massrates and outlet pressure.

   In wellpressure.m, there is lacking some matlab expressions for performing the calculations in the cells (nodes). It is marked with a ? in the code where something is missing.

   Write down the matlab statements that are lacking!

Appendix A – Some Units & Formulas

1 inch =2.54 cm = 0.0254 m

1 feet = 0.3048 m

1 bar = 100000 Pa

1 sg = 1 kg/l (sg - specific gravity)

$M = Q \cdot \rho$ M massrate (kg/s), Q Volumerate (m^3/s), ρ density (kg/m^3)

$Q = v \cdot A$ Q Volumerate (m^3/s), v velocity m/s. A area m^2

$p = \rho \cdot h \cdot 0.0981$ p (bar), ρ density (sg), h – vertical depth (m)

$\frac{P \cdot V}{T} = C$, from Ideal gas law, NB T is in Kelvin and the relation to Celsius is $K = {}^{\circ}C + 273{,}15$

$P \cdot V = C$, Boyles law (temperature is assumed constant)

Appendix B

Main.m

```
% Main program that calls up a routine that uses the bisection
% method to find a solution to the problem f(x) = 0.
% The search intervall [a,b] is specified in the main program.
% The main program calls upon the function bisection which again calls upon
% the function func.

% if error = 1, the search intervall has to be adjusted to ensure
% f(a) x f(b)<0

% Specify search intervall, a and b will be sent into the function
% bisection
 a = 4.0;
 b = 5.0;

% Call upon function bisection which returns the results in the variables
% solution and error.
  [solution,error] = bisection(a,b);
```

Bisection.m

```
function [solution,error] = bisection(a,b)

% The numerical solver implemented here for solving the equation f(x)= 0
% is called Method of Halving the Interval (Bisection Method)

% You will not find exact match for f(x)= 0. Maybe f(x) = 0.0001 in the
end.
% By using ftol we say that if abs(f(x))<ftol, we are satisfied. We can
% also end the iteration if the search interval [a,b] is satisfactory
small.
% These tolerance values will have to be changed depending on the problem
% to be solved.


 ftol = 0.01;

% Set number of iterations to zero. This number will tell how many
 % iterations are required to find a solution with the specified accuracy.

  noit = 0;

  x1 = a;
  x2 = b;
```

```
  f1 = func(x1);
  f2 = func(x2);


%   First include a check on whether f1xf2<0. If not you must adjust your
%   initial search intervall. If error is 1 and solution is set to zero,
%   then you must adjust the search intervall [a,b].


 if (f1*f2)>=0
     error = 1;
     solution = 0;
 else
 % start iterating, we are now on the track.
     x3 = (x1+x2)/2.0;
     f3 = func(x3);

     while (f3>ftol | f3 < -ftol)
        noit = noit +1 ;

        if (f3*f1) < 0
           x2 = x3;
        else
           x1 = x3;
        end

        x3 = (x1+x2)/2.0;
        f3 = func(x3);
        f1 = func(x1);

     end
     error = 0;
     solution = x3;
     noit % This statement without ; writes out the number of iterations to
the screen.
 end
```

func.m

```
function f = func(x)


  f = x^2-4*x+2;
```

Appendix C

```
% Program where the Larsen Cuttings Transport Model is implemented

%  First specify all input parameters:

do = 8.5; % Outerdiameter (in) ( 1 in = 0.0254 m)
di = 5; % Innerdiameter   (in)
rop = 33 % Rate of Penetration - ROP ft/hr (1 ft = 0.3048m)
pv = 15 % Plastic viscosity (cP)
yp = 16 % Yield point (lbf/100ft2)
dcutt = 0.1 % Cuttings diameter (in) (1 inch = 0.0254 m)
mw = 10.833 % Mudweight (ppg - pounds per gallon) 1 ppg = 119.83 kg/m3.
rpm = 80 % rounds per minute
cdens = 19 % cuttings density (ppg - pounds per gallon)
angstart = 50 % Angle with the vertical

% vcut - Cuttings Transport Velocity (CTF in Larsens paper)
% vcrit - Critical Transport fluid velocity (CTFV) in Larsens paper. This
% is the minimum fluid velocity required to maintain a continously upward
% movement of the cuttings.
% vslip - Equivalent slip velocity (ESV) defined as the velocity difference
% between the cuttings and the drilling fluid
% vcrit = vcut+vslip
% All velocities are in ft/s.
% ua - apparent viscosity

% It should be noted that the problem is nested. Vcrit depends on vslip
% which again depends on an updated/correct value for vcrit. An iterative
% approch on the form  x(n+1) = g(x(n)) will be used.


ang = angstart;
vcut = 1/((1-(di/do)^2)*(0.64+18.16/rop));

vslipguess = 3;
vcrit = vcut + vslipguess;

% Find the apparent viscosity (which depends on the "guess" for vcrit)
ua = pv+ (5*yp*(do-di))/vcrit

% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
if (ua <= 53)
 vslip = 0.0051*ua+3.006;
else
 vslip = 0.02554*(ua-53)+3.28;
end

%Now we have two estimates for vslip that can be compared and updated in a
% while loop. The loop will end when the vslip(n+1) and vslip (n) do not
% change much anymore. I.e the iterative solution is found.
n=1;
while (abs(vslip-vslipguess))>0.01
 vslipguess = vslip;
 vcrit = vcut + vslipguess;
% Find the apparent viscosity (which depends on the "guess" for vcrit)
  ua = pv+ (5*yp*(do-di))/vcrit;
```

```
% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
  if (ua <= 53)
   vslip = 0.0051*ua+3.006;
  else
   vslip = 0.02554*(ua-53)+3.28;
  end
  n=n+1;
  vslip  % Take away ; and you will se how vslip converges to a solution
end % End while loop

%
% Cuttings size correction factor: CZ = -1.05D50cut+1.286
 CZ = -1.05*dcutt+1.286
% Mud Weight Correction factor (Buoancy effect)
  if (mw>8.7)
  CMW = 1-0.0333*(mw-8.7)
  else
  CMW = 1.0
  end

% Angle correction factor

CANG = 0.0342*ang-0.000233*ang^2-0.213

vslip = vslip*CZ*CMW*CANG;   % Include correction factors.

% Find final minimum velocity required for cuttings transport (ft/s).

vcrit = vcut + vslip
```

Appendix D – Steady State Model for Two Phase Flow

Conservation of liquid mass

$$\frac{\partial}{\partial z}(A\rho_l\alpha_l v_l) = 0$$

Conservation of gas mass

$$\frac{\partial}{\partial z}(A\rho_g\alpha_g v_g) = 0$$

Conservation of momentum.

$$\frac{\partial}{\partial z}p = -(\rho_{mix}g + \frac{\Delta p_{fric}}{\Delta z})$$

Gas slippage model (simple):

$v_g = Kv_{mix} + S$ (K=1.2, S = 0.5 m/s)

Liquid density model (simple)

$\rho_l(p) = \rho_{lo} + \frac{(p - p_O)}{a_L^2}$, assume water: $\rho_{lo} = 1000$ kg/m³, $p_O = 100000Pa$, $a_L = 1500$ m/s

Gas density model (simple)

$\rho_g(p) = \frac{p}{a_g^2}$, ideal gas: $a_g = 316$ m/s.

Friction model

The friction model presented here is for a Newtonian fluid like water. The general expression for the frictional pressure loss gradient term is given by:

$$\frac{\Delta p_{fric}}{\Delta z} = \frac{2f\rho_{mix}v_{mix}abs(v_{mix})}{(d_{out} - d_{in})} \text{ (Pa/m)}$$

A - (m²)

ρ_i - phase densities (kg/m³), liquid –> i=l, gas ->i =g

v_i - phase velocities (m/s)

μ_i - phase viscosity (Pa s)

p - pressure (Pa)

g – gravity constant 9.81 m/s^2

α_l - phase volume fractions taking values between 0 and 1. $\alpha_l + \alpha_g = 1$.

$\rho_{mix} = \alpha_l \rho_l + \alpha_g \rho_g$ - mixture density

$v_{mix} = \alpha_l v_l + \alpha_g v_g$ - mixture velocity

$\mu_{mix} = \alpha_l \mu_l + \alpha_g \mu_g$ - mixture viscosity

Appendix E – Wellpressure.m

```
function f = wellpressure(pbotguess,gasrate,liquidrate,nopoints,boxlength)

% NB, At first stage we assume that our outlet pressure is 1 Bar (atm
% pressure). This is the physical boundary condtion that we have to ensure
% that out model reaches. If a choke is present. The surface pressure will
% be different. It means that if the choke pressure is 300 000 Pa then the
% variable below should be set to this. You change it her:

 prealsurface  = 300000; % Choke pressure on top of well

% We now start by the deepest node with the pressure we assume: pbotguess
% and the known inlet massrates at bottom. All variables are then found in
% this node.
% For each segment(next node), we calculate the pressure, superficial
% velocities, phase velocities,volume fractions moving upwards.
% We use momentum equation to calculate pressure in next
% node using hydrostatic pressure and the friction model. The superficial
% vecocities and the other variables are found using the two mass
% conservation laws combined with the slip relation.
%

% In the end, we end up with some surface
% rates and a surface outlet pressure. The calculated outlet surface
% pressure should equal the physical outlet condition (now 300000 Pa). We
% can therefore define our wellpressure(pbot)=pcalcsurface-prealsurface.
% The function will be zero if the correct bottomhole pressure is found.

% Set outer/inner diameter of annulus. Define effective flowarea. Assume a
% 7" liner (ID 6.3") and a 3 1/2" drillpipe.
  do = 0.16;
  di = 0.0889;

  flowarea = 3.14/4*(do*do-di*di);

% Specify viscosities [Pa s]. In real life they depend on pressure and temp

  viscl = 0.001;
  viscg = 0.00001;

% Define gas slippage parameters.
  k = 1.2;
  s = 0.55;

% gas gravity constant

   g = 9.81;

 % The mass rate is the same at surface/atmosphere and at bottomhole since
 % we have steady state. This is later
 % used to find the rates at downhole conditions.
```

```
  liqmassratesurf = liquidrate*roliq(100000.0);% Mass rate of injected liq
  liqmassratebhp  = liqmassratesurf; % Massrate injected at bottom of
  % annulus same as injected in drillstring

  gasmassratesurfinj = gasrate*rogas(100000.0); % Mass rate of injected gas
  gasmassratebhpinj  = gasmassratesurfinj; % Massrate injected at bottom of
%annulus same as injected in drillstring

% Here the PI model (reservoir inflow can be turned on )
% by activating the pi formula (remove %)
  prodinx = 0;
%  prodinx = 0.0144/(24*3600); % m3/(s x Pa)

   gasvolratesurface = prodinx*(20000000-pbotguess); % production rate at
%surface conditions
   % Reservoir pressure is 200 bar (20000000 Pa). A bottomhole pressure
   % below that will give production.

   if (gasvolratesurface <0) % Just at check to avoid negative production
      gasvolratesurface = 0;
   end

   gasmassratebhproduction  = gasvolratesurface*rogas(100000.0);
   % massrate at bottom equal to surface
   % This is the massrate flowing from reservoir into the well!

   gasmassratebhp = gasmassratebhpinj+gasmassratebhproduction;
   % Add both injected and produced gasmassrate at bottom


  % Now we loop from the bottom to surface and calculate accross all the
  % segments until we reach the outlet.

  % Define the variables needed. Initialise them first for comp efficiency.
  % vl - liquid velocity, vg -gas velocity,
  % vgs,vls are superficial velocities of gas and liquid.
  % eg-el - phase volume fractions for gas and liquid
  % p - pressure.
  % fricgrad and hydgrad are pressure gradients (Pa/m)due
  % to friction and  hydrostatic pressure components.

  vl = zeros(nopoints,1);
  vg = zeros(nopoints,1);
  vls = zeros(nopoints,1);
  vgs = zeros(nopoints,1);
  eg = zeros(nopoints,1);
  el = zeros(nopoints,1);
  p  = zeros(nopoints,1);
  fricgrad = zeros(nopoints-1,1);
  hydgrad = zeros(nopoints-1,1);


  % Before we loop, we define  all variables at the inlet of the first
  % segment(at bottom). As starting point we use the fact that we know
  % the mass rate of the different phases (same as on top of the well)

  % First find the rates in m3/s (downhole) using that we know the
  % massrates and the densities at the bottom. We use the guessed
  % bottomholepressure to calculate densities which are needed to find the
```

```
% flowrates. Flowrate Q = massrate/density for each phase

liquidratebhp = liqmassratebhp /roliq(pbotguess);
gasratebhp = gasmassratebhp/rogas(pbotguess);

% Find the superficial velocities (m/s)
vls(1) = liquidratebhp/flowarea;
vgs(1) = gasratebhp/flowarea;

% Find Phase velocities. Here we use the gas slip relation
% vg = Kvmix+s = K(vls+vsg)+s or K(vlxel+vgxeg)+S. vls and vgs are
% supeficial velocities and represent the phase velocity multiplied with
% the phase's volume fraction.

vg(1) = k*(vls(1)+vgs(1))+s;
eg(1) = vgs(1)/vg(1);
el(1) = 1-eg(1);
vl(1) = vls(1)/el(1);

% Set pressure equal to guessed pressure at bottom.
p(1) = pbotguess;


% Now we loop across the segments from bottom to top.

sumfric = 0;
sumhyd = 0;

for i =1:nopoints-1

% use the inlet values for each segment to calculate hydrostatic
% and friction pressure across each segment.
 hydgrad(i) = (roliq(p(i))*el(i)+rogas(p(i))*eg(i))*g;
 fricgrad(i) = ...
     dpfric(vl(i),vg(i),el(i),eg(i),p(i),do,di,viscl,viscg);
```

?

```
eg(i+1) = vgs(i+1)/vg(i+1);
el(i+1) = 1-eg(i+1);
vl(i+1) = vls(i+1)/el(i+1);

sumfric = sumfric+fricgrad(i)*boxlength;
sumhyd = sumhyd+hydgrad(i)*boxlength;
end

% We have now reaced the outlet of the well and pout is our calculated
% outlet pressure. This should be equal to the real surface at the outlet
% which was defined to be 3 bar in the beginning of this function. If
% they are equal, f = 0 and a correct bottomhole pressure has been
% guessed for.

pout = p(nopoints);
f = pout-prealsurface;
```