University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

DATE: May 15, 2018

SUBJECT: PET 565 – Core scale modeling and interpretation

TIME: 4 hours

AID: No printed or written means allowed. Definite basic calculator allowed.

THE EXAM CONSISTS OF 5 PROBLEMS ON 4 PAGES

REMARKS: You may answer in English or Norwegian. Exercises 1 - 2 (part A) and exercises 3 - 5 (part B) are given equal weight.

Problem 1.

(a) Consider the conservation law

$$(*) \qquad u_t + f(u)_x = 0, \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(**) \qquad u(x, t = 0) = \phi(x).$$

- Explain by using the *method of characteristics* why a general solution of (*) and (**) takes the form

$$(***) \qquad u(x,t) = \phi\Big(x - f'(u(x,t))t\Big)$$

(b) Based on (***) compute an expression for u_x.
- Explain under what circumstances u_x might blow up (i.e., becomes infinitely large) when we assume that f is convex ($f'' > 0$)

- Verify by direct calculation that (***) satisfies (*) and (**) subject to the condition

$$1 + \phi'(x_0) f''(\phi(x_0))t \neq 0.$$

(c) We assume that $f(u) = \frac{1}{4}u^2$ and

$$\phi(x) = \begin{cases} 2x, & 0 \leq x \leq \frac{1}{2} \\ 1, & \frac{1}{2} < x \leq 1 \\ 3 - 2x, & 1 < x \leq \frac{3}{2}. \end{cases}$$

Compute an expression for characteristics emanating from different $x_0 \in [0, 3/2]$ and identify rarefaction wave and compression wave. As a part of this analysis make a sketch of different characteristics in $x - t$ space and find the breaking time T_b (time for crossing characteristics).

(d) Consider (*) with $f(u) = \frac{1}{4}u^2$ and

$$\phi(x) = \begin{cases} 4, & 0 \leq x < 1; \\ 1, & \text{otherwise} \end{cases}$$

- Consider the two Riemann problems, one at $x = 0$ and the other at $x = 1$. Compute solutions in terms of shock wave and/or rarefaction wave.
- Sketch the solution $u(x, t = 1)$ at time $t = 1$.
- In particular, compute the time T_c when the two waves will start interacting.

(e) For the problem discussed in (d), compute the solution for $t > T_c$
- either by using mathematical relations that characterize the behaviour of the two interacting waves (Rankine-Hugoniot condition, etc) or by using the "Equal-Area Rule"
- What is the solution $u(x, t = 3)$ at time $t = 3$?

Hint: $\int \frac{1}{x-1}\, dx = \ln(x - 1)$ for $x > 1$.

$\frac{2x}{4/3}$

Problem 2. V2014

(a) Mass conservation of water and oil in a 1D reservoir is represented by the following equations:

$$\frac{\partial}{\partial x}\Big[\frac{kk_{rl}}{\mu_l}(\frac{\partial p}{\partial x}+\gamma_l)\Big]=\phi\frac{\partial S_l}{\partial t}, \qquad l=w,o$$

where $\gamma_l=\rho_l g\sin(\alpha)$ accounts for the gravity force.

- list some of the main assumptions for deriving the Buckley-Leverett (BL) model and define the different quantities
- introduce mobility functions λ_l and demonstrate how we can find the following expression for the pressure gradient

$$\frac{\partial p}{\partial x}=-\frac{u_T+\lambda_w\gamma_w+\lambda_o\gamma_o}{\lambda_T}, \qquad \lambda_T=\lambda_w+\lambda_o.$$

- what is u_T and can you express this in terms of some other available variables?

(b) Explain how to obtain the BL formulation

$$\phi\frac{\partial}{\partial t}S+u_T\frac{\partial F(S)}{\partial x}=0, \qquad S=S_w$$

Find the expression for the fractional flow function $F(S)$ when you let $f(S)=\frac{\lambda_w(S)}{\lambda_T(S)}$.

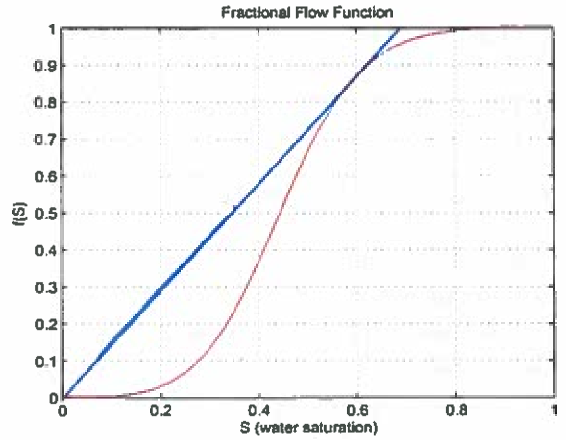
(c) Now, we consider a horizontal reservoir with fractional flow function $f(S)$ as shown in Fig. 1. The BL model takes the following form in dimensionless variables x_D and t_D

$$\frac{\partial S}{\partial t_D}+\frac{\partial f(S)}{\partial x_D}=0.$$

Based on Fig. 1 (next page), compute the solution (saturation distribution) after a time $T=0.5$.

(d) Use the plot of $f'(S)$ and give a sketch of the unphysical solution after a time $T=0.5$ based on the method of characteristics.
- Use the principle of mass conservation and derive the general mathematical expression for the front height S^* satisfied by the physical correct solution.

(e) For the flux function $f(S)$ shown in Fig. 1, we now want to include gravity. It is assumed that $\rho_w>\rho_o$. In particular, consider the following two cases:
(i) upwards dip ($\sin(\alpha)>0$);
(ii) downward dip ($\sin(\alpha)<0$).
- Explain (by sketching an approximate solution) how the solution will change compared to the one computed in (c). Back up your explanation by referring to a sketch of the corresponding fractional flow function for (i) and (ii).



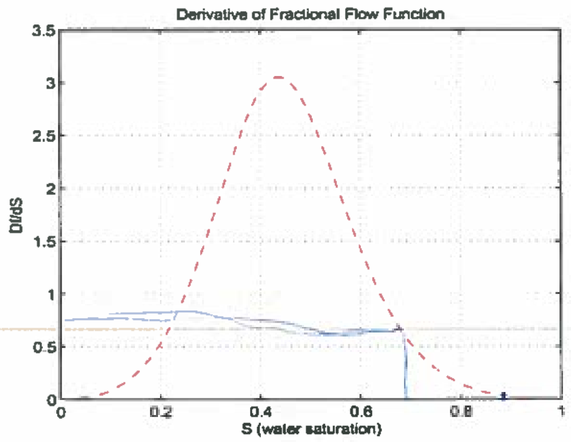


FIGURE 1. Left: $f(S)$. Right $f'(S)$

Problem 3. Halite NaCl (s) dissolves in water, forms free ions Na^+ (aq) and Cl^- (aq) and the complex $NaCl^0$ (aq). The solubility constant for halite is 37.2 and the complex stability constant is 0.167.

a) Calculate the concentration of free ions, concentration of complex and the concentration of dissolved halite in units of mol/L. Assume the activity coefficients are 1. (5p)
b) From a), calculate the ionic strength and the activity coefficients (PS: there is no need to update the answers in a)). (5p)

Problem 4. Consider injection of a species with concentration c that adsorbs with concentration q as described by isotherm methodology. Dispersion is negligible. The isotherm is $q = 1.5c^2$ (q, c both in mol/L).

a) Write the corresponding differential equation. Explain the involved terms. What is the model called? (4p)

Assume initial concentration 0 mol/L and injected concentration 2 mol/L.

b) Sketch the effluent concentration (measured at the outlet end of the system) as function of injected pore volumes (PVs). How many PVs are required to reach steady state? (3p)

After reaching steady state with 2 mol/L the injected concentration is reduced to 1 mol/L.

c) Sketch the effluent concentration vs injected PVs. How many PVs are required to reach steady state? (3p)

Problem 5. Dolomite, $CaMg(CO_3)_2$, dissolves according to:

(1) $$CaMg(CO_3)_2\ (s) \rightleftharpoons Ca^{2+}\ (aq) + Mg^{2+}\ (aq) + 2CO_3^{2-}\ (aq)$$

Consider a dolomite core initially containing equilibrated water. Then, deionized water is injected.

a) Write differential equations for Ca, Mg CO3 and dolomite (d). Include advection and dispersion (if relevant) and source terms $\dot{r}_i$ for each species ($i = ca, mg, co3, d$). (3p)

b) What is the unit of the source terms? (1p) mol/L/s

c) How are the four source terms mathematically related? (2p)

d) Express the dolomite saturation state Ω_d based on the above reaction. (1p)

e) Plot the effluent concentrations of Ca, Mg and CO3 as function of injected pore volumes. Explain the main features and how the concentrations are related. (3p)

$\dot{r} = k_1 C_{ca} + k_2 C_{mg} + K_3 C_{co3} - K_4 C_d$

Formulas. Ionic strength:

(2) $$I = 1/2 \sum_i m_i z_i^2$$

Davies equation:

(3) $$\log \gamma_i = -A(T) z_i^2 \Big(\frac{\sqrt{I}}{1+\sqrt{I}} - 0.3I \Big), \qquad A(25°C) \approx 0.51$$

pH:

(4) $$pH = -log([H^+])$$

Retardation:

(5) $$R_c = 1 + dq/dc, \quad R_f = 1 + \Delta q / \Delta c$$

$x_c = \frac{x_\omega}{R}$