University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

SUBJECT: **PET 510 – Computational Reservoir and Well Modeling**

DATE: **25 november**

TIME: **4 hours**

AID: **Basic calculator is allowed**

THE EXAM CONSISTS OF 6 PROBLEMS ON 5 PAGES AND APPENDIX A - D

REMARKS:
You may answer in English or Norwegian. Exercises 1 and 2 (part A) and exercises 3-6 (part B) are given equal weight.

COURSE RESPONSIBLE: Steinar Evje and Kjell-Kåre Fjelde

Problem 1.

(a) Consider the pressure equation

$$(A1) \qquad \varepsilon u_t = u_{xx}, \qquad x \in \mathbb{R} = (-\infty, +\infty)$$

with initial data

$$(A2) \qquad u(x, t = 0) = u_0(x) = \begin{cases} 0, & x \le 0; \\ 1, & x > 0. \end{cases}.$$

Find an expression for the exact solution of (A1) and (A2) by using the fact that the solution with $\varepsilon = 1$ is given by

$$(A3) \qquad u(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{x}{2\sqrt{t}}} e^{-\theta^2} d\theta.$$

Make use of an appropriate rescaling of the time variable t to get $\hat{t}$ that depends on ε. Derive equation for $v(x, \hat{t})$ where $v(x, \hat{t}) = u(x, t)$.

(b) The task is the same as in (a) but make use of an appropriate rescaling of space variable x to get $\hat{x}$ that depends on ε. Derive equation for $v(\hat{x}, t)$ and make use of (A3) to get the solution of (A1) with initial data (A2).
- Illustrate the effect of ε on the solution by making a rough sketch of the solution $u(x, t)$ for a fixed time t and a "small" value of ε and a "large" value of ε.

(c) In order to derive the solution (A3) of (A1) with initial data (A2) for $\varepsilon = 1$, the idea that is used is to relate the time variable t and the space variable x in a special way. Introduce a variable y that describes this relation and a variable $v(y)$ and rewrite the PDE (partial differential equation) $u_t = u_{xx}$ with initial data (A2) as and ODE (ordinary differential equation) in terms of $v(y)$. Find conditions for v at $y = \pm\infty$.

(d) Consider

$$\begin{aligned} u_t &= u_{xx} + g(u), \qquad x \in (0,1) \\ u(0,t) &= u(1,t) = 0 \\ u(x, t = 0) &= u_0(x) \end{aligned}$$

We want to derive an upper bound for the quantity $\int_0^1 u^2 dx$.
- Find a condition on $g(u)$ which will ensure that $\int_0^1 u^2 dx \le \int_0^1 u_0^2 dx$.
- In particular, give an example of a $g(u)$ that satisfies this condition.

(e) Now, consider a pressure equation (obtained by combing the mass balance equation with Darcy's law) which takes the following form

$$\begin{aligned} (*) \qquad & p_t = \kappa p_{xx}, \qquad x \in (0,1), \qquad \kappa = \frac{k}{\mu \phi_0 (c + c_r)} \\ & p(0,t) = p(1,t) = 1 \\ & p(x, t = 0) = p_0(x) \end{aligned}$$

where k, μ, ϕ_0, c, and c_r are constants (permeability, viscosity, porosity, and compressibility constants) with initial data

$$(**) \qquad p_0(x) = \begin{cases} 1-2x, & 0 \le x < \frac{1}{2}; \\ 2x-1, & \frac{1}{2} \le x \le 1. \end{cases}$$

- Consider a discretization of the domain $[0,1]$ composed of M cells and discretization parameters Δx and Δt. Formulate a discrete version of (*) (explicit scheme). Make sure that the boundary condition is taken into account in the scheme for cell 1 and cell M.
- If $\kappa = 1$, what is the stability condition for this scheme?

(f) Make a sketch of the initial data $p_0(x)$ and the solution $p(x,t)$ at some time $t > 0$ in one and the same figure.

- Assume that $\frac{k}{\mu} = 1$. Make a sketch of the corresponding initial fluid velocity $u_0(x)$ and $u(x,t)$ and explain the motion of fluid and the corresponding change in pressure. Refer to relevant equations when you explain.

- What will be the stationary solution (the solution as time goes to infinity) for pressure p and velocity u?

Problem 2. We consider a transport equation of the form

$$(B1) \qquad u_t + xu_x = b(x,t,u), \qquad x \in (-\infty, +\infty)$$

$$(B2) \qquad u(x,t) = u_0(x) = \exp(-x^2)$$

(a) Compute exact solution when $b(x,t,u) = -u$
- make a sketch of typical characteristics in the $x-t$ coordinate system
- verify that the computed solution satisfies (B1) and (B2)
- consider your solution with the initial data given in (B2) and identify the term in the solution that is responsible for the effect from the source term $b = -u$

(b) We now consider the model (B1) with $b(x,t,u) = 0$. Consider a discretization of the spatial domain $[-5,5]$. Assume that the domain is divided into $2M$ cells. It is assumed that $u_1^{n+1} = u_{2M}^{n+1} = 0$. Formulate a stable discrete scheme for cells $2, \ldots, 2M-1$.

(c) Consider (B1) again where the source term $b(x,t,u) = 0$. Show that the solution u of (B1) and (B2) satisfies the relation

$$\int_{-\infty}^{\infty} u(x,t)dx = e^t \int_{-\infty}^{\infty} u_0(x)dx,$$

if we assume that $xu(x,t) \to 0$ as $x \to \pm\infty$.

(d) Consider (B1) and (B2) with $b(x,t,u) = -u + x$. Compute exact solution and verify that it satisfies (B1) and (B2).
Hint: The solution of an ODE of the form

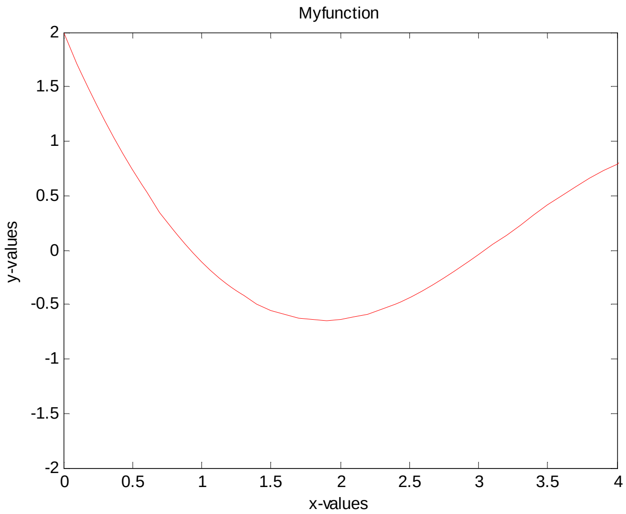
$$\frac{dv}{dt} + v = q(t) \qquad \text{is given by} \qquad v(t) = e^{-t}v_0 + e^{-t}\int_0^t q(s)e^s ds.$$

Exam Part B – Solving Nonlinear Equations & Modelling of Well Flow

There are 12 questions in total. Some formulas, equations and Matlab codes are found in Appendixes. This part constitutes 50 % of exam.

Exercise 3 – Bisection Method

a) We have the function $f(x) = 2e^{-x} - \sin x$. Write down the necessary matlab code to plot the figure below.



b) We want to pick out the smallest positive root of this equation. Fill out the following table and also comment upon how accurate the solution is !

Iteration	x1	x2	x3	f(x1)	f(x2)	f(x3)
1						
2						
3						
4						
5						

c) Explain how you want to change the code in Appendix B in order to find the root which you found in exercise b)

Exercise 4 – Iterative methods

a) Consider the function $f(x) = x^2 - 2x - 3$. This has the roots x_1=-1 and x_2= 3. Consider alternate reformulations on the form $x = g(x)$ and show how we can pick out the two roots by iterations. Use $x_0 = 4$ as starting point.

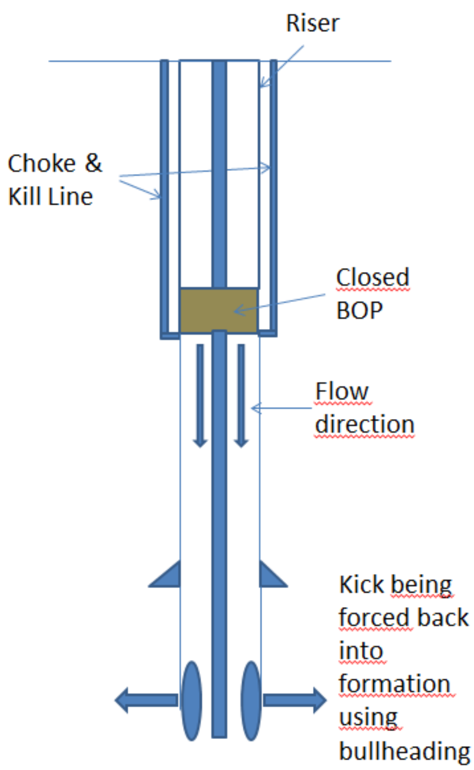
b) Write a matlab code that performs the iterations for the iteration defined by $x = \sqrt{(2x+3)}$

We will be satisfied when the solution satisfies the following requirement $|x_{n+1} - x_n| < 0.001$. Hint: The control structure is the same as the one used in the Matlab code for the Larsen model shown in Appendix C.

Exercise 5 – Well pressures

a) We are at 4000 meters TVD (true vertical depth) in a well and we are circulating with 2000 lpm. The friction in the annulus is 17 bars. The mudweight measured at surface is 1.5 sg. The well is temperature dominated and we expect that this will change the effective downhole mudweight by 0.02 sg. Calculate the effective ECD (in sg) when circulating!

b) Can you estimate what the friction will be if we increase the pump rate to 3000 lpm ?



One well control method is based on pumping the kick back into the formation. This is called bullheading. In this case we will circulate mud down the choke and kill line simultaneously and it is important that the rate is large enough to overcome gas migration in the annulus such that we are able to force the kick downwards. In the following calculations we will assume that the density of the fluid is constant.

c) Use the gas slip relation and show that the minimum required negative liquid velocity (negative means in downward direction) to be able to force the kick downwards is given by the expression: $v_l = \frac{-S}{K\alpha_l}$ and show that if we assume 20 % gas we end up with $v_l = -0.57\frac{m}{s}$ (see Appendix for gas slip relation) for the model assumed here.

d) We will pump 500 lpm down each line simultaneously. Verify by calculation that we will be able to force the kick downward in the annulus. The outer diameter of annulus is 8.5" and the inner diameter is 5".

e) The choke and kill lines are 1000 meter long. The estimated friction gradient is 0.01 Bar/m in these lines. The distance from the BOP to injection zone is 3000 meter and the annulus friction gradient is estimated to be 0,005 Bar/m. The mudweight is 1.5 sg. The well is vertical. The well pressure at the injection zone must be 610 bar to be able to force the kick back. The injection zone is 4000 meter below the rig floor (vertical well).

What must the pump pressure be when the bullheading circulation has been established?

Exercise 6 – Conservation laws

a) What is the main difference between a steady state and transient flow model ? Here you should also give one example of a transient and one example of a steady state flow situation

b) The following shows the transient drift flux mixture momentum equation. In Appendix D, we find the steady state version of it. What kind of physical phenomena does the transient momentum equation describe which the steady state equation don't.

$$\frac{\partial}{\partial t}(A(\rho_l \alpha_l v_l + \rho_g \alpha_g v_g)) + \frac{\partial}{\partial z}(A(\rho_l \alpha_l v_l^2 + \rho_g \alpha_g v_g^2)) + A\frac{\partial}{\partial z}p = -A(\rho_{mix} g + \frac{\Delta p_{fric}}{\Delta z})$$

Appendix A – Some Units & Formulas

1 inch =2.54 cm = 0.0254 m

1 feet = 0.3048 m

1 bar = 100000 Pa

1 sg = 1 kg/l (sg - specific gravity)

$M = Q \cdot \rho$ M massrate (kg/s), Q Volumerate (m^3/s), ρ density (kg/m^3)

$Q = v \cdot A$ Q Volumerate (m^3/s), v velocity m/s. A area m^2

$p = \rho \cdot h \cdot 0.0981$ p (bar), ρ density (sg), h – vertical depth (m)

$\frac{P \cdot V}{T} = C$, from Ideal gas law

$P \cdot V = C$, Boyles law (temperature is assumed constant)

Appendix B

Main.m

```
% Main program that calls up a routine that uses the bisection
% method to find a solution to the problem f(x) = 0.
% The search intervall [a,b] is specified in the main program.
% The main program calls upon the function bisection which again calls upon
% the function func.

% if error = 1, the search intervall has to be adjusted to ensure
% f(a) x f(b)<0

% Specify search intervall, a and b will be sent into the function
% bisection
 a = 4.0;
 b = 5.0;

% Call upon function bisection which returns the results in the variables
% solution and error.
  [solution,error] = bisection(a,b);

 solution % Write to screen.
 error  % Write to screen.
```

Bisection.m

```
function [solution,error] = bisection(a,b)

% The numerical solver implemented here for solving the equation f(x)= 0
% is called Method of Halving the Interval (Bisection Method)

% You will not find exact match for f(x)= 0. Maybe f(x) = 0.0001 in the
end.
% By using ftol we say that if abs(f(x))<ftol, we are satisfied. We can
% also end the iteration if the search interval [a,b] is satisfactory
small.
% These tolerance values will have to be changed depending on the problem
% to be solved.


 ftol = 0.01;

% Set number of iterations to zero. This number will tell how many
 % iterations are required to find a solution with the specified accuracy.

  noit = 0;

  x1 = a;
  x2 = b;
```

```
  f1 = func(x1);
  f2 = func(x2);


%   First include a check on whether f1xf2<0. If not you must adjust your
%   initial search intervall. If error is 1 and solution is set to zero,
%   then you must adjust the search intervall [a,b].


 if (f1*f2)>=0
     error = 1;
     solution = 0;
 else
 % start iterating, we are now on the track.
     x3 = (x1+x2)/2.0;
     f3 = func(x3);

     while (f3>ftol | f3 < -ftol)
        noit = noit +1 ;

        if (f3*f1) < 0
           x2 = x3;
        else
           x1 = x3;
        end

        x3 = (x1+x2)/2.0;
        f3 = func(x3);
        f1 = func(x1);

     end
     error = 0;
     solution = x3;
     noit % This statement without ; writes out the number of iterations to
the screen.
 end
```

func.m

```
function f = func(x)


  f = x^2-4*x+2;
```

Appendix C

```
% Program where the Larsen Cuttings Transport Model is implemented

%  First specify all input parameters:

do = 8.5; % Outerdiameter (in) ( 1 in = 0.0254 m)
di = 5; % Innerdiameter   (in)
rop = 33 % Rate of Penetration - ROP ft/hr (1 ft = 0.3048m)
pv = 15 % Plastic viscosity (cP)
yp = 16 % Yield point (lbf/100ft2)
dcutt = 0.1 % Cuttings diameter (in) (1 inch = 0.0254 m)
mw = 10.833 % Mudweight (ppg - pounds per gallon) 1 ppg = 119.83 kg/m3.
rpm = 80 % rounds per minute
cdens = 19 % cuttings density (ppg - pounds per gallon)
angstart = 50 % Angle with the vertical

% vcut - Cuttings Transport Velocity (CTF in Larsens paper)
% vcrit - Critical Transport fluid velocity (CTFV) in Larsens paper. This
% is the minimum fluid velocity required to maintain a continously upward
% movement of the cuttings.
% vslip - Equivalent slip velocity (ESV) defined as the velocity difference
% between the cuttings and the drilling fluid
% vcrit = vcut+vslip
% All velocities are in ft/s.
% ua - apparent viscosity

% It should be noted that the problem is nested. Vcrit depends on vslip
% which again depends on an updated/correct value for vcrit. An iterative
% approch on the form  x(n+1) = g(x(n)) will be used.

for i = 1:8

ang(i)=angstart+i*5
vcut = 1/((1-(di/do)^2)*(0.64+18.16/rop));

vslipguess = 3;
vcrit = vcut + vslipguess;

% Find the apparent viscosity (which depends on the "guess" for vcrit)
ua = pv+ (5*yp*(do-di))/vcrit

% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
if (ua <= 53)
 vslip = 0.0051*ua+3.006;
else
 vslip = 0.02554*(ua-53)+3.28;
end

%Now we have two estimates for vslip that can be compared and updated in a
% while loop. The loop will end when the vslip(n+1) and vslip (n) do not
% change much anymore. I.e the iterative solution is found.
n=1;
while (abs(vslip-vslipguess))>0.01
 vslipguess = vslip;
 vcrit = vcut + vslipguess;
% Find the apparent viscosity (which depends on the "guess" for vcrit)
  ua = pv+ (5*yp*(do-di))/vcrit;
```

```
% Find vslip based on the "guessed apparent viscosity". This needs to be
% updated until a stable value is obtained. "Iterative approach".
  if (ua <= 53)
   vslip = 0.0051*ua+3.006;
  else
   vslip = 0.02554*(ua-53)+3.28;
  end
  n=n+1;
  vslip  % Take away ; and you will se how vslip converges to a solution
end % End while loop

%
% Cuttings size correction factor: CZ = -1.05D50cut+1.286
 CZ = -1.05*dcutt+1.286
% Mud Weight Correction factor (Buoancy effect)
  if (mw>8.7)
  CMW = 1-0.0333*(mw-8.7)
  else
  CMW = 1.0
  end

% Angle correction factor

CANG = 0.0342*ang(i)-0.000233*ang(i)^2-0.213

vslip = vslip*CZ*CMW*CANG;   % Include correction factors.

% Find final minimum velocity required for cuttings transport (ft/s).

vcrit = vcut + vslip

vcritms = vcrit*0.3048 % Velocity in m/s

Q = 3.14/4*((8.5*0.0254)^2-(5*0.0254)^2)*vcritms % (m3/s)
Q = Q*60*1000  % (lpm)

yrate(i)=Q
end

plot(ang,yrate)
```

Appendix D – Steady State Model for Two Phase Flow

Conservation of liquid mass

$$\frac{\partial}{\partial z}(A\rho_l \alpha_l v_l) = 0$$

Conservation of gas mass

$$\frac{\partial}{\partial z}(A\rho_g \alpha_g v_g) = 0$$

Conservation of momentum.

$$\frac{\partial}{\partial z} p = -(\rho_{mix} g + \frac{\Delta p_{fric}}{\Delta z})$$

Gas slippage model (simple):

$v_g = K v_{mix} + S$ (K=1.2, S = 0.55)

Liquid density model (simple)

$\rho_l(p) = \rho_{lo} + \frac{(p - p_O)}{a_L^2}$, assume water: $\rho_{lo} = 1000$ kg/m^3, $p_O = 100000\,Pa$, $a_L = 1500$ m/s

Gas density model (simple)

$\rho_g(p) = \frac{p}{a_g^2}$, ideal gas: $a_g = 316$ m/s.

Friction model

The friction model presented here is for a Newtonian fluids like water. The general expression for the frictional pressure loss gradient term is given by:

$$\frac{\Delta p_{fric}}{\Delta z} = \frac{2 f \rho_{mix} v_{mix} abs(v_{mix})}{(d_{out} - d_{in})} \quad \text{(Pa/m)}$$

A - (m^2)

ρ_i - phase densities (kg/m^3), liquid – > i=l, gas ->i =g

v_i - phase velocities (m/s)

p - pressure (Pa)

g – gravity constant 9.81 m/s^2

α_i - phase volume fractions taking values between 0 and 1. $\alpha_l + \alpha_g = 1$.

$\rho_{mix} = \alpha_l \rho_l + \alpha_g \rho_g$ - mixture density

$v_{mix} = \alpha_l v_l + \alpha_g v_g$ - mixture velocity