Suggested solution to the final exam.
PART B: Well Testing (50 pt), PET670, December 10, 2018.

Problem 1 (26 pt)

a) There are two flow regimes evident in the data: (1) Early linear flow with half-slope data from the beginning until 0.1 hr (or slightly longer); (2) radial flow from 10 hr (or slightly earlier) and to the end.
(2 + 2 pt)

b) With radial-flow data from 10 hrs to the end, we can use the points p_{wf} = 4941.798 psia at t = 10.0047 hr and p_{wf} = 4931.472 psia at t = 24 hr to determine the semi-log slope. For this drawdown data semi-log analysis with log t may be used to get the slope, m, psi/log-cycle

$$m = -\frac{4931.472 - 4941.798}{\log(24) - \log(10.0047)} = \frac{10.326}{0.38} = 27.17.$$

(2 pt)

From the slope we next get flow capacity, k, md·ft

$$kh = \frac{162.6qB\mu}{m} = -\frac{(162.6)(300)(1.03)(0.65)}{27.17} = \frac{32658.2}{27.17} = 1202.0,$$

and therefore permeability, k, md

$$k = \frac{kh}{h} = \frac{1202.0}{300} = 4.0.$$

(2 pt)

The pressure at 1 hour can be determined by extrapolation from either of the two points used to determine the slope m. We can therefore set p_{1hr}, psia

$$p_{1hr} = 4941.798 - m[\log(1) - \log(10.0047)] =$$
$$= 4941.798 - 27.17[0 - 1.0] = 4968.968.$$

(2 pt)

Then, hydraulic diffusivity may be computed as

$$\frac{k}{\phi\mu c_t r_w^2} = \frac{4}{(0.3)(0.65)(7.5\cdot 10^{-6})(0.3\cdot 0.3)} = \frac{4}{1.317\cdot 10^{-7}} = 3.037\cdot 10^7.$$

The skin value can next be determined from the formula

$$S = 1.151\left(\frac{p_i - p_{1hr}}{m} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.228\right),$$
$$S = 1.151\left(\frac{5000.0 - 4968.968}{27.17} - \log(3.037\cdot 10^7) + 3.228\right) =$$

$$= 1.151(1.142 - 7.482 + 3.228) = -3.58.$$

(2 pt)

The “added” pressure drop at the wellbore can now also be computed as

$$\Delta p_S = \frac{m}{1.151} S = \frac{27.17}{1.151}(-3.58) = -84.508.$$

(2 pt)

c) We can for instance use the points $p_{wf} = 4996.226$ psia at $\Delta t = 0.01$ hr and $p_{wf} = 4988.079$ psia at $\Delta t = 0.1$ hr to determine the slope, m', psi/$\sqrt{\text{hr}}$

$$m' = -\frac{4988.079 - 4996.226}{\sqrt{0.1} - \sqrt{0.01}} = \frac{8.147}{0.316 - 0.1} = 37.72,$$

(2 pt)

for the linear-flow analysis. From this slope we get the half-length, x_f, ft

$$x_f = \frac{4.064 qB}{hm'} \sqrt{\frac{\mu}{k\phi c_t}} = \frac{(4.064)(300)(1.03)}{(300)(37.72)} \sqrt{\frac{0.65}{(4)(0.3)(7.5 \cdot 10^{-6})}} =$$

$$= \frac{1255.78}{11316} \sqrt{\frac{0.65}{9 \cdot 10^{-6}}} = (0.111)(268.74) = 29.83.$$

(2 pt)

If the fracture has infinite conductivity, then we get the skin value

$$S = \ln \frac{2r_w}{x_f} = \ln \frac{(2)(0.3)}{29.83} = \ln(0.0201) = -3.91.$$

If the fracture has uniform flux, then we get

$$S = \ln \frac{er_w}{x_f} = \ln \frac{(2.718)(0.3)}{29.83} = \ln(0.0273) = -3.60.$$

The value from the semi-log analysis is closer to the uniform flux result.
(2 pt)

d) Well flowing pressure after 1 week of production may be estimated as difference of the initial pressure and pressure drawdown:

$$p_{wf}(t) = p_i - m\left(\log t + \log \frac{k}{\phi \mu c_t r_w^2} - 3.228 + \frac{S}{1.151}\right),$$

$$p_{wf}(168) = 5000 - 27.17\left(\log(168) + \log(3.037 \cdot 10^7) - 3.228 + \frac{(-3.58)}{1.151}\right) =$$
$$= 5000 - 27.17\big(2.225 + (7.482 - 3.228 - 3.110)\big) = 5000 - 27.17(2.225 + 1.144) =$$
$$= 5000 - 27.17(3.369) = 5000 - 91.536 = 4908.464.$$

where pressure drawdown, psi

$$p_i - p_{wf} = 91.536\,.$$

(2 pt) Comment: Computed pressure drawdown or flowing pressure (alone) is considered as sufficient result.

If the well will produce at 600 STB/D for the same period of time (1 week), we can calculate increased pressure drawdown based on

$$C = \frac{q_2}{q_1} = \frac{600}{300} = 2,$$
$$p_i - p_{wf,q_2} = C\big(p_i - p_{wf,q_1}\big) = 2(91.536) = 183.072,$$

and flowing pressure

$$p_{wf,q_2}(168) = 5000 - 183.072 = 4816.928.$$

(2 pt) Comment: Computed pressure drawdown or flowing pressure (alone) is considered as sufficient result.

e) Build-up pressure after 24 hours may be computed using

$$p_{ws}(\Delta t) = p_{wf,s} + m\left(\log\frac{\Delta t}{t + \Delta t} + \log t + \log\frac{k}{\phi\mu c_t r_w^2} - 3.228 + \frac{S}{1.151}\right),$$

Therefore, for 24 hours (using also calculation results from d)

$$p_{ws}(24) = 4931.472$$
$$+ 27.17\left(\log(0.5) + \log(24) + \log(3.037 \cdot 10^7) - 3.228 + \frac{(-3.58)}{1.151}\right) =$$
$$= 4931.472 + 27.17\big(-0.301 + 1.380 + (7.482 - 3.228 - 3.110)\big) =$$
$$= 4931.472 + 27.17(1.079 + 1.144) = 4931.472 + 27.17(2.223) = 4991.878$$

(2 pt)

Problem 2 (24 pt)

a) Radial flow regime is evident in the data from 10 hr (or slightly earlier) and to the end.

(2 pt)

b) With radial flow data from 10 hr to the end, we can use the points p_{ws} = 4750.906 psia at Δt = 10.005 hr and p_{ws} = 4751.710 psia at Δt = 24 hr to determine the semi-log slope. For this build-up data semi-log analysis with Horner time $\Delta t/(t+\Delta t)$ may be used to get the slope, m, psi/log-cycle

$$m = \frac{4751.710 - 4750.906}{\log\left(\frac{24}{1+24}\right) - \log\left(\frac{10.005}{1+10.005}\right)} =$$
$$= \frac{0.804}{-0.0177 + 0.0414} = \frac{0.804}{0.0237} = 33.92.$$

(2 pt)

From the slope we next get flow capacity, kh, md·ft

$$kh = \frac{162.6 qB\mu}{m} = \frac{(162.6)(500)(1.03)(0.65)}{33.92} = \frac{54430.35}{33.92} = 1604.7,$$

and therefore permeability, k, md

$$k = \frac{kh}{h} = \frac{1604.7}{300} = 5.4.$$

(2 pt)

The pressure at Δt = 1 hour can be determined by extrapolation from either of the two points used to determine the slope m. We can therefore set p_{1hr}, psia

$$p_{1hr} = 4750.906 + m[\log(1/(1+1)) - \log(10.005/(1+10.005))] =$$
$$= 4750.906 + 33.92[-0.301 + 0.0414] = 4750.906 - 8.806 = 4742.1.$$

(2 pt)

Then, hydraulic diffusivity may be computed

$$\frac{k}{\phi\mu c_t r_w^2} = \frac{5.4}{(0.3)(0.65)(7.5\cdot10^{-6})(0.3\cdot0.3)} = \frac{5.4}{1.317\cdot10^{-7}} = 4.10\cdot10^{7}.$$

The skin value can next be determined from the formula

$$S = 1.151\left(\frac{p_{1hr} - p_{wf,s}}{m} - \log\frac{t}{t+1} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.228\right),$$
$$S = 1.151\left(\frac{4742.1 - 4539.083}{33.92} - \log(0.5) - \log(4.1\cdot10^{7}) + 3.228\right) =$$

$$= 1.151(5.985 + 0.301 - (7.613 - 3.228)) = 1.151(6.286 - (4.385)) = 2.19.$$

(2 pt)

The "added" pressure drop at the wellbore can now also be computed as

$$\Delta p_S = \frac{m}{1.151} S = \frac{33.92}{1.151}(2.19) = 64.539.$$

(2 pt)

c) There is no indication of any boundary effect, so we can use the radius of investigation at the end of the buildup to estimate a minimal circular area based on the radius

$$r_{inv} = 0.0246\sqrt{\frac{kt}{\phi\mu c_t}} = 0.0246\sqrt{\frac{(5.4)(24)}{(0.3)(0.65)(7.5\cdot 10^{-6})}} = 0.0246\sqrt{\frac{129.6}{1.463\cdot 10^{-6}}}$$
$$= 0.0246\sqrt{88.585\cdot 10^{6}} = 0.0246(9411.97) = 231.5$$

This corresponds to the area, ft^2

$$A = \pi r_{inv}^2 = 3.14(231.5)^2 = 168279.$$

(2 pt)

d) With short flow prior to shut-in and no boundary effects seen in buildup data we can use $p*$ as an estimate of the formation pressure. We can again extrapolate the semi-log straight line from the first data point used above and get $p*$, psia

$$p_i = p^* = 4750.906 + m[\log(1) - \log(10.005/(1+10.005))] =$$
$$= 4750.906 + 33.92(0.0414) = 4752.310.$$

(2 pt)

With flow at the same constant rate of 500 STB/D, we can compute drawdown (flowing) pressure after 24 hours using

$$p_{wf}(t) = p_i - m\left(\log t + \log\frac{k}{\phi\mu c_t r_w^2} - 3.228 + \frac{S}{1.151}\right),$$
$$p_{wf}(24) = 4752.31 - 33.92(\log(24) + \log(4.1\cdot 10^{7}) - 3.228 + 2.19/1.151) =$$
$$= 4752.31 - 33.92(1.38 + (4.385) + 1.903) = 4752.31 - 33.92(7.668) =$$
$$= 4752.31 - 260.099 = 4492.211.$$

where pressure drawdown, psi

$$p_i - p_{wf} = 260.099.$$

An alternative interpretation of the task is possible, that the well will produce 24 hours after 1 hour production (25 hours as a sum), so pressure drawdown, psi

$$p_{wf}(25) = 4752.31 - 33.92(\log(25) + \log(4.1 \cdot 10^7) - 3.228 + 2.19/1.151) =$$
$$= 4752.31 - 33.92(1.40 + (4.385) + 1.903) = 4752.31 - 33.92(7.688) =$$
$$= 4752.31 - 260.777 = 4491.533.$$

where pressure drawdown, psi

$$p_i - p_{wf} = 260.777.$$

(2 pt) Comment: Computed pressure drawdown or flowing pressure (alone) is considered as sufficient result.

e) If total compressibility, c_t, will be increased by a factor of 2, the estimated skin, S, will change as

$$\log\frac{k}{\phi\mu c_t r_w^2} + \frac{S}{1.151} = \log\frac{k}{2\phi\mu c_t r_w^2} + \frac{S_{mod}}{1.151} = \log\frac{k}{\phi\mu c_t r_w^2} - \log(2) + \frac{S_{mod}}{1.151},$$
$$\frac{S}{1.151} = -\log(2) + \frac{S_{mod}}{1.151},$$
$$S_{mod} = 1.151\left(\log(2) + \frac{S}{1.151}\right) = 1.151(\log(2)) + S =$$
$$= 1.151(0.301) + S = 0.35 + 2.19 = 2.54,$$
$$\frac{S_{mod}}{S} = \frac{2.54}{2.19} = 1.16.$$

Skin will increase by factor of 1.16.

(4 pt) Comment: this task requires additional derivations and is rated with double points.

If the thickness will be given in [m] instead of [ft], then the thickness in feet should be multiplied by factor of ~3.28. This means that the permeability should be reduced by the same factor. Therefore, permeability, k, md

$$k_{mod} = \frac{k}{3.28} = \frac{5.4}{3.28} = 1.7.$$

(2 pt)

General comments: in a 2-point task, 2 points are given if both equation used and computed result are correct; 1 point if only equation used is correct. An incorrect result of preceding calculations (used in current task) does not change grading of the current task.