LØSNINGSFORSLAG MAT100 XH18

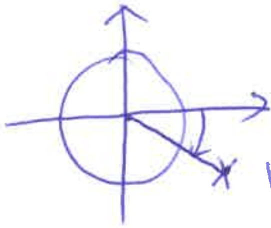
1 a) $z = 2+4i \quad , \quad w = 1-i$

$$z^2 = (2+4i)^2 = 2^2 + 2\cdot 2\cdot 4i + (4i)^2$$
$$= 4 + 16i - 16 = \underline{\underline{-12+16i}}$$

$$w\bar{w} = (1-i)(1+i) = 1^2 - i^2 = 1+1 = \underline{\underline{2}}$$

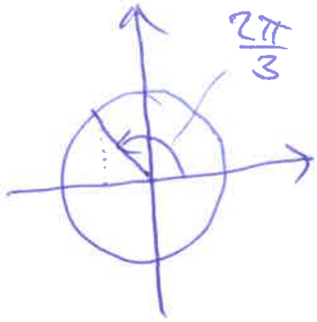
$$\frac{z}{w} = \frac{z\bar{w}}{w\bar{w}} = \frac{(2+4i)(1+i)}{2}$$
$$= \frac{2+2i+4i+4i^2}{2} = \frac{2+6i-4}{2}$$
$$= \frac{-2+6i}{2} = \underline{\underline{-1+3i}}$$

b) $\sqrt{3}-i = 2\cdot\frac{1}{2}(\sqrt{3}-i)$
$$= \underline{\underline{2e^{-\frac{\pi}{6}i}}}$$



$$(\sqrt{3}-i)^{30} = \left(2e^{-\frac{\pi}{6}i}\right)^{30} = 2^{30}e^{-\frac{\pi}{6}i\cdot 30}$$
$$= 2^{30}e^{-5\pi i} = 2^{30}e^{-5\pi i + 6\pi i}$$
$$= 2^{30}e^{\pi i} = 2^{30}(-1) = \underline{\underline{-2^{30}}}$$
$$(= -1073741824)$$

c) 4. RØTTER TIL $-8+8\sqrt{3}i$



$$-8+8\sqrt{3}i = 16\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$= \underline{16e^{i\cdot\frac{2\pi}{3}}} = 2^4 e^{i\frac{2\pi}{3}}$$

<u>RØTTENE:</u>

$$w_1 = 16^{\frac{1}{4}} e^{\frac{\frac{2\pi}{3}i}{4}} = 2e^{\frac{\pi}{6}i} = 2\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}i\right)$$

$$= \underline{\underline{\sqrt{3}+i}}$$

$$w_2 = 16^{\frac{1}{4}} e^{\frac{2\pi}{3}/4\, i + \frac{2\pi}{4}i} = 2e^{\frac{\pi}{6}i + \frac{\pi}{2}i} = 2e^{\frac{2\pi}{3}i}$$

$$= 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \underline{\underline{-1+\sqrt{3}i}}$$

$$w_3 = 16^{\frac{1}{4}} e^{\frac{2\pi}{3}/4 + \frac{4\pi}{4}i} = 2e^{\frac{\pi}{6}i + \pi i} = 2e^{\frac{7\pi}{6}i}$$

$$= 2\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \underline{\underline{-\sqrt{3}-i}}$$

$$w_4 = 16^{\frac{1}{4}} e^{\frac{2\pi}{3}/4\, i + \frac{6\pi}{4}i} = \;\ldots\; = \underline{\underline{1-\sqrt{3}i}}$$

(2) a) $\int (3\sqrt{x}+2e^{-x})dx = \int (3x^{\frac{1}{2}}+2e^{-x})dx$

$= 3\cdot\frac{2}{3}x^{\frac{3}{2}} - 2e^{-x} + C = 2x^{\frac{3}{2}} - 2e^{-x} + C$

b) $\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx$

$u = \ln x$
$v' = x^2$
$v = \frac{1}{3}x^3$
$u' = \frac{1}{x}$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

c) $\int \frac{\cos x + 1}{\sin x + x} dx$

$u = \sin x + x$
$du = (\cos x + 1)dx$

$= \int \frac{du}{u} = \ln|u| + C$

$= \ln|\sin x + x| + C$

d) $\displaystyle\int \frac{5x+13}{(x+2)^2(x-1)}\,dx$

DELBRØK OPPSPALTING:

$$\frac{5x+13}{(x+2)^2(x-1)} = \frac{A}{x-1} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

$$= \frac{A(x+2)^2 + B(x-1)(x+2) + C(x-1)}{(x+2)^2(x-1)}$$

TELLERE MÅ MATCHE:

$$5x+13 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$\underline{x=1}$: $\quad 5\cdot 1+13 = A\cdot(1+2)^2 + B\cdot 0 + C\cdot 0$

$$18 = 9\cdot A \Rightarrow \underline{A=2}$$

$\underline{x=-2}$: $\quad 5\cdot(-2)+13 = A\cdot 0 + B\cdot 0 + C\cdot(-2-1)$

$$3 = -3C \Rightarrow \underline{C=-1}$$

$\underline{x=0}$: $\quad 13 = A\cdot 2^2 + B(-1)\cdot 2 + C\cdot(-1)$

$$13 = 8 - 2B + 1$$

$$2B = 9 - 13 = -4 \Rightarrow \underline{B=-2}$$

$$\int \frac{5x+13}{(x+2)^2(x-1)}dx = \int\left(\frac{2}{x-1} + \frac{-2}{x+2} + \frac{-1}{(x+2)^2}\right)dx$$

$$= 2\ln|x-1| - 2\ln|x+2| + \frac{1}{x+2} + C$$

e) $$\int \frac{1+x}{\sqrt{1-x^2}}dx = \int\left(\frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}\right)dx$$

$$= \sin^{-1}x + \int \frac{x}{\sqrt{1-x^2}}dx$$

$u = 1-x^2$
$du = -2x\,dx$
$-\frac{du}{2} = x\,dx$

$$= \sin^{-1}x + \int \frac{-\frac{du}{2}}{\sqrt{u}}$$

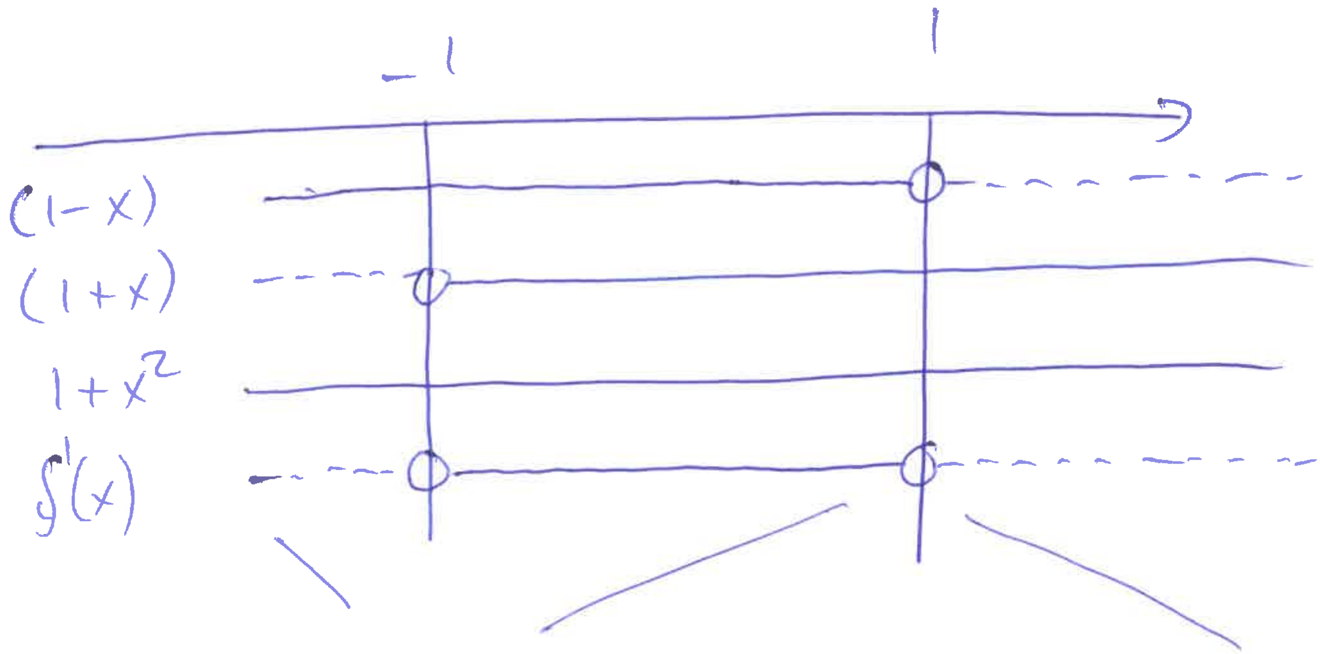
$$= \sin^{-1}x - \int \frac{du}{2\sqrt{u}} = \sin^{-1}x - \sqrt{u} + C$$

$$= \sin^{-1}x - \sqrt{1-x^2} + C$$

③ $f(x) = 2\tan^{-1}x - x$

a) $f'(x) = 2 \cdot \dfrac{1}{1+x^2} - 1 = \dfrac{2 - (1+x^2)}{1+x^2}$

$$= \frac{1-x^2}{1+x^2} = \frac{(1-x)(1+x)}{1+x^2}$$



MINKENDE PÅ $(-\infty, -1]$ OG PÅ $[1, \infty)$

STIGENDE PÅ $[-1, 1]$

MIN VED $x = -1$

$f(-1) = 2\tan^{-1}(-1) - (-1) = -\frac{2\pi}{4} + 1$

$= -\frac{\pi}{2} + 1$

$\left(-1, -\frac{\pi}{2} + 1\right)$

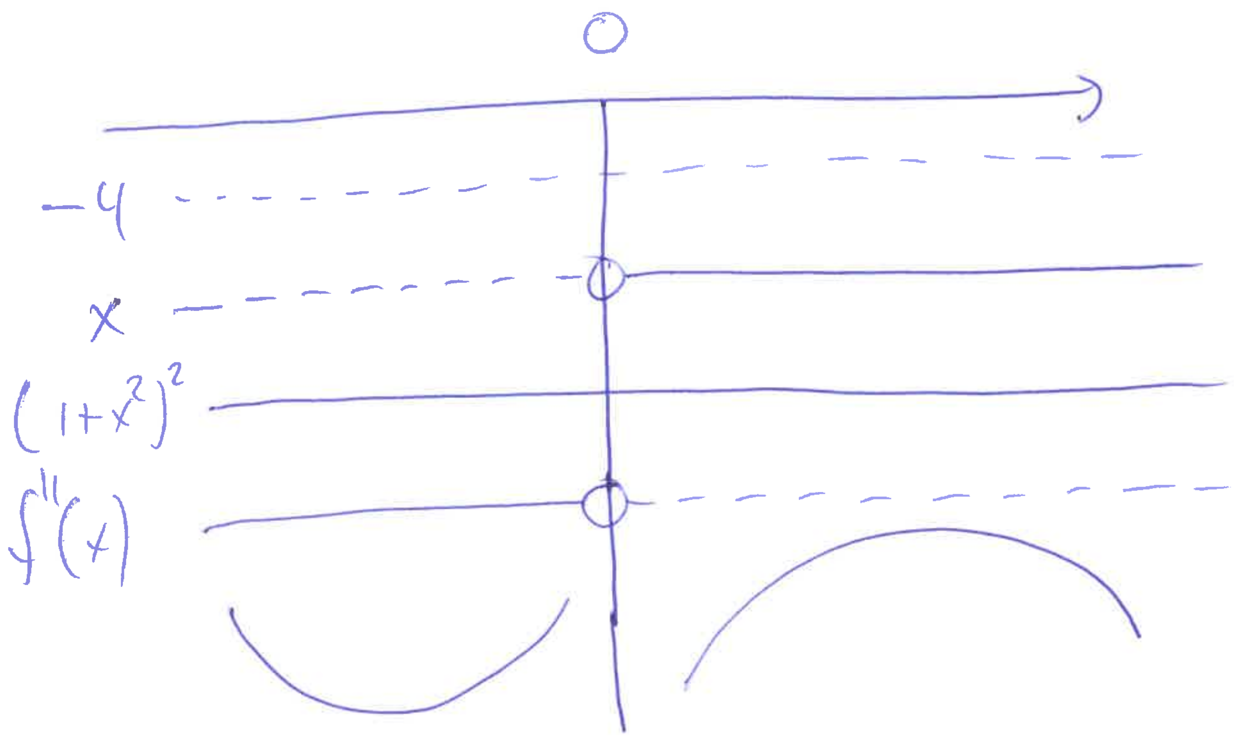
MAX VED $X=1$

$$f(1) = 2\tan^{-1}(1) - 1 = \frac{\pi}{2} - 1$$

$$\left(1, \frac{\pi}{2} - 1\right)$$

b) $$f''(x) = 2\cdot\left(\frac{1}{1+x^2}\right)' = 2\cdot(-1)\,\frac{2x}{(1+x^2)^2}$$

$$= -\frac{4x}{(1+x^2)^2}$$



KRUMMER OPP PÅ $[-\infty, 0]$

KRUMMER NED PÅ $[0, \infty)$

VENDEPUNKT: $(0, f(0)) = (0, 0)$

c) $$\lim_{x\to\infty}\left(f(x)-(-x+\pi)\right)=\lim_{x\to\infty}\left(2\tan^{-1}x-x+x+\pi\right)$$

$$=\lim_{x\to\infty}\left(2\tan^{-1}x-\pi\right)=2\cdot\frac{\pi}{2}-\pi=\underline{0}$$

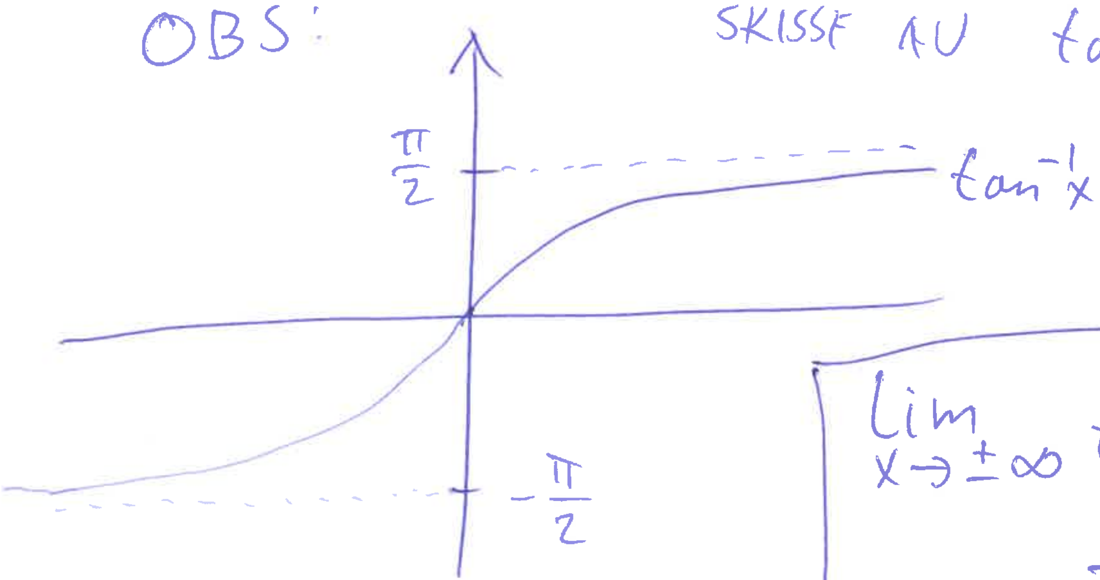
$\Rightarrow$ $\underline{\underline{y=-x+\pi \text{ ER EN SKRÅ ASYMP.}}}$

$$\lim_{x\to\infty}\left(f(x)-(-x-\pi)\right)=\lim_{x\to\infty}\left(2\tan^{-1}x+\pi\right)$$

$$=2\cdot\left(-\frac{\pi}{2}\right)+\pi=\underline{0}$$

$\Rightarrow$ $\underline{\underline{y=-x-\pi \text{ ER EN SKRÅ ASYMP.}}}$

OBS: SKISSE AV $\tan^{-1}x$.



$$\boxed{\lim_{x\to\pm\infty}\tan^{-1}x=\pm\frac{\pi}{2}}$$

④ a) $xy^3 - 2yx^3 = 4$

IMPLISITT DERIVASJON:

$$\frac{d}{dx}\left(xy^3 - 2yx^3\right) = \frac{d}{dx}(4)$$

$$y^3 + x \cdot 3y^2 \frac{dy}{dx} - 2\frac{dy}{dx}x^3 - 2y \cdot 3x^2 = 0$$

$$\left(3xy^2 - 2x^3\right)\frac{dy}{dx} = -y^3 + 6yx^2$$

$$\frac{dy}{dx} = \frac{-y^3 + 6yx^2}{3xy^2 - 2x^3}$$

$$\left.\frac{dy}{dx}\right|_P = \frac{-2^3 + 6 \cdot 2 \cdot 1^2}{3 \cdot 1 \cdot 2^2 - 2 \cdot 1^3} = \frac{-8 + 12}{12 - 2}$$

$$= \frac{4}{10} = \underline{\frac{2}{5}}$$

TANGENT: $y = ax + b$

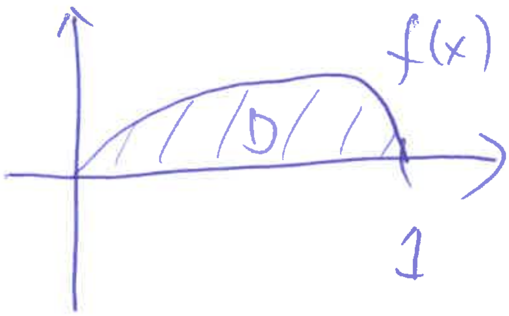
$a = \frac{2}{5}$

GÅR IGJENNOM $(1,2)$:

$$2 = \frac{2}{5} \cdot 1 + b \Rightarrow b = 2 - \frac{2}{5} = \underline{\frac{8}{5}}$$

$$\underline{\underline{y = \frac{2}{5}x + \frac{8}{5}}}$$

b) $f(x) = x\sqrt{1-x^2}$



1) AREAL:

$$A = \int_0^1 f(x)\,dx = \int_0^1 x\sqrt{1-x^2}\,dx$$

$$\boxed{\begin{aligned} u &= 1-x^2 \\ du &= -2x\,dx \end{aligned}}$$

$$= \int_1^0 \sqrt{u}\left(-\frac{du}{2}\right) = \left(-\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_1^0$$

$$= -\frac{1}{3}0^{3/2} - \left(-\frac{1}{3}\cdot 1^{3/2}\right) = \underline{\underline{\frac{1}{3}}}$$

2) OMDREININGSLEGEMET:

$$V = \pi\int_0^1 (f(x))^2\,dx = \pi\int_0^1 x^2(1-x^2)\,dx$$

$$= \pi\int_0^1 (x^2-x^4)\,dx = \pi\left(\frac{1}{3}x^3 - \frac{1}{5}x^5\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{3}\cdot 1^3 - \frac{1}{5}1^5 - \left(\frac{1}{3}0^3 - \frac{1}{5}\cdot 0^5\right)\right)$$

$$= \pi\left(\frac{1}{3}-\frac{1}{5}\right) = \pi\left(\frac{5}{15}-\frac{3}{15}\right) = \underline{\underline{\frac{2}{15}\pi}}$$

5 a) $y'' - 4y' + 5y = 0$

KARAK. LIGN.

$r^2 - 4r + 5 = 0$

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} = \underline{2 \pm i}$$

$$\Rightarrow y = Ae^{2x}\cos x + Be^{2x}\sin x$$

b) $y'' - 4y' + 5y = \sin x$.

HOMOGEN LIGNING SOM I a).

$y_h =$ (SVAR FRA a)).

GJETTER PÅ PARTIKULÆR LØSNING:

$y_p = C\sin x + D\cos x$

$y_p' = C\cos x - D\sin x$

$y_p'' = -C\sin x - D\cos x$

$$y_p'' - 4y_p' + 5y_p = -C\sin x - D\cos x$$
$$-4(C\cos x - D\sin x)$$
$$+5(C\sin x + D\cos x)$$

$$= \sin x \cdot (-C + 4D + 5C)$$
$$+ \cos x \cdot (-D - 4C + 5D)$$

$$= 4(D+C)\sin x + 4(D-C)\cos x$$

$$= \sin x$$

$$\Rightarrow 4(D+C) = 1, \quad 4(D-C) = 0$$

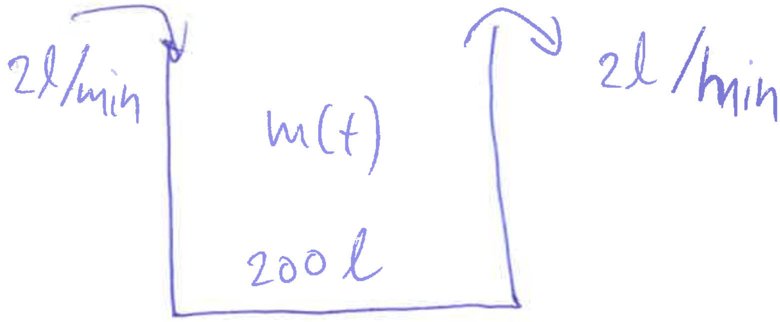
$$\Rightarrow D = C = \frac{1}{8}$$

$$y_p = \frac{1}{8}\sin x + \frac{1}{8}\cos x$$

LØSN:

$$y(x) = y_h + y_p = Ae^{2x}\cos x + Be^{2x}\sin x$$
$$+ \frac{1}{8}\sin x + \frac{1}{8}\cos x$$

6



a) TETTHET: $\rho = \dfrac{\text{MENGDE}}{\text{VOLUM}}$.

$$\frac{dm}{dt} = \left(\rho_{\text{INN}} \cdot \binom{\text{FERSKVANN}}{\text{INN}}\right) - \left(\rho_{\text{UT}} \cdot \binom{\text{VANN}}{\text{UT}}\right)$$

$$= 0 \cdot 2 - \frac{m}{200} \cdot 2 = \underline{-\frac{m}{100}}$$

INITIAL BETINGELSE:

VED START: $m(t) = 35 \cdot 200 = 7000$

$$\Rightarrow \quad \text{IVP:} \begin{cases} \dfrac{dm}{dt} = -\dfrac{m}{100} \\ m(0) = 7000 \end{cases}$$

b) $$\frac{dm}{dt} = -\frac{m}{100}$$ SEPARABEL.

$$\frac{dm}{m} = -\frac{dt}{100}$$

$$\int \frac{dm}{m} = -\int \frac{dt}{100}$$

$$\ln|m| = -\frac{t}{100} + C$$

$$m = D \cdot e^{-\frac{t}{100}}$$

$$m(0) = 7000 = D \cdot e^{0} = D$$

$$\Rightarrow \underline{\underline{m(t) = 7000\, e^{-\frac{t}{100}}}}$$

2,0% SALT KONSENTRASJON $\Rightarrow$ 20 g/l.

$$\Rightarrow m = 25 \cdot 200 = 4000$$

$$\cancel{7000 = 5000\, e^{-\frac{t}{100}}}$$

$$4000 = 7000\, e^{-\frac{t}{100}}$$

$$e^{\frac{t}{100}} = \frac{7000}{4000} = \frac{7}{4}$$

$$\frac{t}{100} = \ln\frac{7}{4}$$

$$t = 100 \cdot \ln\frac{7}{4} \simeq 0.56 \cdot 100 \approx \underline{56}$$

DVS ETTER 56 MINUTTER ER SALTKONSENTRASJ[ONEN]

$\underline{\underline{2,0\%}}$