FACULTY OF SCIENCE AND TECHNOLOGY

English text

SUBJECT: PET660 Reservoir simulation

DATE: March 9, 2017

TIME: 4 hours

AID: No printed or written aid allowed. Definite basic calculator allowed.

All problem points are given equal weights except the single point of Problem 3 which is given double weight.

THE EXAM CONSISTS OF 7 PROBLEMS ON 5 PAGES + APPENDIX

REMARKS:

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Problem 1

a)
Determine the order of the approximation

$$u_x \approx \frac{u_{i-2} - 4u_{i-1} + 3u_i}{2\Delta x} \quad .$$

.

b)
Determine stability criterion for the difference approximation

$$\frac{u_{i+1}^n - u_i^n}{\Delta x} = \frac{u_i^{n+1} - u_i^n}{\Delta t}$$

of $$u_x = u_t$$

Problem 2

a)
What are the main differences in model input data between a Black Oil and a Compositional model?

b)
Given a grid for a 3D reservoir where mobile water and free gas is present initially.
Which Black Oil model input data are required to compute initial equilibrium state for the given grid?

Problem 3

Consider the 3 x 2 grid

Δx_1 Δx_2 Δx_3	Δx_1 (Δx_2) Δx_3	Δx_1 (Δx_2) (Δx_3)
Δx_1 Δx_2 (Δx_3)	Δx_1 (Δx_2) (Δx_3)	Δx_1 (Δx_2) (Δx_3)

Variables in parenthesis are treated explicitly in an Adaptive Implicit Method (AIM).
Use the example above to outline the main steps in the AIM solution procedure.
Show matrix structures at each step.

Problem 4

a)
The flow of oil between block *i* and *i*+1 in a three-phase Black Oil model is given by

$$F = CT_o(p_{i+1} - p_i),$$

where C is a constant. The i-direction is assumed horizontal, i.e. no gravity term is present.
Write definition of C and T_o.

b)
Assume pressure relation $p_{i+1} > p_i$. Moreover block *i*+1 is under saturated (no free gas).
Use upstream weighting.
Linearize *F*.

Problem 5

a)
Consider a 2-phase water/oil system with zero capillary pressure.
Derive the IMPES pressure equation to be used for solving pressures.

b)
Show how new saturations are computed after pressure update for the case considered in a).

Problem 6

Consider a 1D horizontal reservoir where two fluids water and oil are flowing. The cross section A is constant. A uniform grid with N blocks of length Δx will be used for the numerical solution. Time step length Δt is constant. The fluids are incompressible and volume factors are set equal to 1. Capillary pressure is zero.
The reservoir is initially filled with oil. Water is injected in block N with constant reservoir volume rate Q (Q a positive number) and a producer is located in block 1.

a)
With the assumptions above write discretized mass balance equations.

b)
Suppose viscosities are constant and equal and that relative permeabilities are straight lines, $k_{rw}(x) = x$, $k_{ro}(x) = 1-x$. Moreover, the reservoir is homogeneous and incompressible. Use upstream evaluation.
Show that the equations for mass balance can be written

$$\text{water} \quad (S)_{i+1}(p_{i+1} - p_i) - (S)_i(p_i - p_{i-1}) + q_{w,i} = C\Delta_t(S)$$
$$\text{oil} \quad (1-S)_{i+1}(p_{i+1} - p_i) - (1-S)_i(p_i - p_{i-1}) + q_{o,i} = C\Delta_t(1-S)$$

for $i = 2,\dots,N-1$.

Write definitions of S, C and $q_{l,i}$, $l = o, w$.

Use no flow exterior boundaries and write mass balance equations for blocks 1 and N.
Write expressions for $q_{l,1}$, *and* $q_{l,N}$, $l = o, w$.
Hint: Use phase mobilities to compute production rates.

c)
Show that pressure differences can be written

$$p_{i+1} - p_i = B \; , \; i = 1,\dots, N-1.$$

Write definition of B.

d)
Saturation equations are obtained by substituting pressure differences in the water equations.
Show that explicit water equations for blocks $i = 2,\dots,N-1$ can be written

$$D\frac{\Delta t}{\Delta x}(S_{i+1}^n - S_i^n) = \Delta_t S \, , \; i = 2,\dots,N-1 . \qquad (*)$$

Write definition of the constant *D*.

Write equations corresponding to (*) for blocks 1 and *N*.

e)
Set N = 5.
Use equations in d) with maximal stable time step length, i.e. $\frac{D\Delta t}{\Delta x} = 1$, to compute solution after 5 steps.

Problem 7

Give linear equations $A\vec{x} = \vec{b}$.

Algorithm for ORTHOMIN with restart:

$$- \ given \ \ \vec{x}^0 and \ set \ \ \vec{r}^{\,0} = \vec{b} - A\vec{x}^0, \ \vec{q}^{\,0} = \vec{r}^{\,0} \qquad (*)$$

$$for \ \ k = 0 \ \ to \ \ convergence \ \ do:$$

$$- \ compute$$

$$\omega^k = \frac{\vec{r}^{\,k} \circ A\vec{q}^{\,k}}{A\vec{q}^{\,k} \circ A\vec{q}^{\,k}}$$

$$- \ set \ \ \vec{x}^{\,k+1} = \vec{x}^{\,k} + \omega^k \vec{q}^{\,k}, \ \vec{r}^{\,k+1} = \vec{r}^{\,k} - \omega^k A\vec{q}^{\,k}$$

$$- \, generate \; numbers$$

$$\alpha_i^k = -\frac{A\vec{r}^{\,k+1} \circ A\vec{q}^{\,i}}{A\vec{q}^{\,i} \circ A\vec{q}^{\,i}}, \quad i = 0 \ldots, k$$

$$- \ set \ \ \vec{q}^{\,k+1} = \vec{r}^{\,k+1} + \sum_{i=0}^{k} \alpha_i^k \vec{q}^{\,i}$$

$$- \ if \ \ k+1 = N_{cycle}, \ set \ \ \vec{x}^{\,0} = \vec{x}^{\,N_{cycle}}, \ \vec{r}^{\,0} = \vec{r}^{\,N_{cycle}}$$

$$and \ go \, to \ \ (*)$$

a)
Give a short definition of the symbols involved. No derivation required.

b)

Given a linear system of equations

$$\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad (*)$$

Use ORTHOMIN with initial estimate $x = 0$, $y = 0$ to compute the estimate after one iteration.

APPENDIX

Let J denote the complex unit, $J = \sqrt{-1}$, a, b and ϕ real numbers.

$$| a + Jb |^2 = a^2 + b^2$$
$$e^{J\phi} = \cos\phi + J\sin\phi$$
$$e^{J\phi} + e^{-J\phi} = 2\cos\phi$$
$$e^{J\phi} - e^{-J\phi} = 2J\sin\phi$$
$$\cos^2\phi + \sin^2\phi = 1$$
$$\cos(-\phi) = \cos\phi$$
$$\sin(-\phi) = -\sin\phi$$
$$1 - \cos\phi = 2\sin^2(\phi/2)$$

Let A_i denote values of a parameter distributed in a grid with block lengths Δx_i .

Arithmetic mean:

$$\frac{\Delta x_i A_i + \Delta x_{i+1} A_{i+1}}{\Delta x_i + \Delta x_{i+1}} .$$

Harmonic mean:

$$\frac{A_i A_{i+1}(\Delta x_i + \Delta x_{i+1})}{A_{i+1}\Delta x_i + A_i \Delta x_{i+1}} .$$