University of Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

English

SUBJECT: PET 660 Reservoir simulation

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Solution.

Problem 1

a)
Taylor expansions.

$$u(x-\Delta x) = u(x) - u'(x)\Delta x + \frac{1}{2}u''(x)\Delta x^2 - \frac{1}{3!}u'''(\xi_1)\Delta x^3 \qquad (1)$$

$$u(x-2\Delta x) = u(x) - 2u'(x)\Delta x + 2u''(x)\Delta x^2 - \frac{8}{3!}u'''(\xi_2)\Delta x^3 \qquad (2)$$

Combine equations (2) - 4*(1).

$$u(x-2\Delta x) - 4u(x-\Delta x) = -3u(x) + 2u'(x)\Delta x + R\Delta x^3,$$

i.e.

$$u_x = \frac{u_{i-2} - 4u_{i-1} + 3u_i}{2\Delta x} + O(\Delta x^2)$$

b)
The error function

$$\xi^n e^{iJ\theta}, \quad J = \sqrt{-1}, \ \theta \ arbitrary,$$

inserted in the difference equations

$$\gamma\xi^n e^{J(i-1)\phi} - \gamma\xi^n e^{Ji\phi} = \xi^{n+1}e^{Ji\phi} - \xi^n e^{Ji\phi}, \qquad \gamma = \frac{\Delta t}{\Delta x^2} \quad .$$

Division by $\xi^n e^{iJ\theta}$

$$\gamma(e^{J\phi} - 1) = \xi - 1$$

$$\xi = 1 - \gamma(1-\cos(\phi)) + J\gamma\sin(\phi)$$

Stability if $|\xi|^2 \le 1$.

$$|\xi|^2 = 1 - 2\gamma(1-\cos(\phi)) + \gamma^2(1-2\cos(\phi)+\cos^2(\phi)) + \gamma^2\sin^2(\phi)$$
$$= 1 - 2\gamma(1-\cos(\phi)) + 2\gamma^2(1-\cos(\phi)) = 1 + (1-\cos(\phi))(\gamma-1) = 1 + 2\sin^2(\phi/2)(\gamma-1)$$

Requirement for stability: $\gamma \le 1$

Problem 2

a)
Main differences in fluid input data.
For Black Oil model fluid data are specified using tables for viscosities, volume factors, solution ratio and reference densities.
For compositional computations an equation of state is used to compute fluid properties. Hence, parameters like critical temperatures and pressures for each component are needed.

b)
Fluid densities and a reference pressure are used to compute equilibrium pressure distribution.
To compute initial saturations the position of fluid contacts (WOC, GOC) must be specified.
In addition critical water saturation and capillary pressures are used.

Problem 3

Explicit unknowns in transmissibility terms are evaluated at step n.
Corresponding matrix elements are set to 0.

Structure of linear equations:

x x x x x x x x x	x x x x x x		x x x x x x		
x x x x x x x x x	x x x x x x x x x	x x x		x x x	
	x x x x x x	x x x x x x x x x			x x x
x x x x x x x x x			x x x x x x x x x	x x x	
	x x x x x x		x x x x x x	x x x x x x x x x	x x x
		x x x		x x x	x x x x x x x x x

Gauss – Jordan reduction:

x x x	x x x x x x		x x x x x x		
x x x x x x x x x	x x x	x x x		x x x	
	x x x x x x	x x x			x x x
x x x x x x x x x			x x x	x x x	
	x x x x x x		x x x x x x	x x x	x x x
		x x x		x x x	x x x

Reordering, explicit unknown first.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
x								**x**	**x**	**x**			**x**			**x**	
	x										**x**	**x**					**x**
		x									**x**	**x**					**x**
			x					**x**	**x**	**x**						**x**	
				x							**x**	**x**		**x**	**x**		**x**
					x						**x**	**x**		**x**	**x**		**x**
						x							**x**			**x**	
							x						**x**			**x**	
								x			**x**	**x**		**x**	**x**		
									x		**x**	**x**		**x**	**x**		
										x	**x**	**x**		**x**	**x**		
								x	**x**	**x**	**x**		**x**			**x**	
								x	**x**	**x**		**x**	**x**			**x**	
											x	**x**	**x**				**x**
								x	**x**	**x**				**x**		**x**	
								x	**x**	**x**					**x**	**x**	
											x	**x**		**x**	**x**	**x**	**x**
													x			**x**	**x**

Solution:

D	**A**
	B

$$D\vec{x}_1 + A\vec{x}_2 = \vec{r}_1$$
$$B\vec{x}_2 = \vec{r}_2$$

$\vec{x}_1$ explicit unknowns, $\vec{x}_2$ implicit unknowns

- solve $B\vec{x}_2 = \vec{r}_2$
- substitution $\vec{x}_1 = D^{-1}(\vec{r}_1 - A\vec{x}_2)$.

Problem 4

a)

$$T_o = \left(\frac{k_{ro}}{\mu_o B_o}\right)_{i+\frac{1}{2}}$$

$$C = \frac{k_x A}{\Delta x}$$

b)
Block i+1 is under saturated. Unknowns:

$$F(p_i, p_{i+1}, p_{s,i+1}, S_{w,i+1}) = C \frac{k_{ro}(S_{w,i+1})}{\mu_o(p_{i+1}, p_{s,i+1}) B_o(p_{i+1}, p_{s,i+1})}(p_{i+1} - p_i)$$

Linearization:

$$\frac{\partial F^k}{\partial p_i}\Delta p_i^k + \frac{\partial F^k}{\partial p_{i+1}}\Delta p_{i+1}^k + \frac{\partial F^k}{\partial p_{s,i+1}}\Delta p_{s,i+1}^k + \frac{\partial F^k}{\partial S_{w,i+1}}\Delta S_{w,i+1}^k + F^k$$

Problem 5

a)
Example, 2-phase oil/water problem

Time step n -> n+1

Equations:

$$\Delta T_w \Delta \psi_w + q_{w,i} = \frac{V_i}{\Delta t}\Delta_t(\varphi S_w / B_w) = C_i \varphi_i^{n+1} \frac{1}{B_{w,i}^{n+1}} S_{w,i}^{n+1} - A_w$$

$$\Delta T_o \Delta \psi_o + q_{o,i} = \frac{V_i}{\Delta t}\Delta_t(\varphi S_o / B_o) = C_i \varphi_i^{n+1} \frac{1}{B_{o,i}^{n+1}} S_{o,i}^{n+1} - A_o$$

$$C_i = \frac{V_i}{\Delta t}$$

Use $(S_w)_i^{n+1} + (S_o)_i^{n+1} = 1$

$$\Delta T_w \Delta \psi_w + q_{w,i} = C_i \varphi_i^{n+1} \frac{1}{B_{w,i}^{n+1}} S_{w,i}^{n+1} - A_w$$

$$\Delta T_o \Delta \psi_o + q_{o,i} = C_i \varphi_i^{n+1} \frac{1}{B_{o,i}^{n+1}} (1 - S_{w,i}^{n+1}) - A_o$$

Multiply water eq. by $(B_w)_i^{n+1}$ and oil eq. by $(B_o)_i^{n+1}$ and add.

Pressure equation:

$$(B_w)_i^{n+1}\Delta T_w\Delta\psi_w+(B_o)_i^{n+1}\Delta T_o\Delta\psi_o+(B_w)_i^{n+1}q_{w,i}+(B_o)_i^{n+1}q_{o,i}$$
$$=C_i\varphi_i^{n+1}-(B_w)_i^{n+1}A_w-(B_o)_i^{n+1}A_o \qquad (*)$$

where saturation dependent terms in T_l, q_l (rel. perm., cap.. press.) are evaluated explicitly (step n).

b)
Explicit update of saturation:

Water equation with new pressures obtained by solving pressure equation:

$$C_i\varphi_i^{n+1}\frac{1}{B_{w,i}^{n+1}}S_{w,i}^{n+1}-A_w=\Delta T_w\Delta\psi_w+q_{w,i}$$

$$S_{w,i}^{n+1}=\frac{B_{wi}^{n+1}}{\varphi_i^{n+1}C_i}(\Delta T_w\Delta\psi_w+q_{w,i}+A_w)$$

Saturation dependent terms in T_w and q_w are evaluated explicitly (step n).

Problem 6

a)
Discretized mass balance equations

$$\left(\frac{kk_{rl}}{\mu_l}\right)_{i+\frac{1}{2}}\frac{A}{\Delta x}(p_{i+1}-p_i)-\left(\frac{kk_{rl}}{\mu_l}\right)_{i-\frac{1}{2}}\frac{A}{\Delta x}(p_i-p_{i-1})+Q_{l,i}=\frac{A\Delta x}{\Delta t}\Delta_t(\varphi S_l),\quad l=w,o$$

b)
Homogeneous, incompressible reservoir, upstream evaluation, constant viscosity μ:

$$\frac{Ak}{\mu\Delta x}(k_{rl})_{i+1}(p_{i+1}-p_i)-\frac{Ak}{\mu\Delta x}(k_{rl})_i(p_i-p_{i-1})+Q_{l,i}=\frac{A\Delta x\varphi}{\Delta t}\Delta_t(S_l),\quad l=w,o\,.$$

Relative permeabilities straight lines and $S=S_w$. For i = 2,…,N-1

water $\quad S_{i+1}(p_{i+1}-p_i)-S_i(p_i-p_{i-1})+q_{w,i}=C\Delta_t(S)$

oil $\quad (1-S)_{i+1}(p_{i+1}-p_i)-(1-S)_i(p_i-p_{i-1})+q_{o,i}=C\Delta_t(1-S)$

where $C = \dfrac{\Delta x^2 \mu \varphi}{k \Delta t}$ and $q_{l,i} = 0$.

Using closed exterior boundaries:

- for block 1 the term with $(p_i - p_{i-1})$ vanishes
- for block N the term with $(p_{N+1} - p_N)$ vanishes.

Block 1:

water $S_2(p_2 - p_1) + q_{w,1} = C\Delta_t(S)$

oil $(1-S)_2(p_2 - p_1) + q_{o,1} = C\Delta_t(1-S)$

Block N:

water $-S_N(p_N - p_{N-1}) + q_{w,N} = C\Delta_t(S)$

oil $-(1-S)_N(p_N - p_{N-1}) + q_{o,N} = C\Delta_t(1-S)$

Moreover,

$$q_{w,N} = \frac{\Delta x \mu}{Ak} Q,\ q_{o,N} = 0,\ q_{w,1} = -\frac{\Delta x \mu S}{Ak} Q,\ q_{o,1} = -\frac{\Delta x \mu (1-S)}{Ak} Q\,.$$

c)

Adding equations for block N:

$$p_N - p_{N-1} = \frac{\Delta x \mu}{Ak} Q$$

Adding equations for block i, i = 2,…, N-1:

$$p_i - p_{i-1} = p_{i+1} - p_i\,.$$

By recursion i from N-1 to 2:

$$p_{i+1} - p_i = B\ ,\ i = 1,\ldots,N-1\,.$$

$$B = \frac{\Delta x \mu}{Ak} Q$$

d)

Substitution of pressure differences in water equations i, i = 2,…N-1:

$$\frac{B}{C}(S_{i+1}^n - S_i^n) = \Delta_t S$$

Now $\frac{B}{C} = \frac{Q\Delta t}{A\varphi\Delta x}$ and

$$D\frac{\Delta t}{\Delta x}(S_{i+1}^n - S_i^n) = \Delta_t S\text{, } i = 2,\ldots,N\text{-}1\,. \qquad (*)$$

with $D = \frac{Q}{A\varphi}$.

Equation 1:

$$\frac{B}{C}S_2 - \frac{\Delta x \mu S}{CAk}Q = \Delta_t(S)$$

$$D\frac{\Delta t}{\Delta x}S_2 - DQ\frac{\Delta t}{\Delta x}S_1 = \Delta_t(S)$$

Equation *N*:

$$-\frac{B}{C}S_N + \frac{\Delta x \mu}{CAk}Q = \Delta_t(S)$$

$$D\frac{\Delta t}{\Delta x}S_N + DQ\frac{\Delta t}{\Delta x} = \Delta_t(S)$$

e)

Maximum time step length if $D\frac{\Delta t}{\Delta x} = 1$.

Equations for boundary block 1 and 5 are similar to equations (*) using no flow exterior boundaries and adding well terms.

Equation *N*: $-S_N^n + K_N = \Delta_t(S)$

where $K_N = \frac{Q\Delta t}{\varphi A \Delta x} = D\frac{\Delta t}{\Delta x} = 1$.

Hence

$$S_N^{n+1} = 1$$

Equation 1:

$$S_2^n + K_1 = \Delta_t(S)$$

where $K_1 = -\frac{Q\Delta t}{\varphi A\Delta x}S_1^n = -S_1^n$.

Hence

$$S_1^{n+1} = S_2^n$$

From (*) : $S_i^{n+1} = S_{i+1}^n, i = 2,\ldots N-1$.

e)
N = 5 and explicit method with maximal stable time step length.

Equations

$$S_5^{n+1} = 1$$
$$S_4^{n+1} = S_5^n$$
$$S_3^{n+1} = S_4^n$$
$$S_2^{n+1} = S_3^n$$
$$S_1^{n+1} = S_2^n$$

step \ block	1	2	3	4	5
0	0	0	0	0	0
1	0	0	0	0	1
2	0	0	0	1	1
3	0	0	1	1	1
4	0	1	1	1	1
5	1	1	1	1	1

Problem 7

a)
Algorithm for ORTHOMIN with restart:

$\vec{x}^k$, solution at step k
$\vec{q}^k$, directional vector at step k

$\vec{x}^0$, initial estimate

$\vec{r}^k = \vec{b} - A\vec{x}^k$, residual at step k

° , inner product (dot product, scalar product) of vectors

N_{cycle}, number of iterations before restart (ECLIPSE NSTACK input parameter).

b)

Solution:

$$\vec{r}^{\,0} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\vec{q}^{\,0} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$A\vec{q}^{\,0} = \begin{pmatrix} 7 \\ -1 \end{pmatrix},$$

$$\omega^0 = 0.4$$

$$\vec{x}^1 = \vec{x}^{\,0} + \omega^0 \vec{q}^{\,0} = \begin{pmatrix} 1.2 \\ 0.4 \end{pmatrix}$$